

Bonds

- **Bond is security issued by the borrower to the lender.**
- **Typically it's a debt security.**
- **Issuer owes the holders a debt and, depending on the terms of the bond, is obliged to pay them interest (the coupon) or to repay the principal at a later date, termed the maturity.**
- **Interest is usually payable at fixed intervals (semiannual, annual, sometimes monthly).**
- **Bond is a form of loan or IOU (I owe you): the holder of the bond is the lender (creditor), the issuer of the bond is the borrower (debtor)**

Coupon Rate - It is the rate of periodic interest payment.

Usually this rate is fixed throughout the life of the bond.

It can also vary with a money market index, such as LIBOR or MIBOR , or it can be even more exotic.

There could be zero coupon bonds as well.

Par Value - It is the value issuers pays to the holder on the maturity of the bond.

It is also known as face value.

Generally bonds with coupon as issued (sold) at par or at face value.

Zero coupon bonds are issued at a discount.

Yield to Maturity or Redemption yield (YTM)

- YTM is the rate of return anticipated on a bond if it is held until the maturity date.
- YTM is considered a long-term bond yield expressed as an annual rate.
- The calculation of YTM takes into account the current market price, par value, coupon interest rate and time to maturity.
- It is equivalent to the internal rate of return (IRR) of a bond.

Example

- A bond with a par value of Rs.1000 with coupon rate 8% (semi-annual payments) might be sold to a buyer at Rs.1000 for 10 years.

Observations:

1. Bond issued at par. (No discount as coupon is paid.)
2. Periodic payment of Rs.40 every 6 months.

Example

- A bond issued at Rs. 463 of par value of Rs.1000 with no coupon for 10 years.

Observations:

1. Bond issued at discount. (As coupon is not paid.)

Bond Prices

Present Value of an Ordinary Annuity

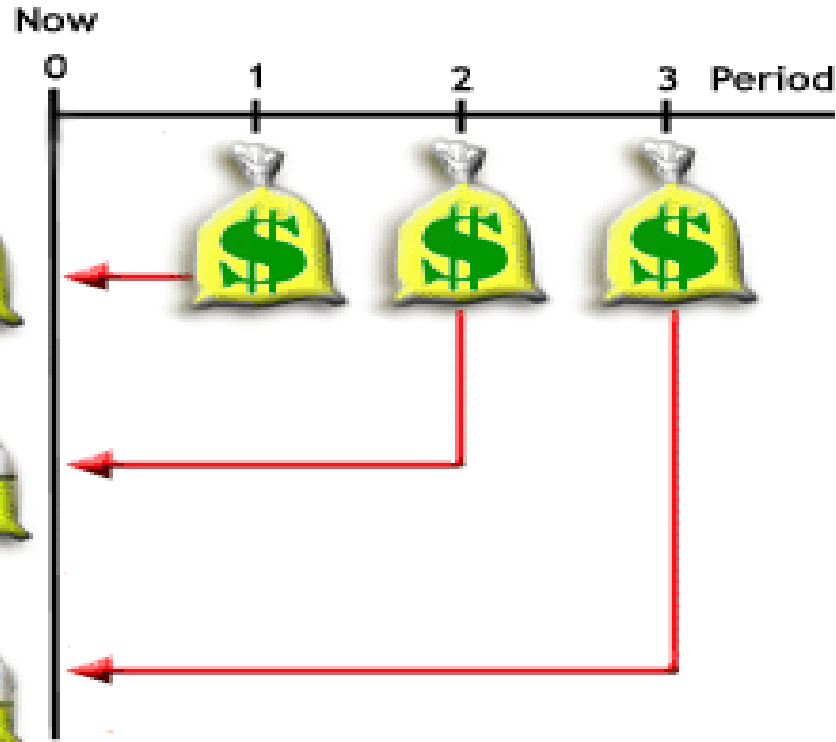
$$= \frac{\text{Coupon}(\$)}{(1 + \text{interest rate})}$$



$$= \frac{\text{Coupon}(\$)}{(1 + \text{interest rate})^2}$$



$$= \frac{\text{Coupon}(\$)}{(1 + \text{interest rate})^3}$$



Sum of the above equals the present value of an annuity formula:

$$PV = PMT \times \left[\frac{1 - (1 + i)^{-n}}{i} \right]$$

Where: PV = Present Value
 PMT = Coupon Payment
 i = Interest rate
 n = Number of periods

Bond Prices

$$\text{Bond Price} = \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + \dots + \frac{C}{(1+i)^n} + \frac{M}{(1+i)^n}$$

