Applicable Mathematics
"Probability"

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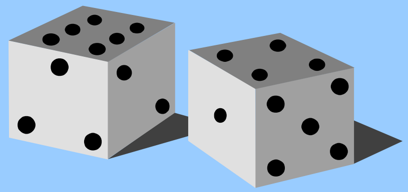


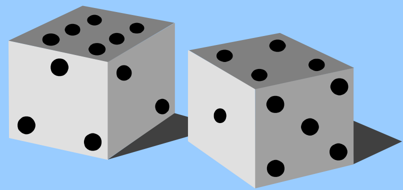
Definitions

Probability is the mathematics of chance.

It tells us the relative frequency with which we can expect an event to occur

The greater the probability the more likely the event will occur.

It can be written as a fraction, decimal, percent, or ratio.

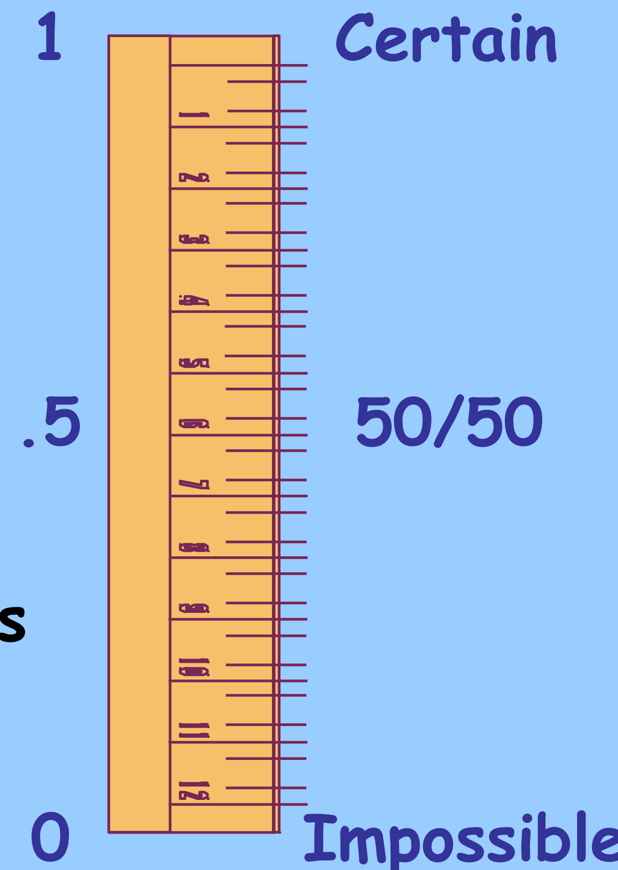


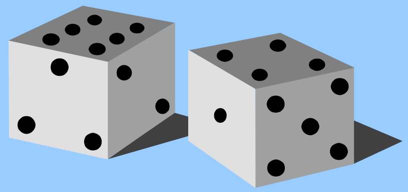
Definitions

Probability is the numerical measure of the likelihood that the event will occur.

Value is between 0 and 1.

Sum of the probabilities of all events is 1.



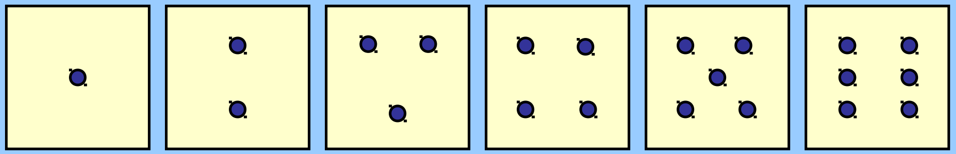


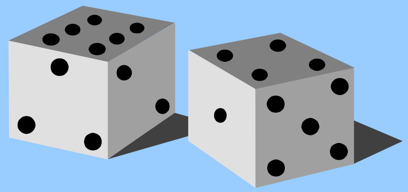
Definitions

A probability experiment is an action through which specific results (counts, measurements, or responses) are obtained.

The result of a single trial in a probability experiment is an outcome.

The set of all possible outcomes of a probability experiment is the sample space, denoted as S.
e.g. All 6 faces of a die: S = { 1 , 2 , 3 , 4 , 5 , 6 }

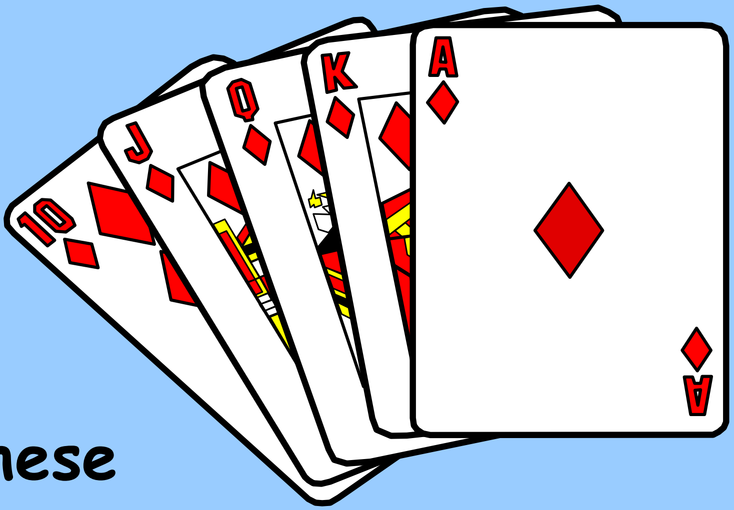




Definitions

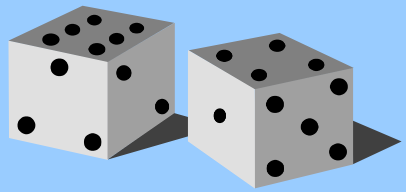
Other Examples of Sample Spaces may include:

Lists
Tables
Grids
Venn Diagrams
Tree Diagrams



May use a combination of these

Where appropriate always display your sample space

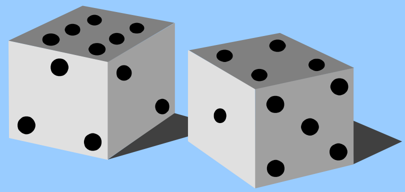


Definitions

An event consists of one or more outcomes and is a subset of the sample space.

Events are often represented by uppercase letters, such as A, B, or C.

Notation: The probability that event E will occur is written P(E) and is read
"the probability of event E."



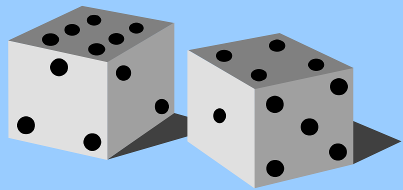
Definitions

- The Probability of an Event, E:

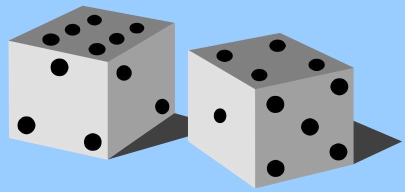
$$P(E) = \frac{\text{Number of Event Outcomes}}{\text{Total Number of Possible Outcomes in S}}$$

Consider a pair of Dice

- Each of the Outcomes in the Sample Space are random and equally likely to occur.

e.g. P() = $\frac{2}{36} = \frac{1}{18}$

(There are 2 ways to get one 6 and the other 4)



Definitions

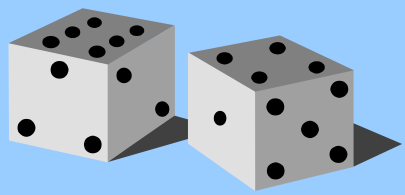
There are three types of probability

1. Theoretical Probability

Theoretical probability is used when each outcome in a sample space is equally likely to occur.

$$P(E) = \frac{\text{Number of Event Outcomes}}{\text{Total Number of Possible Outcomes in S}}$$

The Ultimate probability formula ☺



Definitions

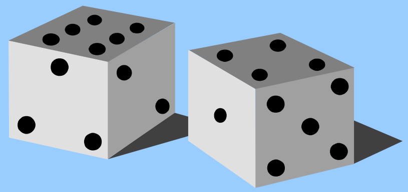
There are three types of probability

2. Experimental Probability

Experimental probability is based upon observations obtained from probability experiments.

$$P(E) = \frac{\text{Number of Event Occurrences}}{\text{Total Number of Observations}}$$

The experimental probability of an event E is the relative frequency of event E



Definitions

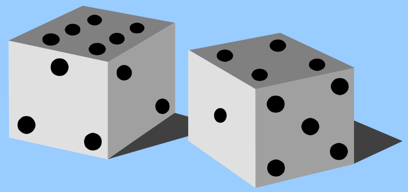
There are three types of probability

3. Subjective Probability

Subjective probability is a probability measure resulting from intuition, educated guesses, and estimates.

Therefore, there is no formula to calculate it.

Usually found by consulting an expert.

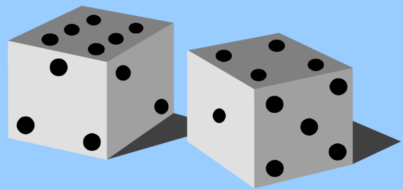


Definitions

Law of Large Numbers.

As an experiment is repeated over and over, the experimental probability of an event approaches the theoretical probability of the event.

The greater the number of trials the more likely the experimental probability of an event will equal its theoretical probability.



Complimentary Even

The complement of event E is the set of all outcomes in a sample space that are not included in event E.

The complement of event E is denoted by $E' \ or \ \overline{E}$

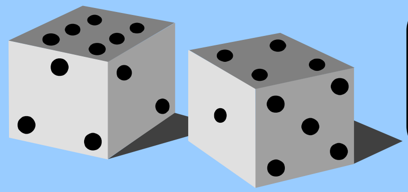
Properties of Probability:

$$0 \le P(E) \le 1$$

$$P(E) + P(\overline{E}) = 1$$

$$P(E) = 1 - P(\overline{E})$$

$$P(\overline{E}) = 1 - P(E)$$



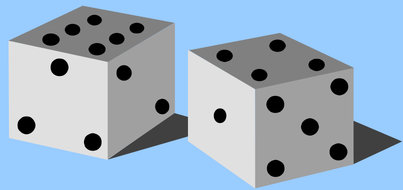
The Multiplication R

If events A and B are independent, then the probability of two events, A and B occurring in a sequence (or simultaneously) is:

$$P(A\,and\,B) = P(A \cap B) = P(A) \times P(B)$$

This rule can extend to any number of independent events.

