Determinants

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Determinants

- Ascalerquantityobtained by expanding the elements of a square matrix with respect to the elements of any row or column is called a determinant of that matrix.
- Consider the following square matrix anditsdeterminant.
- $[a_{11}a_{12}]$ $|a_{11}a_{12}|$
- $[a_{21}a_{22}]$ $|a_{21}a_{22}|$ $|A| = a_{11}a_{22} - a_{12}a_{21}$

Evaluation of a determinant

- The value of the determinant is obtained by expanding it with the help of the elements of any row or column.
- Multiply each elementofthe first row (or column) by the determinant obtained after removing the row and column to which it belongs.Thesigns of the products are taken alternatively positive and negative.

Illustrative Example

- Evaluate the following determinant

$$\begin{vmatrix} -15 & -2 \\ 2 & 10 \\ 2 & 7 & 3 \end{vmatrix}$$

- $|A| = -1\begin{vmatrix} 10 \\ 7 & 3 \end{vmatrix} - 5\begin{vmatrix} 2 & 0 \\ 2 & 3 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 2 & 7 \end{vmatrix}$
- $|A| = -1(1\times3-0\times7)-5(2\times3-0\times2)-2(2\times7-2\times1)$
- $=-1(3) -5 (6) -2 (12)$
- $=-3- 30 -24$
- $=-57$

Solving linear equations

- Solve the linear equations
- $2x + 3y = 5$
- $3x - 2y = 1$
- $D = \begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix} = 2(-2)-3(3) = -4-9=-13$
- $D_1 = \begin{vmatrix} 5 & 3 \\ 1 & -2 \end{vmatrix} = -13$ and $D_2 = -13$
- $X = D_1/D = 1$, $y = D_2/D = 1$
- This is called Cramer's rule

Problem to solve

- Solve the following system of equations by means of determinants
- $x+y+z-7=0$
- $x+2y+3z-16=0$
- $x+3y+4z-23=0$
- Calculate $D = -1, D_1=0, D_2=-5, D_3=-2$
- $x = D_1/D$
- $y = D_2/D$
- $z = D_3/D$

Problemonlinear equations

- An amount of Rs. 5000 is put into three investments attheratesofinterest of 6%, 7% and 8% per annum respectively. The total annual income is Rs. 358. If the combined income from the first two investment is Rs. 70 morethantheincomefromthe third, find the amount of each investment by using determinants.

Problem to solve

- A company produces three products everyday. Their total production on a certain day is 45 tons. It is found that the production of third product exceeds the production of first product by 8 tons while the total production of first and third product is twice the production of second product. Determine the production level of each product using Crammer's rule.

Problem for Practice

- One unit of food 1 contains 100 units of vitamins, 60 units of minerals and 80 calories.One unit of food2contains150units of vitamins, 60 units of minerals and180calories.One unit of food3contains90units of vitamins,40units of minerals and100calories. Diet requirement for a patient is 1100 unitsof vitamins,500units of minerals and1200 calories. Find out by determinant method how many units of each food should bemixed to form the diet which would meet the requirement exactly.

Rememberplanesuse matrix of longitude and latitude to fly and to reach the specified destination