C = coupon payment

n = number of payments

i or r = interest rate, or required yield or the YTM

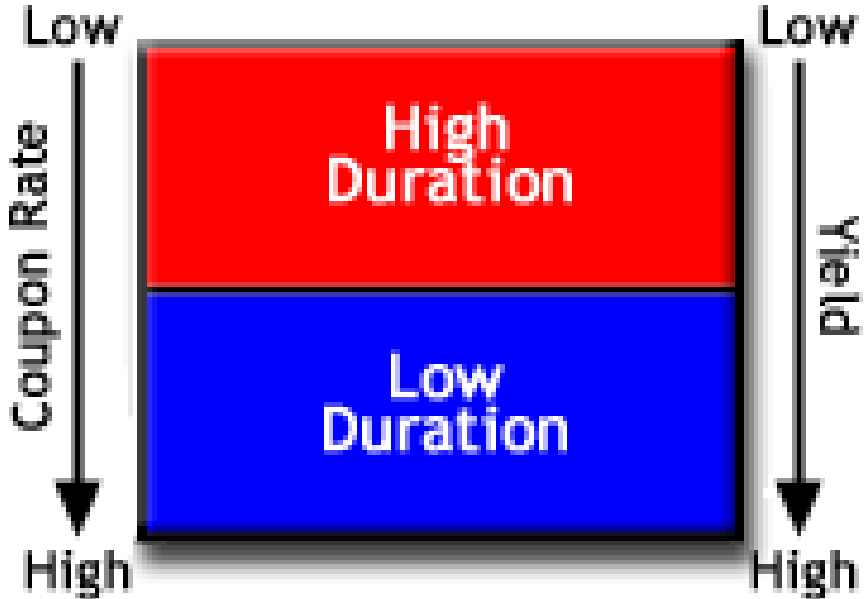
M = value at maturity, or par value

$$\text{Bond Price} = C * \frac{\left[1 - \left[\frac{1}{(1+i)^n} \right] \right]}{i} + \frac{M}{(1+i)^n}$$

Duration

- It is a measurement of how long, in years, it takes for the price of a bond to be repaid by its internal cash flows.
- It is an important measure for investors to consider, as bonds with higher durations carry more risk and have higher price volatility than bonds with lower durations.
- Duration is expressed as a number of years.
- Zero-Coupon Bond – Duration is equal to its time to maturity.
- Vanilla Bond - Duration will always be less than its time to maturity.

Duration, Coupon rate & YTM



Duration

- **Rising interest rates mean falling bond prices, while declining interest rates mean rising bond prices.**
- **Duration is a measure of the sensitivity of the price (the value of principal) of a bond to a change in interest rates.**
- **Duration is expressed as a number of years.**
- **Fortunately for investors, this indicator is a standard data point provided in the presentation of comprehensive bond and bond mutual fund information.**
- **The bigger the duration number, the greater the interest-rate risk or reward for bond prices.**
- **It is a common misconception among non-professional investors that bonds and bond funds are risk free. They are not.**

Duration

- Investors need to be aware of two main risks that can affect a bond's investment value: credit risk (default) and interest rate risk (rate fluctuations).
- The duration indicator addresses the latter issue.(i.e. interest rate risk)

Types of Duration

- There are four main types of duration calculations, each of which differ in the way they account for factors such as interest rate changes and the bond's embedded options or redemption features.
- Macaulay duration
- Modified duration
- Effective duration
- Key-rate duration

Macaulay duration

- The formula usually used to calculate a bond's basic duration is the Macaulay duration, which was created by Frederick Macaulay in 1938, although it was not commonly used until the 1970s.
- Macaulay duration is calculated by adding the results of multiplying the present value of each cash flow by the time it is received and dividing by the total price of the security.

$$\text{Macaulay Duration} = \frac{\sum_{t=1}^n \frac{t * C}{(1+i)^t} + \frac{n * M}{(1+i)^n}}{P}$$

$$C * \frac{\left[1 - \frac{1}{(1+i)^n} \right]}{i} + \frac{M}{(1+i)^n}$$

$$\text{Macaulay Duration} = \frac{\sum_{t=1}^n \frac{t * C}{(1+i)^t} + \frac{n * M}{(1+i)^n}}{C * \left[\frac{1 - \left[\frac{1}{(1+i)^n} \right]}{i} \right] + \frac{M}{(1+i)^n}}$$