Two events are independent if the occurrence of the first event does not affect the probability of the occurrence of the second event. More on this later

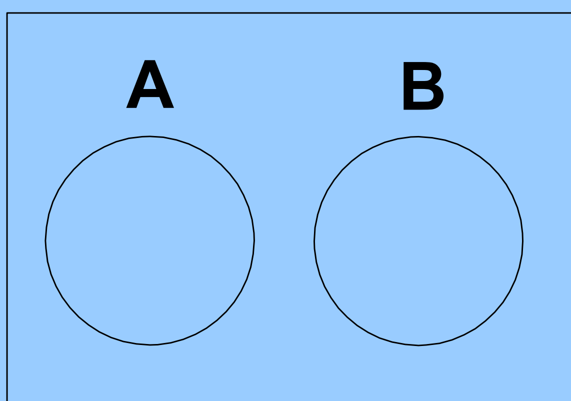


Mutually Exclusive

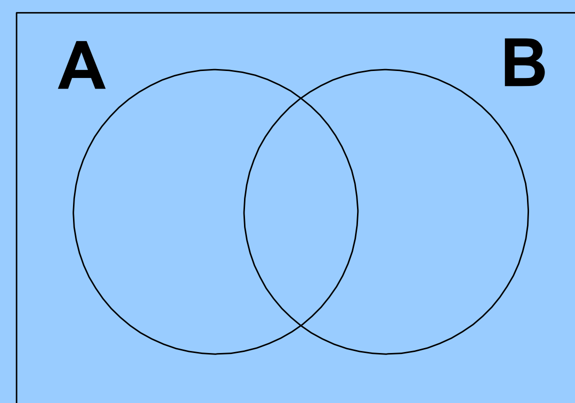
Two events A and B are mutually exclusive if and only if:

$$P(A \cap B) = 0$$

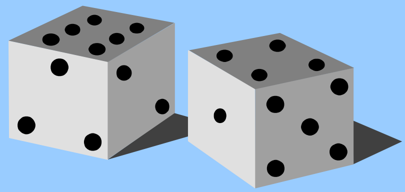
In a Venn diagram this means that event A is disjoint from event B.



A and B are M.E.



A and B are not M.E.



The Addition Rul

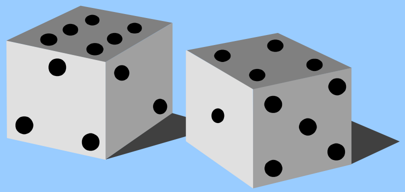
The probability that at least one of the events A or B will occur, P(A or B), is given by:

$$P(A \, or \, B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If events A and B are mutually exclusive, then the addition rule is simplified to:

$$P(A \, or \, B) = P(A \cup B) = P(A) + P(B)$$

This simplified rule can be extended to any number of mutually exclusive events.



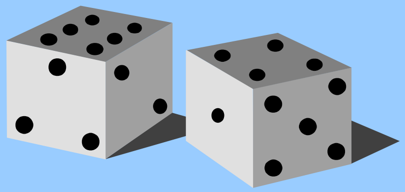
Conditional Probabil

Conditional probability is the probability of an event occurring, given that another event has already occurred.

Conditional probability restricts the sample space.

The conditional probability of event B occurring, given that event A has occurred, is denoted by P(B|A) and is read as "probability of B, given A."

We use conditional probability when two events occurring in sequence are not independent. In other words, the fact that the first event (event A) has occurred affects the probability that the second event (event B) will occur.

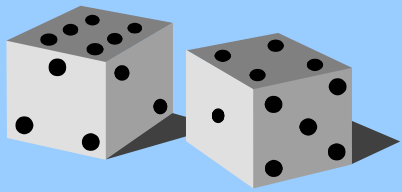


Conditional Probabil

Formula for Conditional Probability

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \quad or \quad P(B \mid A) = \frac{P(B \cap A)}{P(A)}$$

Better off to use your brain and work out conditional probabilities from looking at the sample space, otherwise use the formula.



Conditional Probabil

e.g. There are 2 red and 3 blue counters in a bag and, without looking, we take out one counter and do not replace it.

The probability of a 2nd counter taken from the bag being red depends on whether the 1st was red or blue.

Conditional probability problems can be solved by considering the individual possibilities or by using a table, a Venn diagram, a tree diagram or a formula.

Harder problems are best solved by using a formula together with a tree diagram.

e.g. 1. The following table gives data on the type o
grouped by petrol consumption, owned by 100 people.

	Low	Medium	High	Total
Male	12	33	7	
Female	23	21	4	
				100

One person is selected at random.

L is the event "the person owns a low rated car"

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L is the event "the person owns a low rated car"

F is the event "a female is chosen".

Find (i) $P(L)$ (ii) $P(F \cap L)$ (iii) $P(F|L)$

There is no need for a Venn diagram or a formula to solve this type of problem.

We just need to be careful which row or column we look at.

Solution:

	Low	Medium	High	To
Male	12	33	7	
Female	23	21	4	
	35			100

Find (i) $P(\text{L})$ (ii) $P(\text{F} \cap \text{L})$ (iii) $P(\text{F}|\text{L})$

(i) $P(\text{L}) = \dfrac{\cancel{35}^{\,7}}{\cancel{100}_{\,20}} = \dfrac{7}{20}$ (Best to leave the answers as fractions)

Solution:

	Low	Medium	High	To
Male	12	33	7	
Female	23	21	4	
				100

Find (i) $P(L)$ (ii) $P(F \cap L)$ (iii) $P(F|L)$

(i) $P(L) = \frac{\cancel{35}^{\,7}}{\cancel{100}_{\,20}} = \frac{7}{20}$

(ii) $P(F \cap L) = \frac{23}{100}$

The probability of selecting a female with a low rated car.

Solution:

	Low	Medium	High	To
Male	12	33	7	
Female	23	21	4	
	35			100

Find (i) $P(L)$ (ii) $P(F \cap L)$ (iii) $P(F|L)$

(i) $P(L) = \frac{\not{35}^{\,7}}{\not{100}_{\,20}} = \frac{7}{20}$

(ii) $P(F \cap L) = \frac{23}{100}$

(iii) $P(F|L) = \frac{23}{35}$

The sample space is restricted from 100 to 35.

We must be careful with the denominators in (ii) and (iii). Here we are given the car is low rated. We want the total of that column.

Solution:

	Low	Medium	High	To
Male	12	33	7	
Female	23	21	4	
				100

Find (i) $P(L)$ (ii) $P(F \cap L)$ (iii) $P(F|L)$

(i) $P(L) = \frac{\cancel{35}^{\,7}}{\cancel{100}_{\,20}} = \frac{7}{20}$

(ii) $P(F \cap L) = \frac{23}{100}$

(iii) $P(F|L) = \frac{23}{35}$

Notice that

$P(L) \times P(F|L) = \frac{\cancel{7}^{\,1}}{20} \times \frac{23}{\cancel{35}_{\,5}} = \frac{23}{100}$

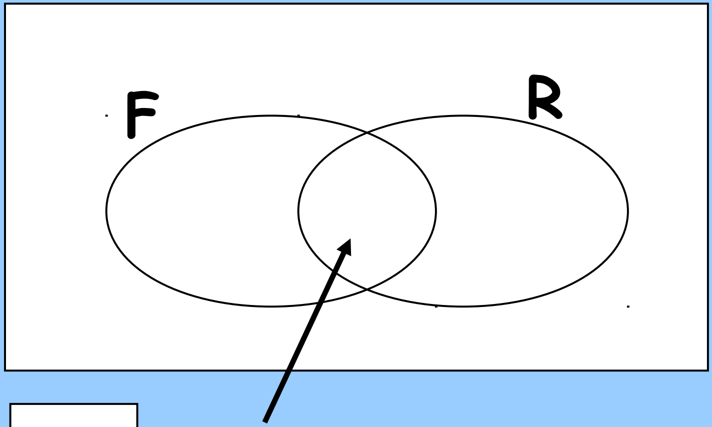
$= P(F \cap L)$

So, $P(F \cap L) = P(F|L) \times P(L)$

e.g. 2. I have 2 packets of seeds. One contains 20 seeds and although they look the same, 8 will give red flowers and 12 blue. The 2nd packet has 25 seeds of which 15 will be red and 10 blue.

Draw a Venn diagram and use it to illustrate the conditional probability formula.

Solution: Let R be the event " Red flower " and
F be the event " First packet "



Red in the 1st packet

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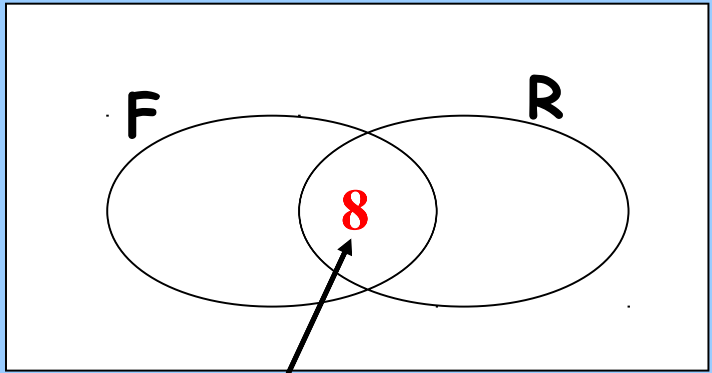

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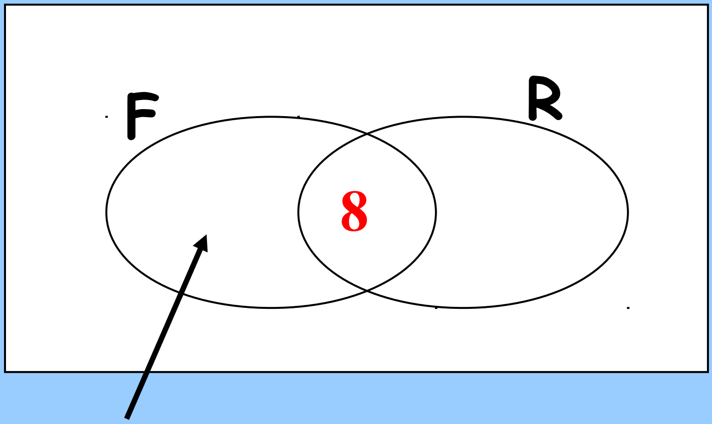

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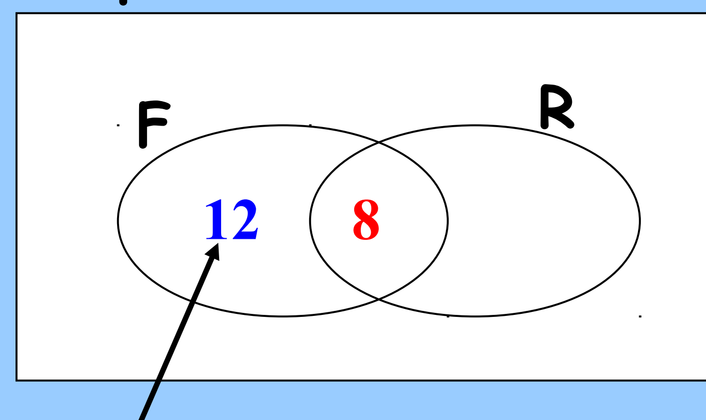

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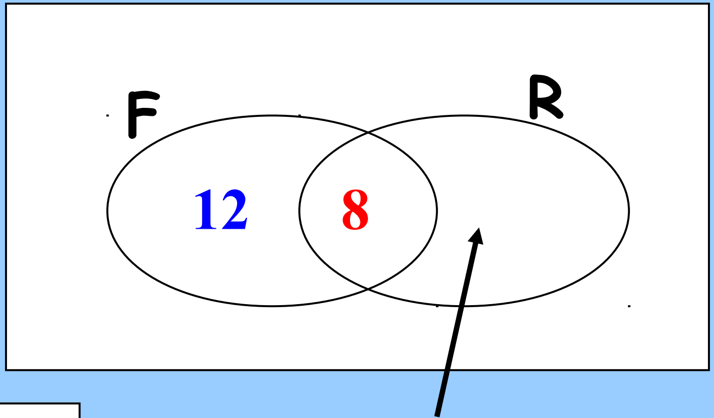

