Integral Calculus

Like differentiation integration is yet another important mathematical concept that is useful in business matters.

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Integration

Integration for all practical purposes is the reverse of differentiation.

Here are some fundamental formulae of integration.

$\int x^n dx = x^{n+1}/n+1 + C$

$\int 1/x dx = \log |x| + C$

$\int e^x dx = e^x + C$

$\int a^x dx = a^x/\log a + C$

Note that every time the constant C is added. It is because the derivative of a constant is zero.

Some Rules

$\int x^5 dx = x^{5+1}/5+1 + C = x^6/6 + C$

$\int \sqrt{x} dx = \int x^{1/2} dx$

$= x^{1/2 +1}/1/2+1 + C$

$=2/3\ x^{3/2} + C$

$\int 1/x^3 dx = \int x^{-3} dx$

$= x^{-3+1}/-3+ 1 + C$

$=-1/2\ x^2 + C$

Illustrative Example

The marginal cost function of a product is given by MC = $50 - 2x + 3x^2$. Find the total cost function and average cost function if cost of producing 5 units is 360.

Given MC =$dC/dx = 50 - 2x + 3x^2$

$dC = (50 - 2x + 3x^2)dx$

$\int dC = \int (50 - 2x + 3x^2)dx$

$C = 50x - x^2 + x^3 + k$

Substituting $x = 5$ we get $C = 350 + k$

Equating this to 360, we get $k = 10$

Hence $C = 50x - x^2 + x^3 + 10$ and

Average cost $C/x = 50 - x + x^2 + 10/x$

Problems to Solve

If the marginal cost of a product is equal to half of its average cost. Show that the fixed cost is zero. If the cost of producing 9 units of the product is Rs. 60, find the cost function.

The marginal cost function of manufacturing x pairs of shoes is $6+10x-6x^2$. The total cost of producing a pair of shoes is Rs. 12. Find the total and average cost functions.

Profit

- The marginal cost C'(x) and the marginal revenue R'(x) are given by
- C'(x) = 20 +x/20 and R'(x) = 30. If the fixed cost is 200 determine the maximum profit and number of items produced for this profit.
- Let P denote the profit function
- Hence dP/dx = dR/dx – dC/dx = 10 -x/10
- dP = (10 -x/20) dx, Integrating both sides we get

$P = 10x - X^2/40 + k$ Given fixed cost as 200 that means P is 200 when x is zero.

Substituting these values we get k = -200 Thus the equation becomes

$P = 10x - X^2/40 - 200$ For P to be maximum dP/dx should be zero.

- Hence x-x/20= 0 Therefore x = 200, substituting we get P = 800

Optimum Time

- The revenue and cost rate of a firm are given as
- $R'(t) = 8\text{-}t1/2$ and $C'(t) = 2 + 2t^{1/2}$ where t is measured in years and R and C are measured in lakhs of rupees. Determine how long the operation should continue and the profit that can be generated during this period.
- By equating $R'(t) = C'(t)$ we get $t = 4$ that is four years
- By integrating $p(t) = R'(t) – C'(t)$ we get profit during this period as 18 means Rupees eighteen lakhs.

Consumer Producer Surplus

If we refer only to demand curve along with the equilibrium point, we will find that there are some consumers who can pay a price higher than the equilibrium price and thus consumers have been benefited because of equilibrium points . The benefit received by them is known as consumer's surplus.

There are certain producers who are willing to sell their product even at a price lower than the equilibrium price. Due to equilibrium price they are benefited. This is known as producer's surplus.

Illustrative Example

The demand functions for a commodity is given by p = 75 -3x, Find the consumers surplus corresponding to the equilibrium price = 15.

Given $p_0 = 15$

$X_0 = 75 - p_0/3 = 75 - 15/3 = 20$

$CS = \int D(x).dx - x_0 p_0$

$= \int (75 - 3x)dx - (20*15) = 75x - 3/2\, x^2 - 300$

$= 75(20) - 3/2\,(20)^2 - 300 = 1500 - 600 - 300 = 600$

Hence consumer surplus is 600.

Problem to Solve

When the price of a given commodity averaged Rs. 150, the company sold 10 units. When the price dropped to an average of Rs. 80, 20 units were sold by the same company. It has also been observed when the price was Rs. 80, 20 units were available for sale. When the price reached Rs. 60, 15 units were available for sale. Find the demand and supply functions, assuming both are linear. Also determine consumer's and producer's surplus.

Integration leads to beauty