Modified Duration

- **Modified duration is a modified version of the Macaulay model that accounts for changing interest rates.**
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- **For bonds without any embedded features, bond price and interest rate move in opposite directions, so there is an inverse relationship between modified duration and an approximate 1% change in yield.**
- **Because the modified duration formula shows how a bond's duration changes in relation to interest rate movements, the formula is appropriate for investors wishing to measure the volatility of a particular bond.**

$$\text{Modified Duration} = \left[\frac{\text{Macaulay Duration}}{\left(1 + \frac{\text{yield - to - maturity}}{\text{number of coupon periods per year}} \right)} \right]$$

$$\text{Modified Duration} = \left[\frac{\text{Macaulay Duration}}{\left(1 + \frac{\text{YTM}}{n} \right)} \right]$$

- **The modified duration formula discussed above assumes that the expected cash flows will remain constant, even if prevailing interest rates change;**
- **This is also the case for option-free fixed-income securities. On the other hand, cash flows from securities with embedded options or redemption features will change when interest rates change.**
- **For calculating the duration of these types of bonds, effective duration is the most appropriate.**
- **Effective duration requires the use of binomial trees to calculate the option-adjusted spread (OAS).**

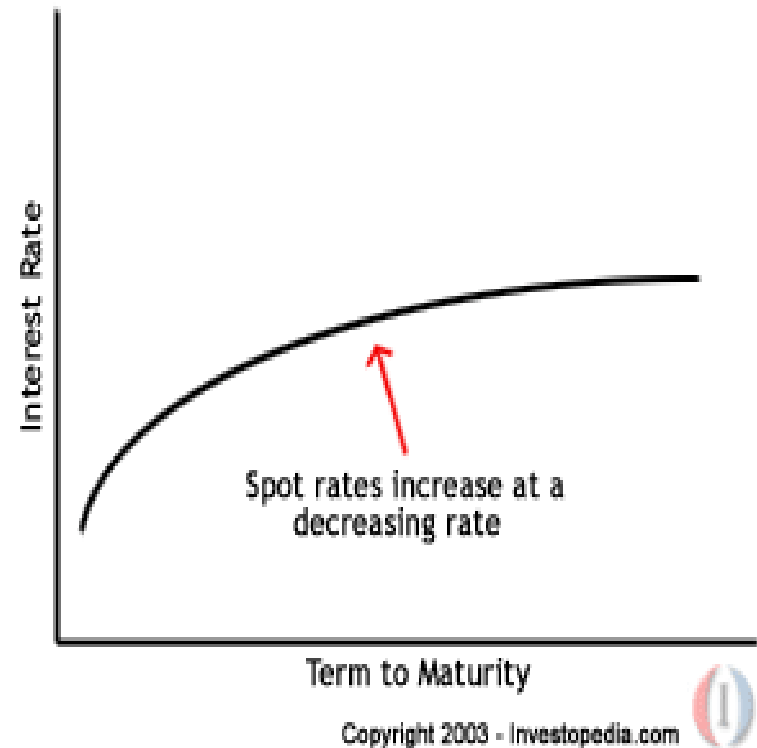
- The final duration calculation to learn is key-rate duration, which calculates the spot durations of each of the 11 "key" maturities along a spot rate curve
- These 11 key maturities are at the three-month and one, two, three, five, seven, 10, 15, 20, 25, and 30-year portions of the curve.
- In essence, key-rate duration, while holding the yield for all other maturities constant, allows the duration of a portfolio to be calculated for a one-basis-point change in interest rates.
- The key-rate method is most often used for portfolios such as the bond ladder, which consists of fixed-income securities with differing maturities. Here is the formula for key-rate duration:

$$\frac{\text{Price of security after a 1\% decrease in yield} - \text{Price of security after a 1\% increase in yield}}{2 * (\text{Initial price of security}) * 1\%}$$

Term Structure of Interest Rates

Term Structure Of Interest Rates

- The relationship between interest rates or bond yields and different terms or maturities
- The term structure of interest rates is also known as a yield curve and it plays a central role in an economy.
- The term structure reflects expectations of market participants about future changes in interest rates and their assessment of monetary policy conditions.



- In general terms, yields increase in line with maturity, giving rise to an upward sloping yield curve or a "normal yield curve."
- One basic explanation for this phenomenon is that lenders demand higher interest rates for longer-term loans as compensation for the greater risk associated with them, in comparison to short-term loans.
- Occasionally, long-term yields may fall below short-term yields, creating an "inverted yield curve" that is generally regarded as a indication of recession.

- **Real Interest Rate = Nominal Interest Rate - (Expected or Actual) Inflation**
- **For example if RBI sets repo rate at 8% and CPI inflation is at 7.5% then real interest rate is 0.5%.**
- **Real Interest rates should be positive so as to encourage savings in financial assets.**