Red in the 2nd packet

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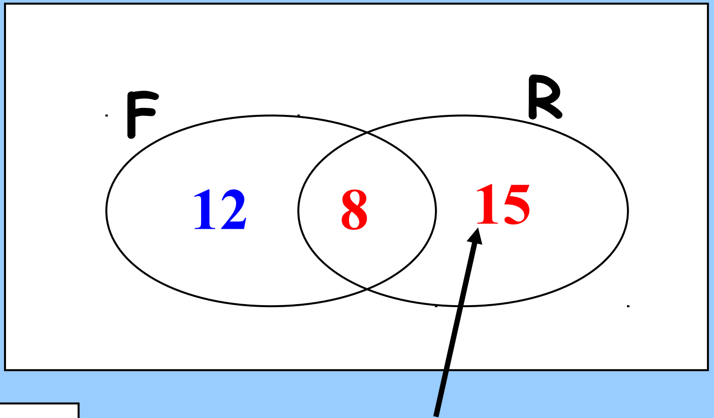

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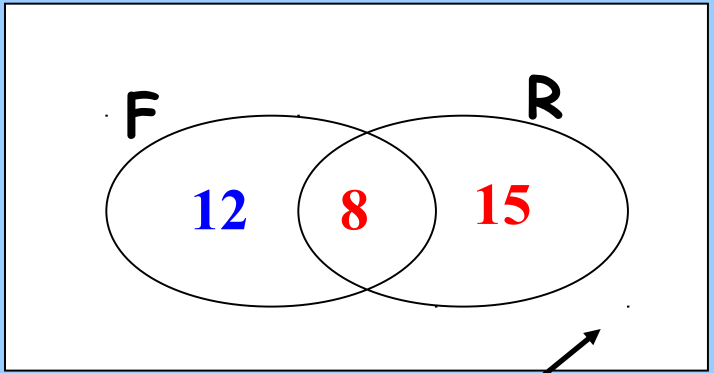

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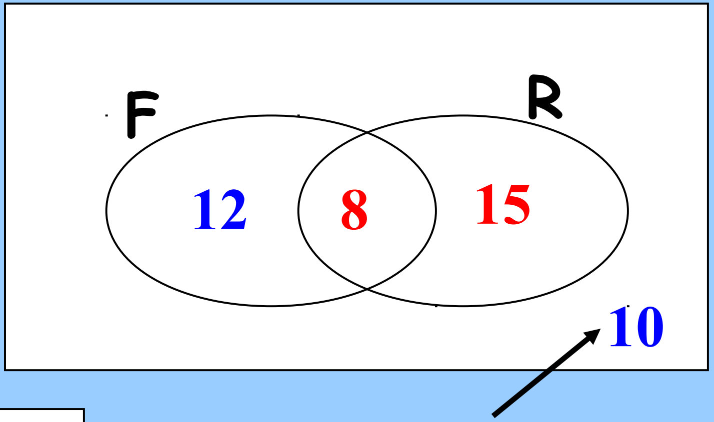

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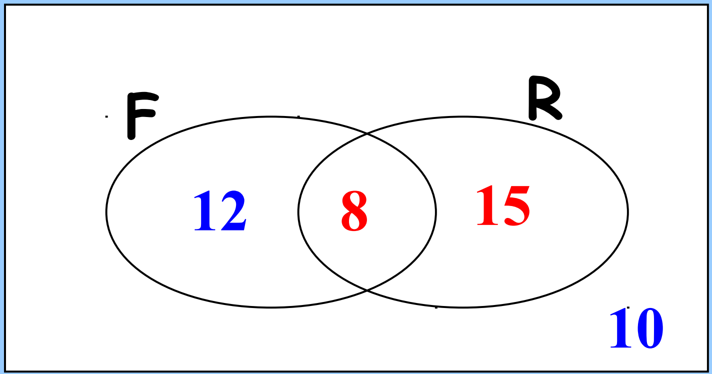

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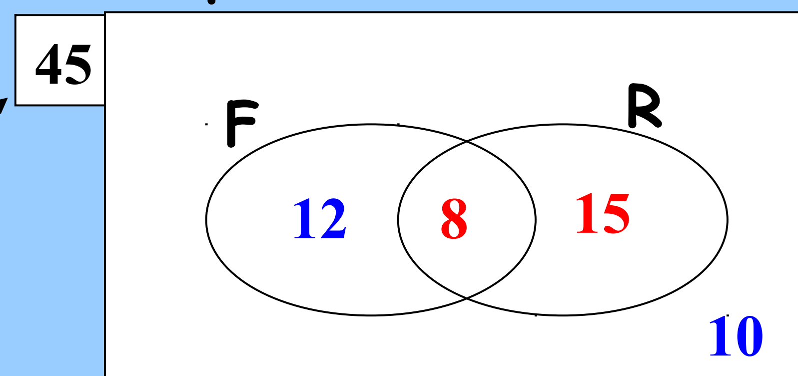

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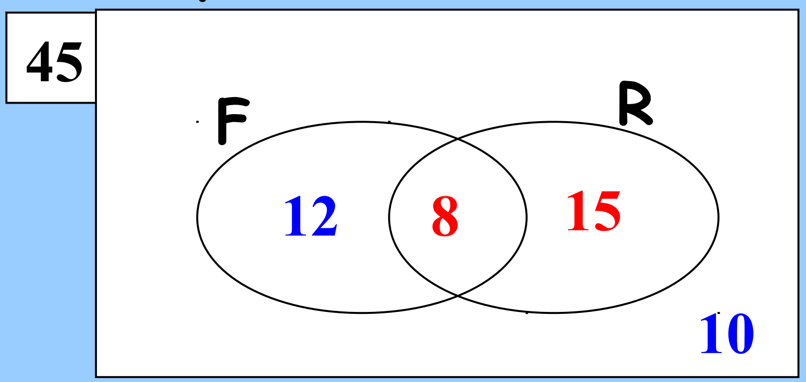

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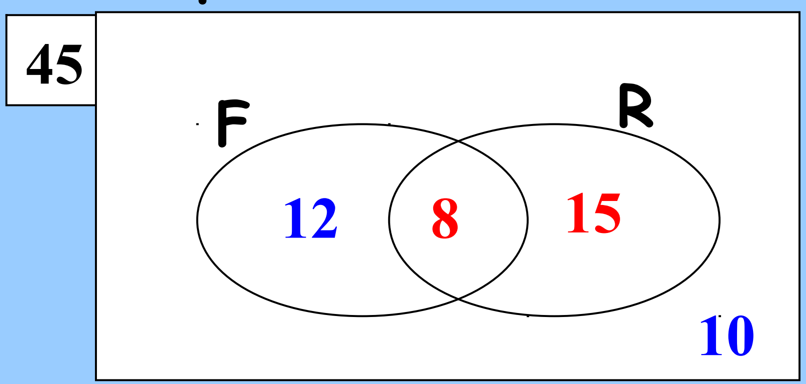
$P(R \cap F) =$



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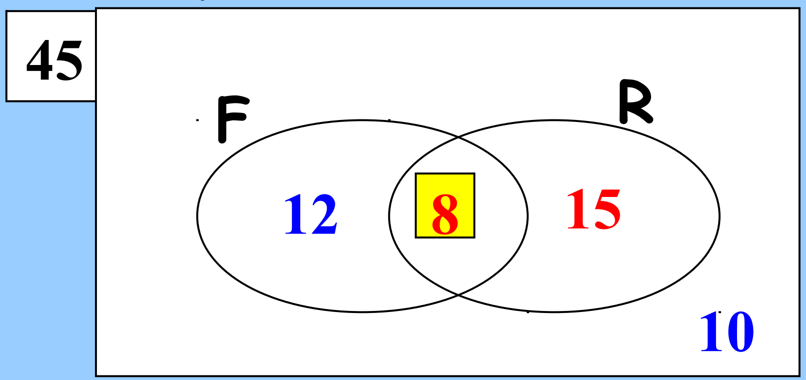
$$P(R \cap F) = \frac{8}{}$$



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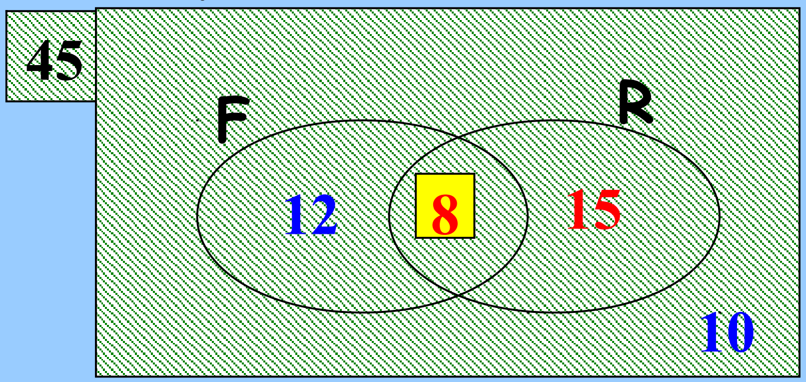
$$P(R \cap F) = \frac{8}{45}$$



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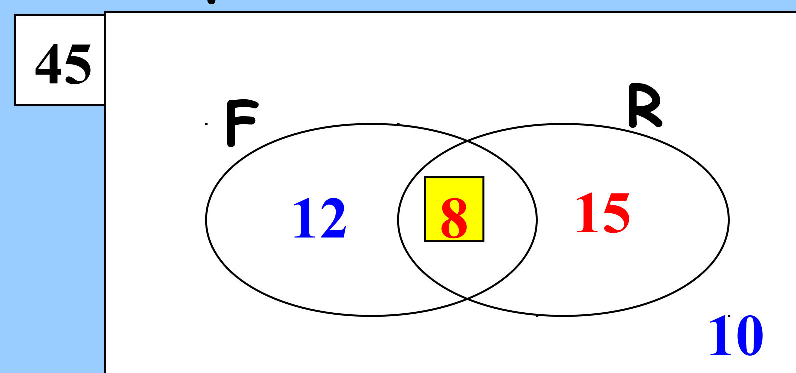
$$P(R \cap F) = \frac{8}{45}$$

$$P(R|F) = \frac{8}{\;}$$



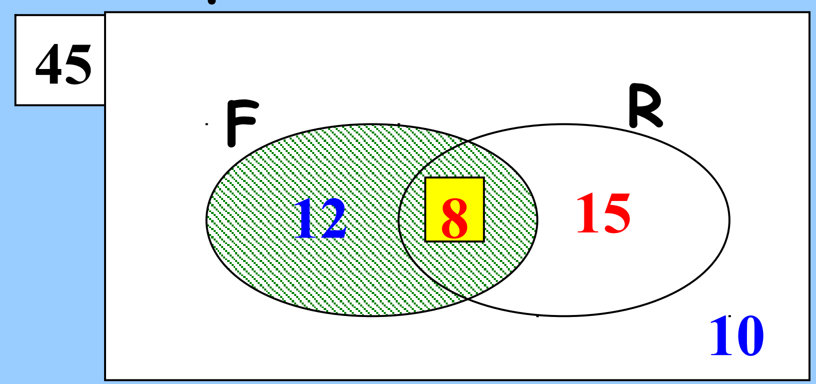
e.g. 2. I have 2 packets of seeds. One contains 20 seeds and although they look the same, 8 will *give* red flowers and 12 blue. The 2nd packet has 25 seeds of which 15 will be red and 10 blue.

