Differential Calculus

An important mathematical concept that has tremendous applications in business matters.

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Derivative

Derivative of f(x) is defined as Lim f(x+h)-f(a)/h and is shown as dy/dx

Physical meaning: The derivative can be considered as the rate of change. It is related to a function just as velocity is related to the distance travelled by a moving particle.

Geometric meaning: The derivative of a function at a point is the slope of the tangent to the curve y=f(x) at the point {a, f(a)}.

Some Rules

$d/dx(C) = 0$ (The first derivative of a constant is zero)

$d/dx(x^n) = nx^{n-1}$ (Take n out and decrease the index by one)

$d/dx\{\log f(x)\} = 1/f(x)*f(x)$ (Take inverse, multiply by first derivative)

$d/dx\{e^{f(x)}\} = e^{f(x)} * f'(x)$ (The function multiplied by first derivative)

Differentiate the following expressions

$X + 8$

$X^2 + X + 24$

$\sqrt{x} - 20$

Equilibrium Point

Suppose two graphs are plotted: one showing the demand function and another showing the supply function. The point at which these two curves intersect is known as equilibrium point. The price and quantity at equilibrium point are known as equilibrium price and equilibrium quantity respectively. Mathematically speaking equilibrium point can be achieved by equating demand and supply functions.

Illustrative Example

Find the equilibrium point if the demand function is $p = -8x/100 + 12$ and the supply function is $p = 2x/100 + 2$.

At equilibrium point demand and supply equations intersect. Hence, we can equate these two equations.

$-8x/100 + 12 = 2x/100 + 2$

$-10x/100 = -10$

Hence $x = 100$

Putting this value in the equation $p = 2x/100 + 2$ we get the value of equilibrium price p as 4.

Problem to Solve

Find the equilibrium point from the following supply and demand equations.

1. $p = 2x/35 + 5$, $p = -3x + 10$ respectively
2. $x = 20 - 3p - p^2$, $x = p - 1$ respectively

Hint: Equate two equations and solve to find values of x and p. In the second case solve the equation to get p first and then obtain the value of x by substituting the value of p.

Marginal and Average Cost

- Marginal Cost = d C (x)/dx (First derivative of total cost C (x))
- Average Cost = C (x)/x (total cots divided by number of units)
- d/dx (AC) = 1/x {MC – AC}
- When AC is a increasing function then d/dx (AC)>0
- I/x (MC-AC)> 0 Hence MC>AC
- When AC is a decreasing function then MC < AC.

Illustrative Example

The manufacturer's total cost function is given as $C(x) = 2X^2 + 3x + 1000$

- Find Average cost function, marginal cost function and marginal cost when 5 units are produced.
- Average cost $C(x)/x = 2x + 3 + 1000/x$
- Marginal cost $d/dx\,[C(x)]/dx = 4x + 3$
- Marginal cost when 6 units are produced is
- $4(5) + 3 = 23$

Revenue Function

- Revenue of a company depends on its sale. Thus R (x) = p x
- Where p is the price per unit and x is the number of units sold.
- If there is a competition among industries to sell the same product then the value of p remains constant. Hence dR/dx = p
- that is MR = p
- If the company has monopoly then dR/dx = p + x.dp/dx
- that is MR = p + x.dp/dx

Illustrative Example

- The monopolist has a demand function $x = 5 - 2p$ find Total revenue, average revenue and marginal revenue.
- Revenue = $px = (5-x)x/2$
- Average Revenue = Total/x = $(5 - x)x/2x = 5/2 - x/2$
- Marginal Revenue = $d(TR)/dx = d/dx\,(5-x)x/2 = 5/2 - x$

Maxima and Minima

At maximum or at minimum point the first derivative of a function is zero.

This property can be used to find out the maxima and minima of a given function. Steps involved are as follows:

Step 1: Put $y = f(x)$

Step 2: Find dy/dx

Step 3: Equate dy/dx to zero and solve the equation.

Step 4: Get the second derivative and substitute these values, note the sign of the value. If the value is positive then it is the point of minima, if the value is negative then it is the point of maxima.

Illustrative Example

Find the maxima and minima for the following function.

$f(x) = 41 + 24x – 18\ x^2$

We have $f(x) = 41 + 24x - 18\ x^2$

$F'\ (x) = 0 + 24 – 18 * 2 * x$

$= 24\ -36x$

Equating this equation to zero we get

$24-36x = 0$, Hence $x = 2/3$

Now $f''(x) = -36$ which is negative. Hence $x = 2/3$ is a point of maxima.

Now Find the maxima and minima of the function $f(x) = 2/x – 2/x^2$

Illustrative Example

A company charges Rs. 550 for a transistor set on orders of 50 or less sets. The charge is reduced by Rs. 5 per set for each set ordered in excess of 50. Find the largest size order company should allow so as to receive a maximum revenue.

Translating the verbal information into symbolic mode we get

$R = (50 + x)(550 - 5x)$

$= 27500 + 300x - 5x^2$

$dR/dx = 300 - 10x$

Equating dR/dx to zero we get $x = 30$.

Thus the maximum revenue is obtained when $(50+30) = 80$ sets are ordered.

Problem to Solve

Tour operator charges Rs. 136 per passenger for 100 passengers with a refund of Rs. 4 for each 10 passengers in excess of 100. Determine the number of passengers that will maximize the amount of money the tour operator receives.

Hint: Prepare the equation, get its first derivative, equate it with zero and solve the equation to get the value of x. Get the second derivative to confirm that it was the point of maxima.