

Time Value of Money

Time Value of Money

- 1 crore rupee's today is more than 1 crore rupee's 5 years from now.

Why????

- Since it could be invested on some **interest or return generating instrument** for the 5 year period.
- Also due to **inflation in most cases** what one could buy today from the same amount would be more as compared to 5 years from now.

Common Terminologies related to Time Value of Money

- Present Value (PV) – i.e. Today's Value
- Future Value (FV) – i.e. Value in the future
- No of years or Number of time frames (n)
- Rate of Return or Discounting rate (r)
- Annuity (A)– i.e. Constant Cash flow year after year for number of years

Relation between PV & FV

$$FV = PV \cdot (1+r)^n$$

- Above equation assumes annual compounding & one time cash flow.
- In case of Semiannual compounding the equation would change to

$$FV = PV \cdot (1+r/2)^{2n}$$

- In general, if year is divided in small parts of by dividing it say 't' times then n should be multiplied by 't' times.

$$FV = PV \cdot (1+r/t)^{t \cdot n}$$

- In case of continuous compounding above equation will change to,

$$FV = PV \cdot e^{(rn)}, \text{ where } e=2.71828$$

Example

Q. What is the future value of Rs.10000 invested today at the end of 7 years, given the investment is to earn 9% rate of return, in the following cases ?

1. Annual Compounding
2. Semi Annual Compounding
3. Quarterly Compounding
4. Continuous compounding

Given

PV=10000

t=7

r=9%

FV=???

Solution

$$FV1 = 10000 * (1+9\%)^7 = 18280.39$$

$$FV2 = 10000 * (1+9\%/2)^{(7*2)} = 10000 * (1.045)^{14} = 18519.45$$

$$FV3 = 10000 * (1+9\%/4)^{(7*4)} = 10000 * (1.0225)^{28} = 18645.45$$

$$FV1 = 10000 * (2.71828)^{(7*9\%)} = 18776.11$$

Relation between FV & PV

$$PV = FV / ((1+r)^n)$$

- Above equation assumes annual compounding & one time cash flow.
- In case of Semiannual compounding the equation would change to

$$PV = FV / ((1+r/2)^{2n})$$

- In general, if year is divided in small parts of by dividing it in say 't' times then n should be multiplied by 't' times.

$$PV = FV / ((1+r/t)^{t*n})$$

- In case of continuous compounding above equation will change to,

$$PV = FV / (e^{(rn)}), \text{ where } e = 2.71828$$

Example

Q. What is the present value of Rs.1000000 which you would get at the end of 10 years, given the average inflation in the interim is say 8% per annum, in the following cases ?

1. Annual Compounding
2. Semi Annual Compounding
3. Quarterly Compounding
4. Continuous compounding

Given

FV=1000000

t=10

r=8%

PV=???

Solution

$$PV1 = 1000000 / ((1+8\%)^{10}) = 463193.5$$

$$PV2 = 1000000 / ((1+8\%/2)^{(10*2)}) = 1000000 / ((1.04)^{20}) = 456386.9$$

$$FV3 = 1000000 / ((1+8\%/4)^{(10*4)}) = 10000 / ((1.02)^{40}) = 452890.4$$

$$FV1 = 1000000 / (2.71828)^{(10*8\%)} = 449329$$

FV of the annuity

$$FVA = A \frac{(1+r)^n - 1}{r}$$

r

Annual cash flows of fixed amount A.

Example

What is the future value of an annuity which pays Rs.25000 per year at the end of each of the next 15 years, given the investment is to earn 11% rate of return ?

Given

A=25000

t=15

r=11%

FVA=???

Solution

$$FVA = 25000 \frac{(1+11\%)^{15} - 1}{11\%}$$

11%

=860133.97

PV of the annuity or perpetuity

$$PVA = A \cdot \frac{1 - (1+r)^{-n}}{r}$$

Annual cash flows of fixed amount A for n number of years.

$$PVP = \frac{A}{r}$$

r

Annual cash flows of fixed amount A for infinite number of years.

Example

What is the present value of an annuity which pays Rs. 25000 per year at the end of each of the next 15 years, given the investment is to earn 11% rate of return? What if this investment is done till infinity?

Given

$A=25000$

$t=15$

$r=11\%$

$PVA1=???$

$PVA2=???$

Solution

$$PVA1=25000*(1-(1/1+11\%)^{15})/11\%$$

$$= 179771.74$$

$$PVA2=\underline{25000}$$

11%

$$=227272.73$$