Draw a Venn diagram and use it to illustrate the conditional probability formula.

Solution: Let R be the event " Red flower " and
F be the event " First packet "

$$P(R \cap F) = \frac{8}{45}$$

$$P(R|F) = \frac{8}{20} \qquad P(F) =$$



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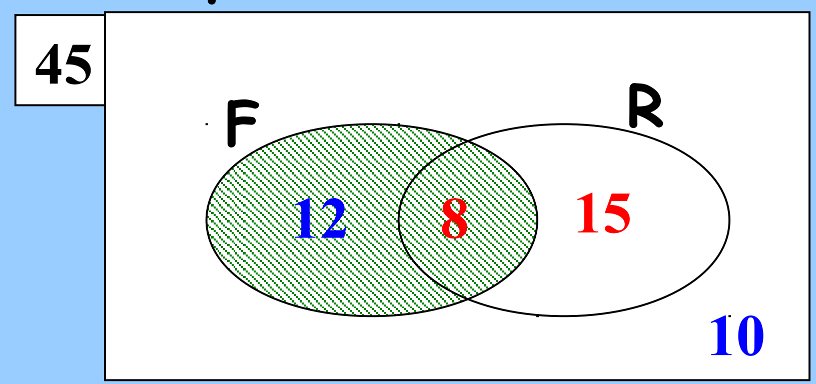

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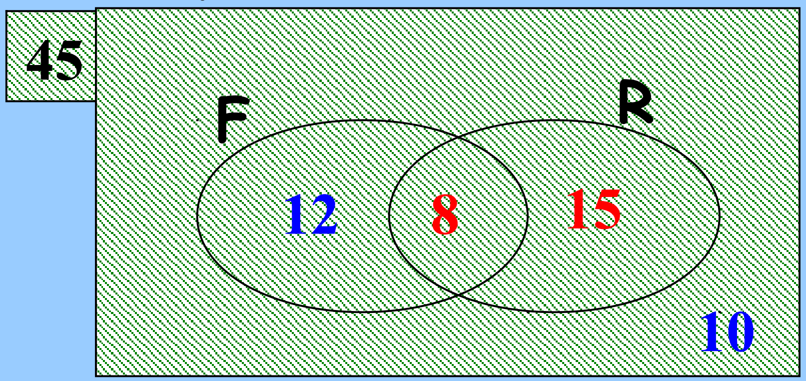

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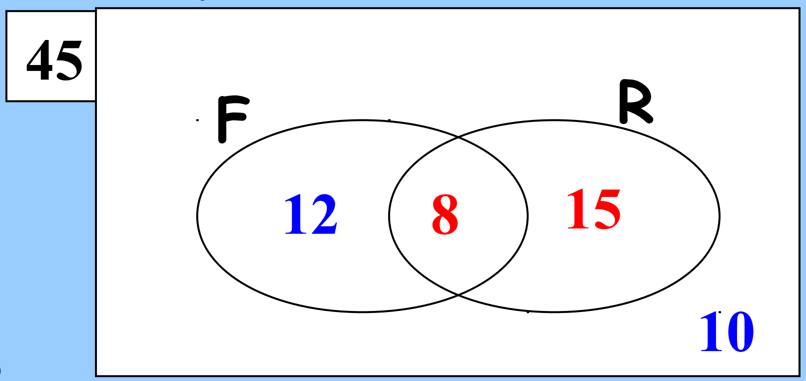
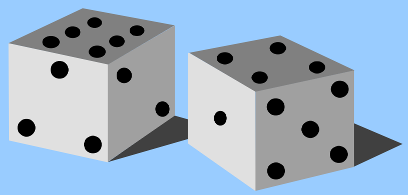


$$P(R \cap F) = \frac{8}{45}$$

$$P(R|F) = \frac{8}{20} \qquad P(F) = \frac{20}{45}$$

$$\Rightarrow P(R|F) \times P(F) = \frac{8}{{}_1\cancel{20}} \times \frac{{}^1\cancel{20}}{45} = \frac{8}{45}$$

So, $P(R \cap F) = P(R|F) \times P(F)$



Probability Tree Diagrams

The probability of a complex event can be found using a probability tree diagram.

1. Draw the appropriate tree diagram.

2. Assign probabilities to each branch. (Each section sums to 1.)

3. Multiply the probabilities along individual branches to find the probability of the outcome at the end of each branch.

4. Add the probabilities of the relevant outcomes, depending on the event.

e.g. 3. In November, the probability of a man getti
to work on time if there is fog on the M6 is $\frac{2}{5}$.

If the visibility is good, the probability is $\frac{9}{10}$.

The probability of fog at the time he travels is $\frac{3}{20}$.

(a) Calculate the probability of him arriving on time.

(b) Calculate the probability that there was fog given that he arrives on time.

There are lots of clues in the question to tell us we are dealing with conditional probability.

e.g. 3. In November, the probability of a man getti
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(a) Calculate the probability of him arriving on time.

(b) Calculate the probability that there was fog given that he arrives on time.

There are lots of clues in the question to tell us we are dealing with conditional probability.

Solution: Let T be the event " getting to work on time "
Let F be the event " fog on the M6 "

Can you write down the notation for the probabilities that we want to find in (a) and (b)?

(a) Calculate the probability of him arriving on time.

(b) Calculate the probability that there was fog given he arrives on time. $P(F|T)$

Can you also write down the notation for the three probabilities given in the question?

" the probability of a man getting to work on time if there is fog is $\frac{2}{5}$ " $P(T|F) = \frac{2}{5}$

Not foggy

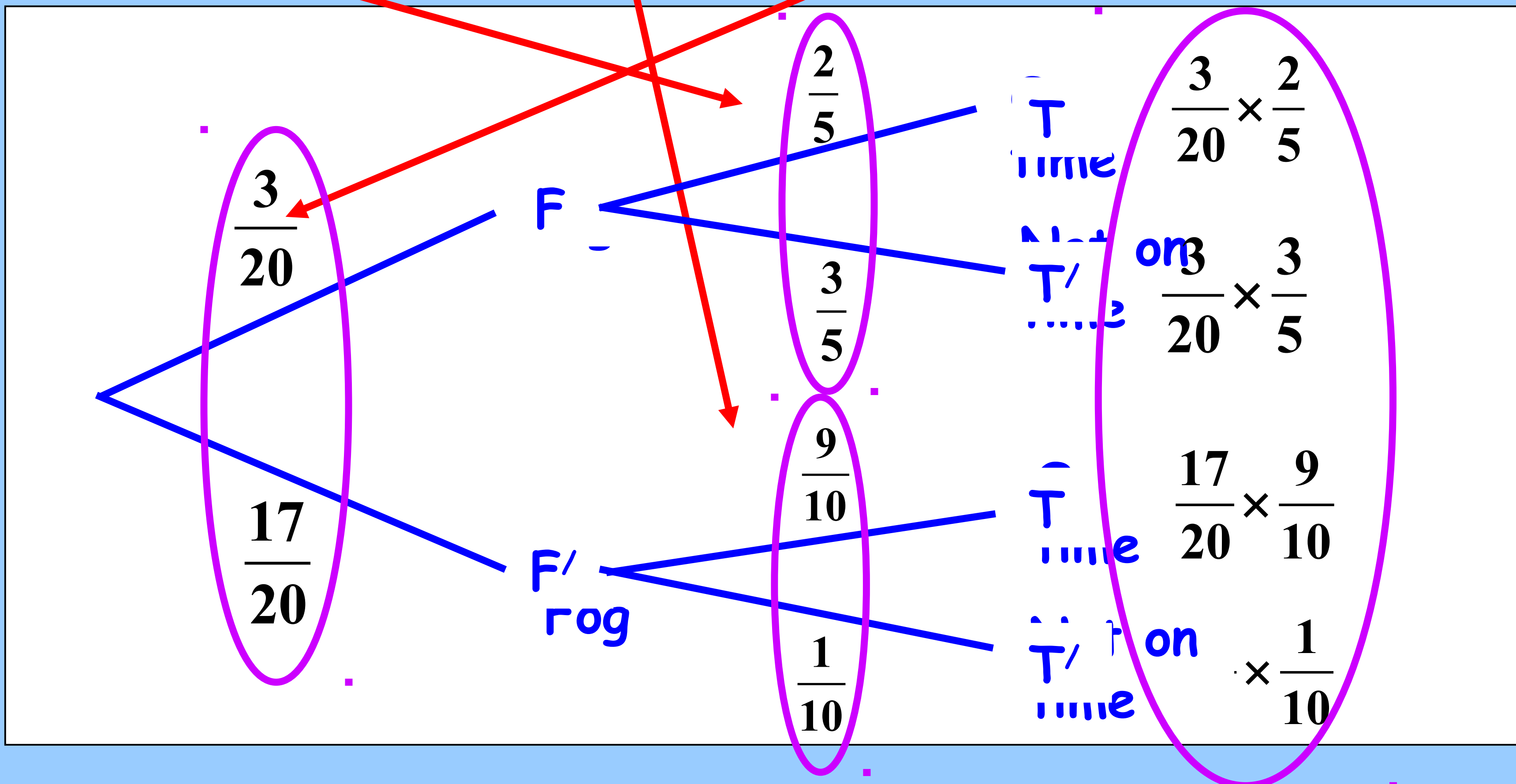
" If the visibility is good, the probability is $\frac{9}{10}$ ". $P(T|F') = \frac{9}{10}$

" The probability of fog at the time he travels is $\frac{3}{20}$ ".

$P(F) = \frac{3}{20}$

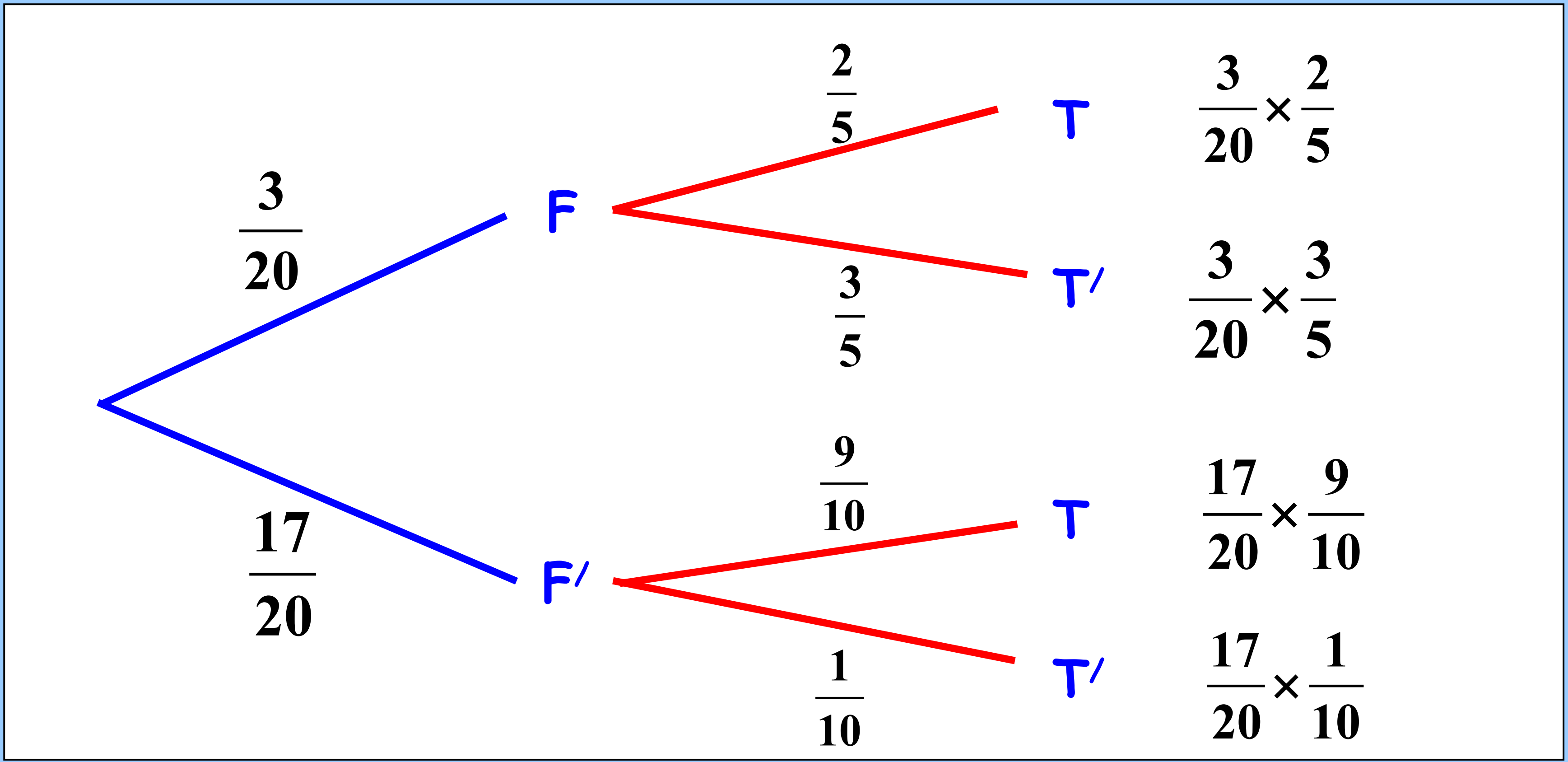
This is a much harder problem so we draw a tree diagram.

$P(T|F) = \frac{2}{5}$ $P(T|F') = \frac{9}{10}$ $P(F) = \frac{3}{20}$



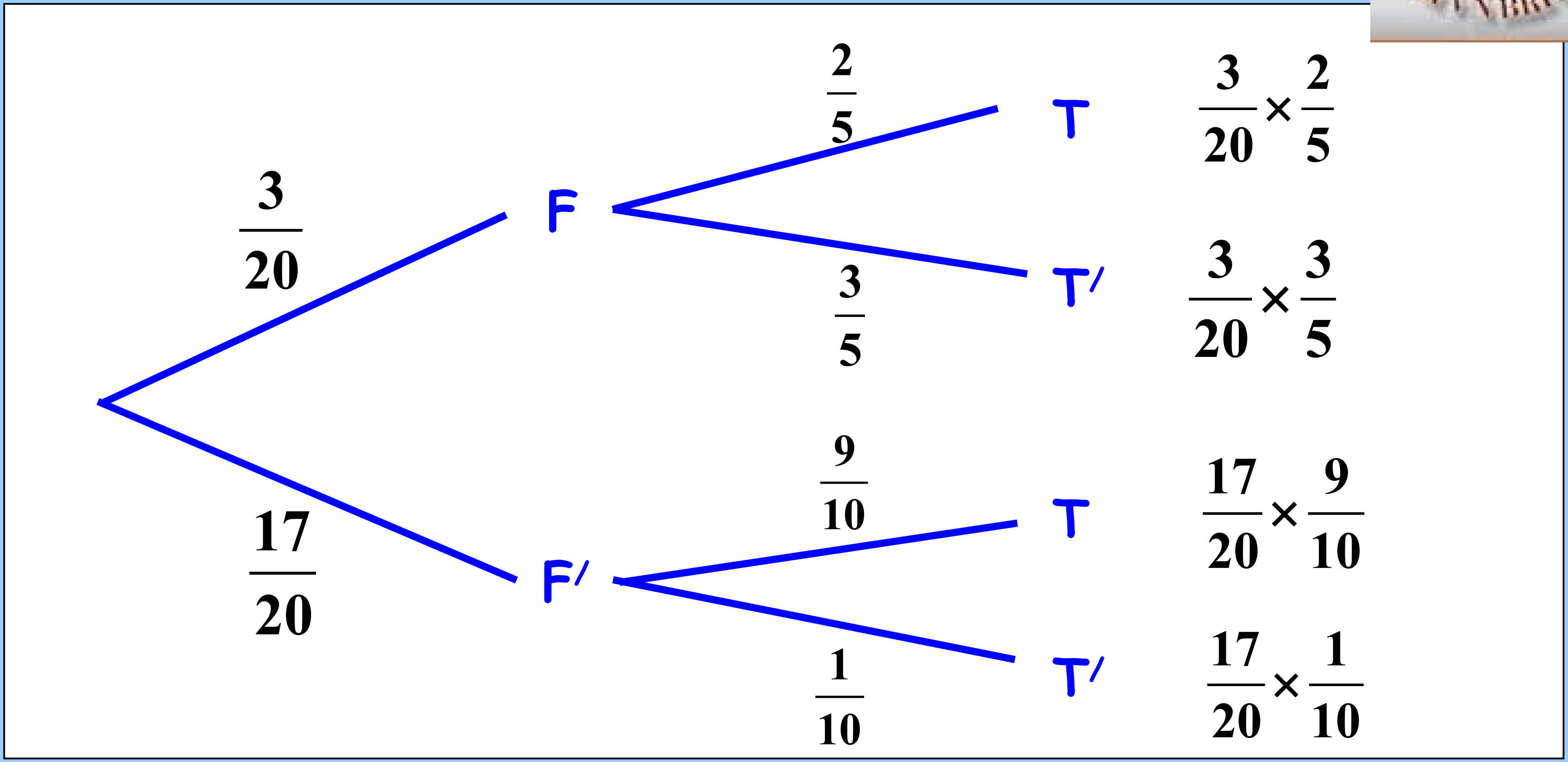
Each section sums to 1

$P(T|F) = \frac{2}{5}$ $P(T|F') = \frac{9}{10}$ $P(F) = \frac{3}{20}$



Because we only reach the 2nd set of branches after the 1st set has occurred, the 2nd set must represent conditional probabilities.

(a) Calculate the probability of him arriving on time.



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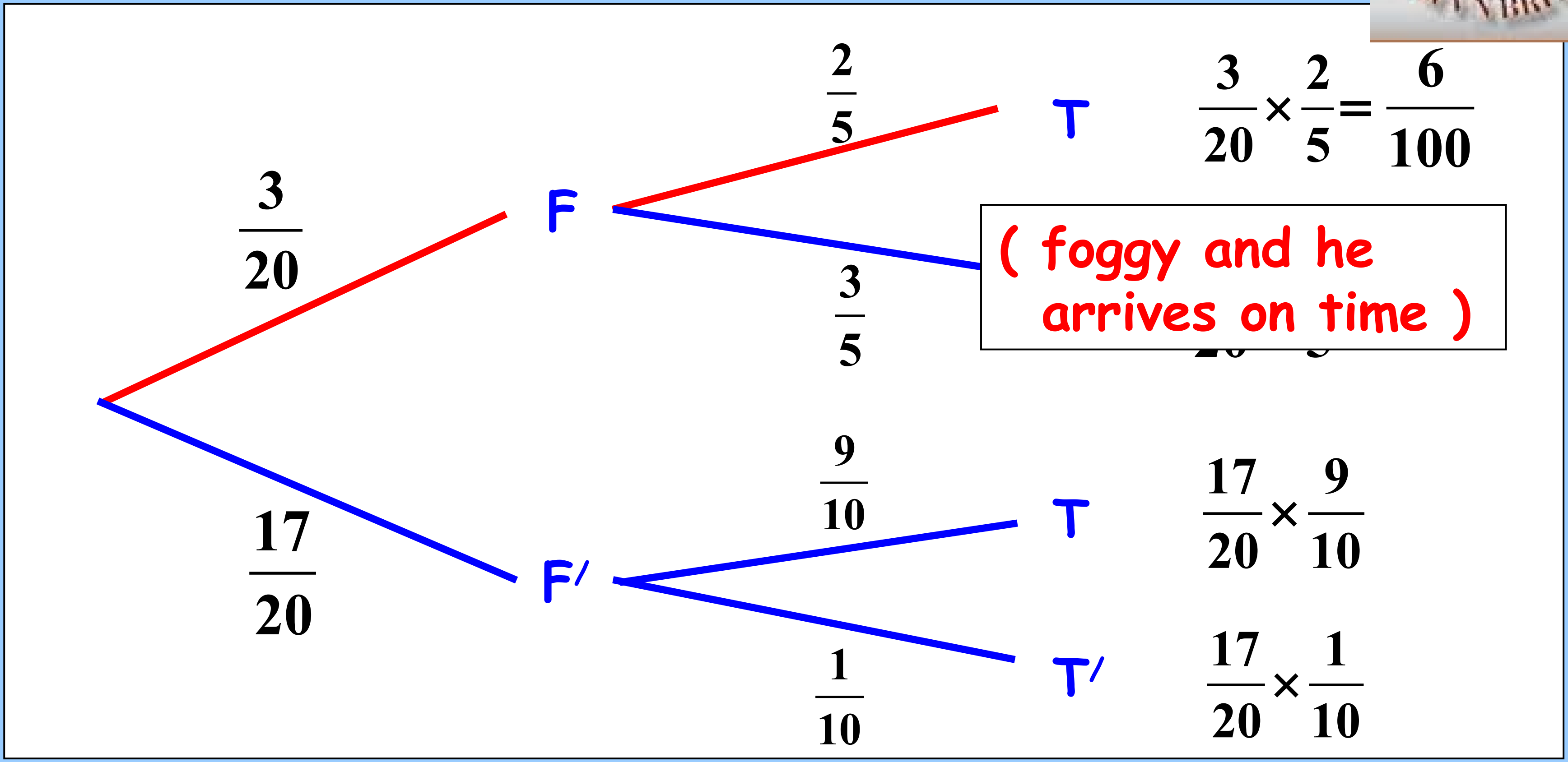

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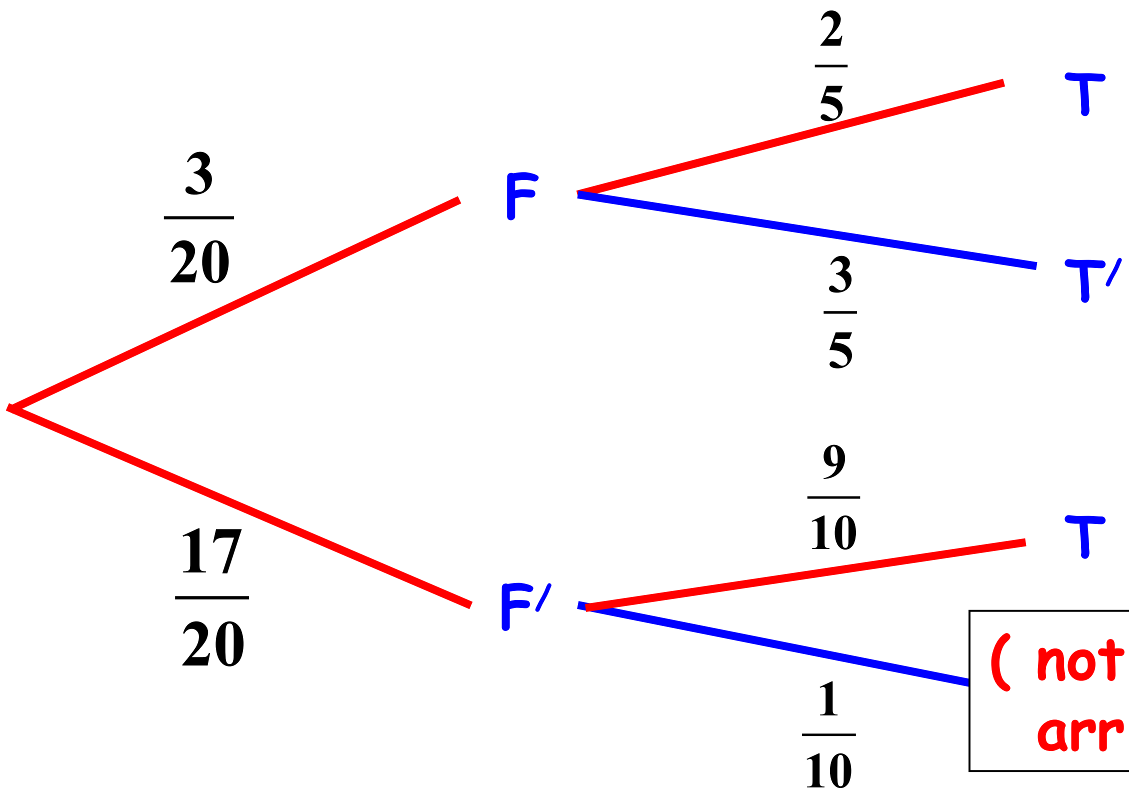


$$\frac{3}{20} \times \frac{2}{5} = \frac{6}{100}$$

$$\frac{3}{20} \times \frac{3}{5}$$

$$\frac{17}{20} \times \frac{9}{10} = \frac{153}{200}$$

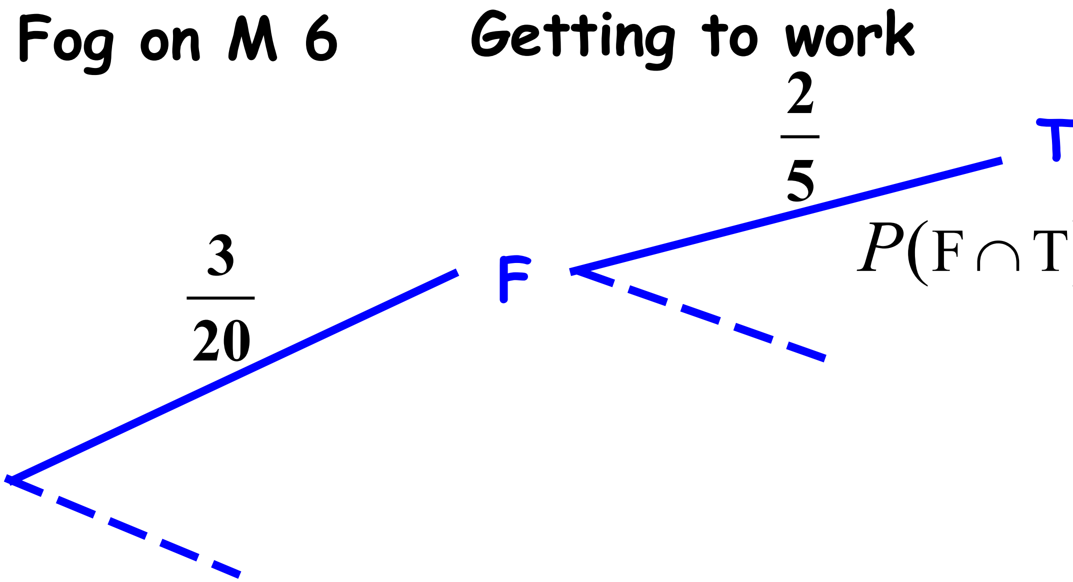
(not foggy and he arrives on time)

$$P(\mathrm{T}) = P(\mathrm{F} \cap \mathrm{T}) + P(\mathrm{F}^{/} \cap \mathrm{T}) = \frac{6}{100} + \frac{153}{200} = \frac{\not{165}^{\,33}}{\not{200}_{\,40}} = \frac{33}{40}$$

(b) Calculate the probability that there was fog given he arrives on time.

We need $P(\mathrm{F}|\mathrm{T})$

$$P(\mathrm{F}|\mathrm{T}) = \frac{P(\mathrm{F}\cap\mathrm{T})}{P(\mathrm{T})}$$



$$P(\mathrm{F}\cap\mathrm{T}) = \frac{3}{20}\times\frac{2}{5} = \frac{6}{100}$$

$$P(\mathrm{F}|\mathrm{T}) = \frac{P(\mathrm{F}\cap\mathrm{T})}{P(\mathrm{T})}$$

From part (a), $P(\mathrm{T}) = \frac{33}{40}$

$$\Rightarrow \quad P(\mathrm{F}|\mathrm{T}) = \frac{6}{100} \div \frac{33}{40} = \frac{\cancel{6}^{\,2}}{\cancel{100}_{\,5}}\times\frac{\cancel{40}^{\,2}}{\cancel{33}_{\,11}} \quad \Rightarrow \quad P(\mathrm{F}|\mathrm{T}) = \frac{4}{55}$$

Eg 4. The probability of a maximum temperature of 2
more on the 1st day of Wimbledon (tennis competition!)
has been estimated as $\frac{3}{8}$. The probability of a particular Aussie player winning on the 1st day if it is below $28°$ is estimated to be $\frac{3}{4}$ but otherwise only $\frac{1}{2}$.

Draw a tree diagram and use it to help solve the following:

(i) the probability of the player winning,

(ii) the probability that, if the player has won, it was at least $28°$.

Solution: Let T be the event " temperature $28°$ or more "

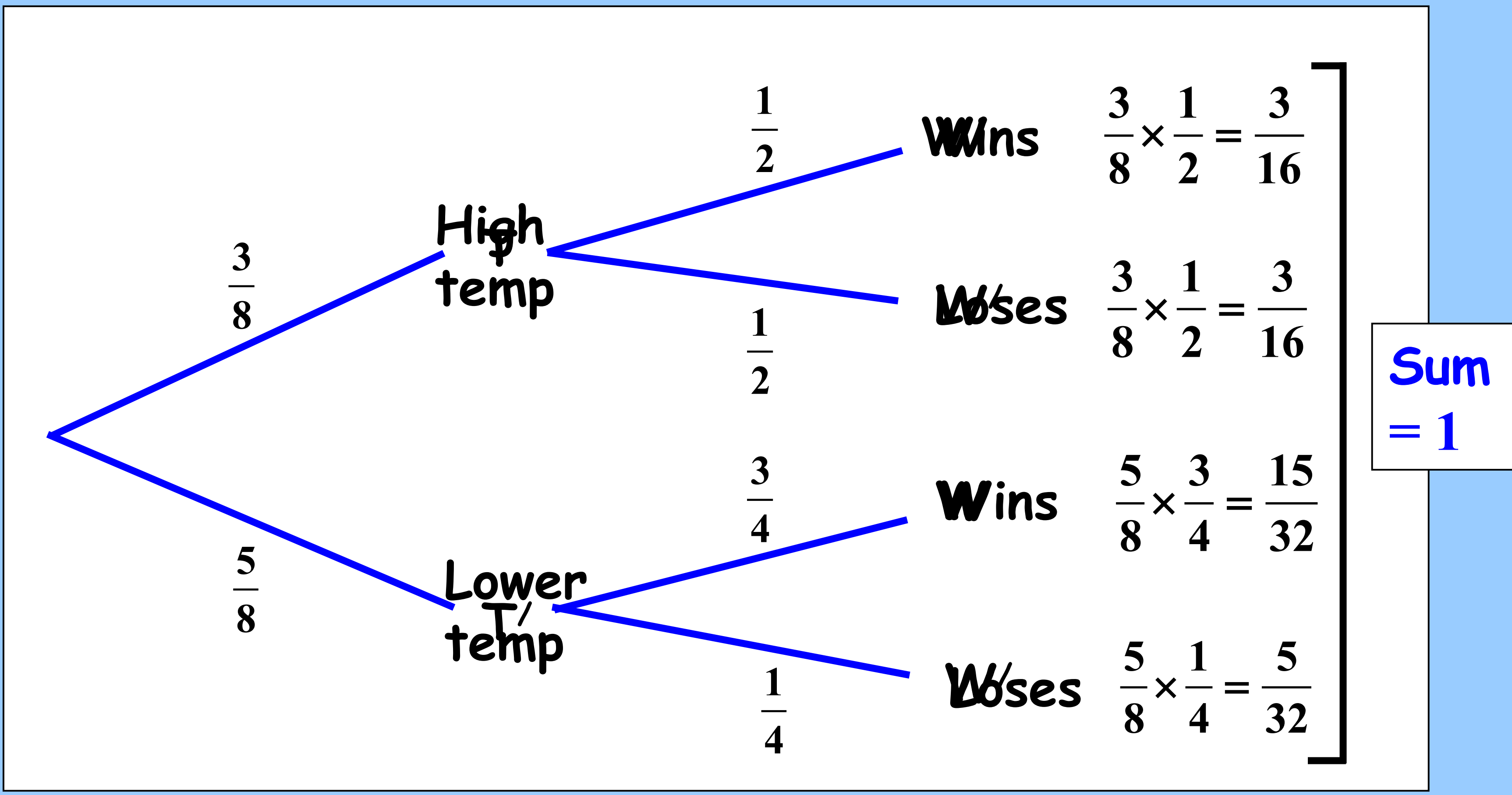
Let W be the event " player wins "

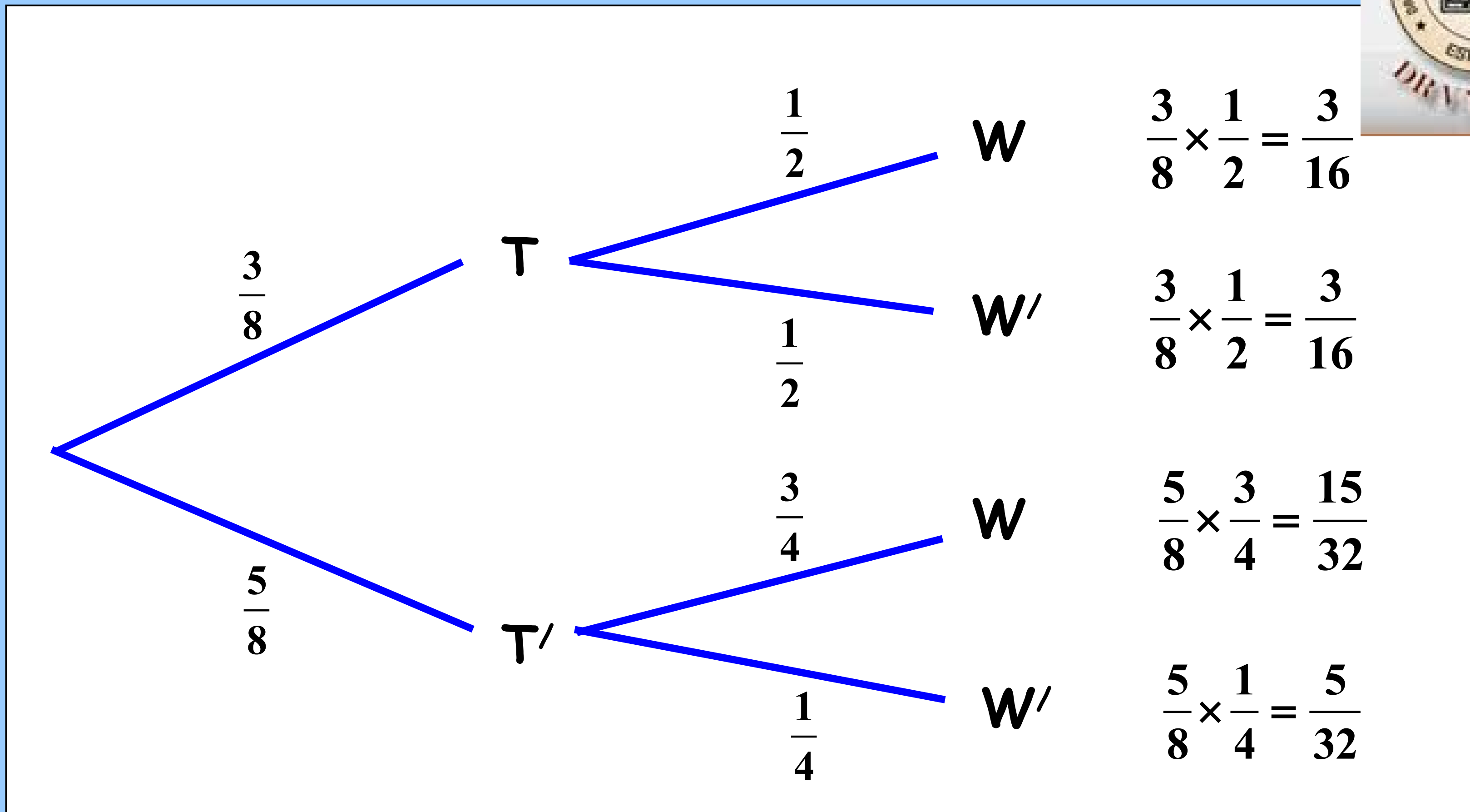
Then, $P(T) = \frac{3}{8}$ $P(W|T') = \frac{3}{4}$ $P(W|T) = \frac{1}{2}$

Let T be the event " temperature 28° or more "

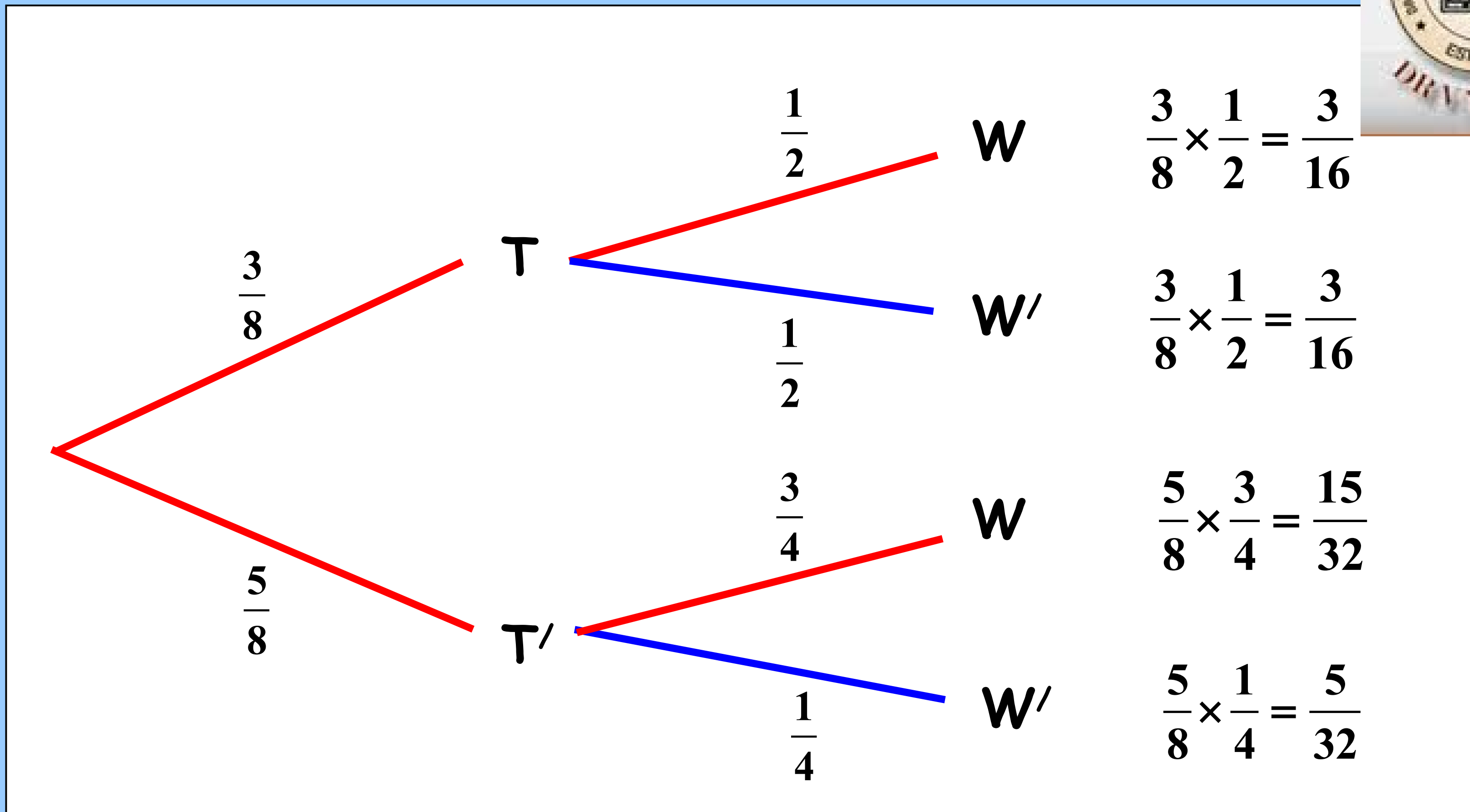
Let W be the event " player wins "

Then, $P(T) = \frac{3}{8}$ $P(W|T') = \frac{3}{4}$ $P(W|T) = \frac{1}{2}$

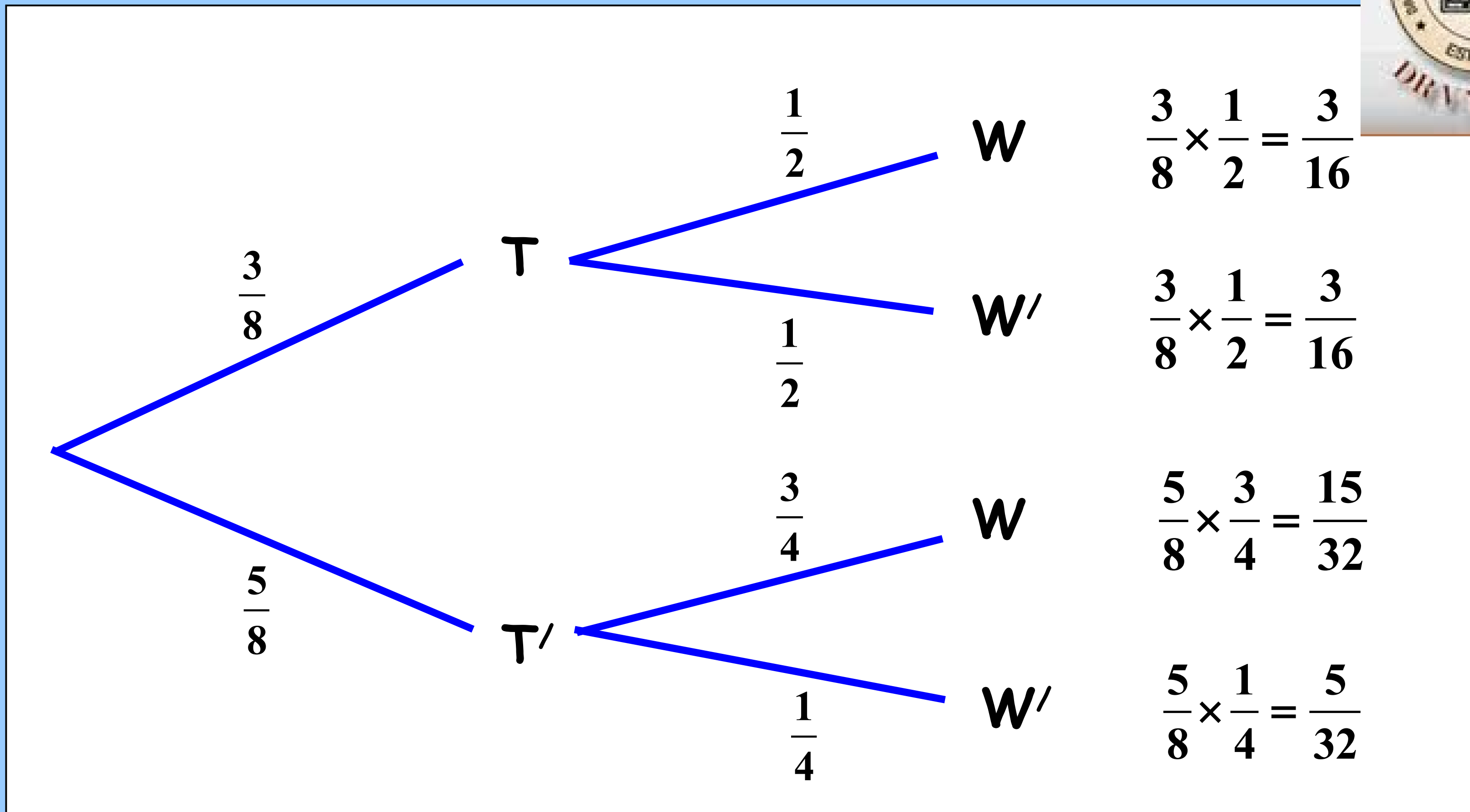




(i) $$P(\mathrm{W}) = P(\mathrm{T} \cap \mathrm{W}) + P(\mathrm{T}' \cap \mathrm{W})$$

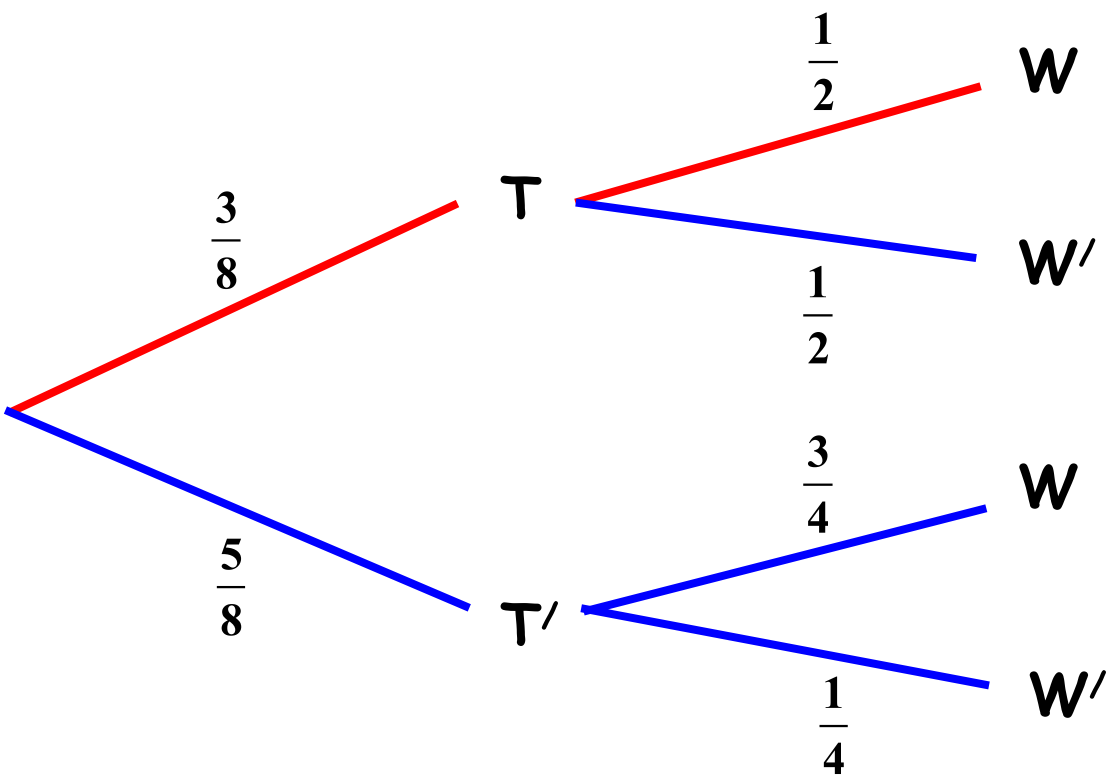


(i) $P(W) = P(T \cap W) + P(T' \cap W) = \frac{3}{16} + \frac{15}{32} = \frac{6+15}{32} = \frac{21}{32}$



$$P(\mathrm{W}) = \frac{21}{32}$$

(ii) $$P(\mathrm{T} \mid \mathrm{W}) = \frac{P(\mathrm{T} \cap \mathrm{W})}{P(\mathrm{W})}$$



$$\frac{3}{8}\times\frac{1}{2}=\frac{3}{16}$$

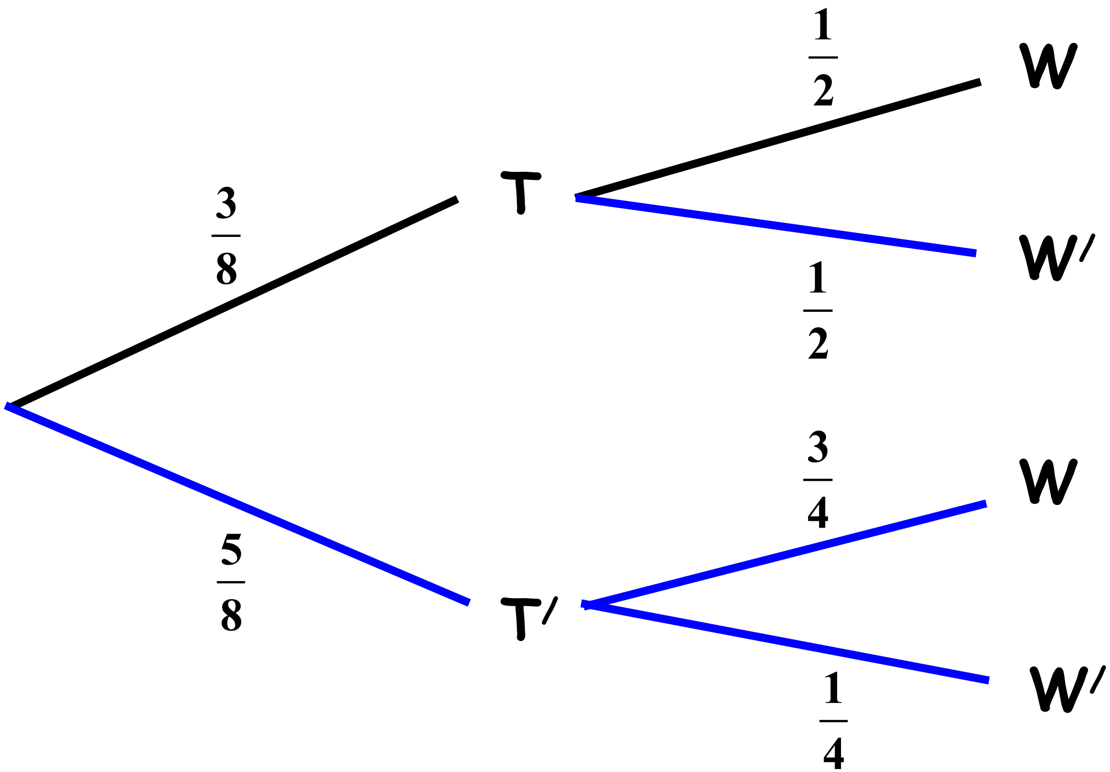
$$\frac{3}{8}\times\frac{1}{2}=\frac{3}{16}$$

$$\frac{5}{8}\times\frac{3}{4}=\frac{15}{32}$$

$$\frac{5}{8}\times\frac{1}{4}=\frac{5}{32}$$

$$P(W)=\frac{21}{32}$$

(ii) $P(\mathrm{T}\mid \mathrm{W})=\dfrac{P(\mathrm{T}\cap \mathrm{W})}{P(\mathrm{W})} \Rightarrow P(\mathrm{T}\mid \mathrm{W})=\dfrac{3}{16}\div\dfrac{21}{32}$



$$\frac{3}{8}\times\frac{1}{2}=\frac{3}{16}$$

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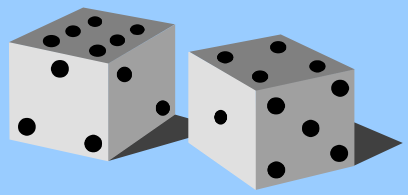
$$\frac{5}{8}\times\frac{3}{4}=\frac{15}{32}$$

$$\frac{5}{8}\times\frac{1}{4}=\frac{5}{32}$$

$$P(W)=\frac{21}{32}$$

(ii) $$P(T\,|\,W)=\frac{P(T\cap W)}{P(W)} \Rightarrow P(T\,|\,W)=\frac{3}{16}\div\frac{21}{32}=\frac{\cancel{3}^{1}}{\cancel{16}_{1}}\times\frac{\cancel{32}^{2}}{\cancel{21}_{7}}=\frac{2}{7}$$



Independent Even

We can deduce an important result from the conditional law of probability:

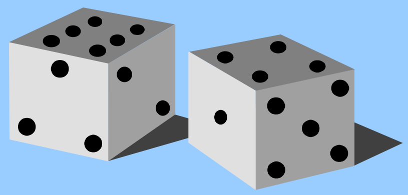
If B has no effect on A, then, $P(A|B) = P(A)$ and we say the events are independent.

(The probability of A does not depend on B.)

So, $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

becomes $P(A) = \dfrac{P(A \cap B)}{P(B)}$

or $P(A \cap B) = P(A) \times P(B)$



Independent Even

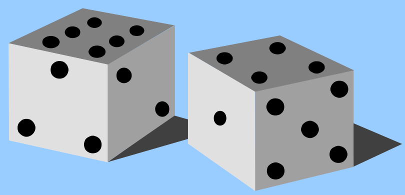
Tests for independence

$$P(A|B) = P(A)$$

$$P(B|A) = P(B)$$

or

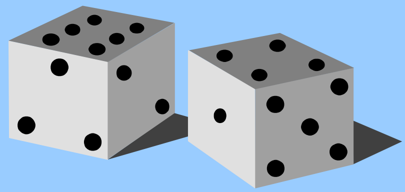
$$P(A \cap B) = P(A) \times P(B)$$



Expected Value

Suppose that the outcomes of an experiment are real numbers called $x_1, x_2, x_3, \ldots, x_n$
and suppose that these outcomes have probabilities $p_1, p_2, p_3, \ldots, p_n$ respectively. Then the expected value of x, E(x), of the experiment is:

$$E(x) = \sum_{i=1}^{n} x_i p_i$$



Expected Value

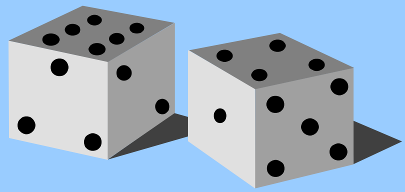
Example

At a raffle, 1500 tickets are sold at $2 each for four prizes of $500, $250, $150, and $75. What is the expected value of your gain if you play?

Gain	$498	$248	$148	$73	-$2
P(x)	$\frac{1}{1500}$	$\frac{1}{1500}$	$\frac{1}{1500}$	$\frac{1}{1500}$	$\frac{1496}{1500}$

$$E(x) = 498 \times \frac{1}{1500} + 248 \times \frac{1}{1500} + 148 \times \frac{1}{1500} + 73 \times \frac{1}{1500} + -2 \times \frac{1496}{1500}$$

$$= -\$1.35$$



Odds

When one speaks about the odds in favour of an event, they are actually stating the number of favourable outcomes of an event to the number of unfavourable outcomes of the event, assuming that the outcomes are equally likely.

The odds in favour of event E are:
$n(E):n(\bar{E})$ $P(E):P(E)$

The odds against event E are
$n(\bar{E}):n(E)$ $P(E):P(E)$

If the odds in favour of E are a:b, then

$$P(E) = \frac{a}{a+b}$$

Applicable Mathematics
"Probability"