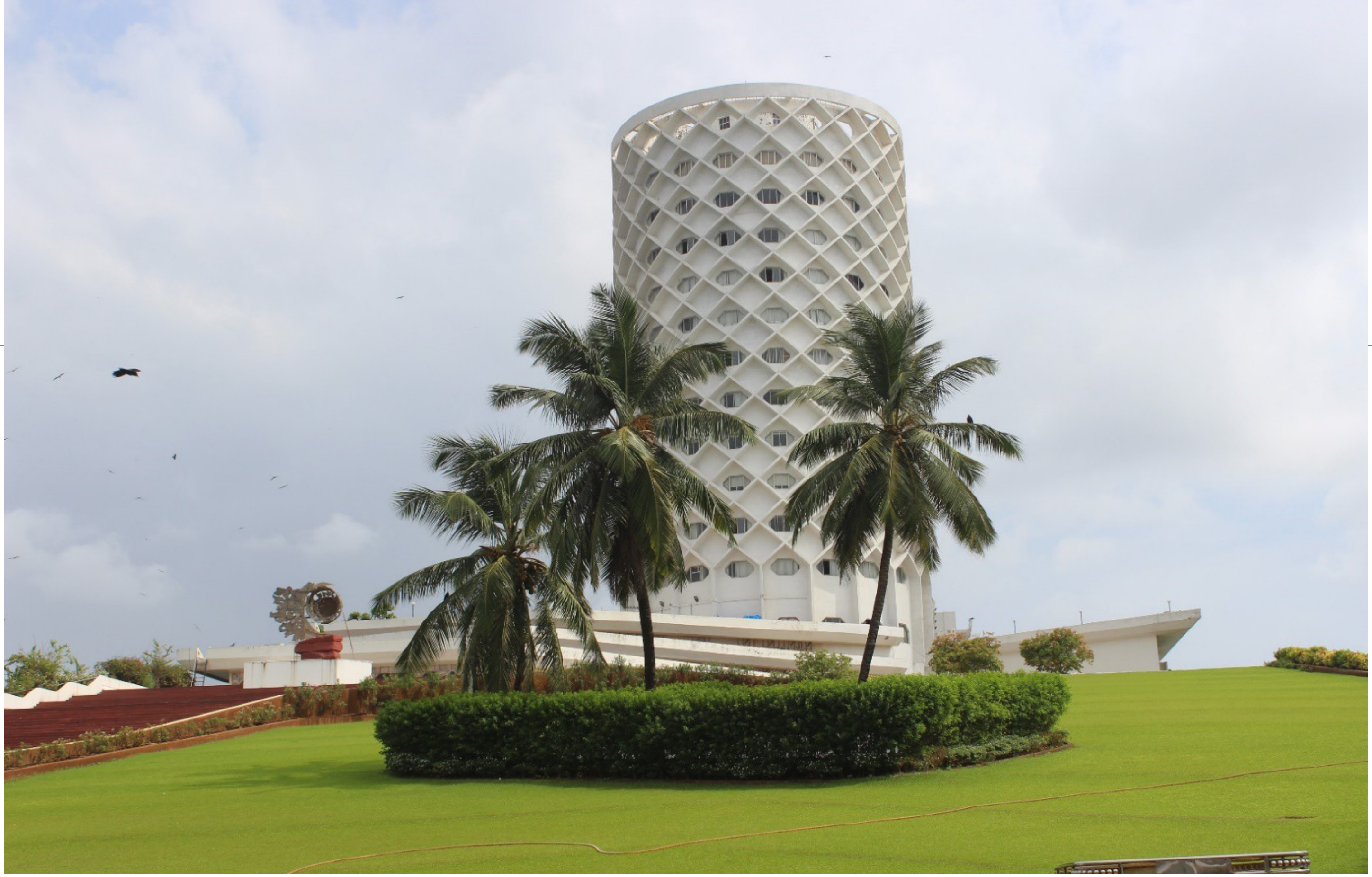

Hypothesis Testing

S CAgarkar

VN BRIMS, Thane

Introduction

- Suppose a mall manager tells us that the average work efficiency of her employees is 90%. How can we test the validity of her statement?
- Taking a sample of workers from the mall we can work out efficiency. If it is close to 90% we may accept her statement. If it is not then we reject her hypothesis.
- We cannot accept or reject a hypothesis about a population parameter simply by intuition. Instead, we need to learn mathematical techniques to decide objectively, on the basis of sample information.

Two tailed tests of means

- This test focuses on two tails of a normal curve and tries to see if the sample mean lies between the upper and lower limits calculated from hypothetical mean.
- A manufacturer supplies a rear axles to trucks that should withstand 80,000 pounds psi in stress test. Long experience indicates that SD of the strength is 4000 pound psi. Selecting a sample 100 axles they find that the mean stress is 79,600 psi. Should axles be accepted at 0.05 significance level?
- Given $\mu = 80,000$, $\sigma = 4000$, $n = 100$, $x = 79,600$ we need to calculate SE as well as UL & LL.

Continued

- $SE = \sigma \div \sqrt{n} = 4000 \div \sqrt{100} = 400$
- The z value for 0.05 significance level is 1.96
- $UL = \mu + 1.96\ SE$
- $= 80000 + 1.96 * 400 = 80{,}784$ pounds psi
- $LL = \mu - 1.96\ SE$
- $= 80000 - 1.96 * 400 = 79{,}216$
- Thus the upper level of stress is 80,784 psi while the lower level of stress is 79,216 psi. The observed mean falls between these two limits. Hence manufacturer should accept the claim and the production run from the factory.

Problem to solve

- Atlas Sporting Goods has implemented a special promotion for its propane stove and it suspects that the promotion may have resulted in a price change for customer. Atlas knows that before the promotion began the average retail price of the stove was $34 with SD $4.2. Atlas sampled 25 of its retailers after the promotion began and found the mean price for stove was $32.4. At the 0.05 significance level using a two tailed test does Atlas have reason to believe that the average retail price to the consumer has changed? (Ans. No).

One tailed tests of means

- Suppose a hospital uses large quantities of packaged doses of a particular drug. The individual dose of the drug is 100 cc. If you give excess drug the body will pass it on. If you give less then there is no desired effect. The hospital has purchased a huge stock and wants to know if it is acceptable. The hospital inspects 50 doses and finds that mean is 99.75 and SD is 2cc.
- Given: μ_{Ho} = 100, σ=2, n=50 and x=99.75

Continued

- $SE = \sigma/\sqrt{n}$ = 0.2829cc

- In this case we calculate LL as we have to ensure that the amount is not significantly lower than 100 cc. We must use one tailed test. At 90 percent confidence level the value of z is 1.28.
- $LL = \mu_{Ho} - 1.28\ SE = 99.64$

- The hospital should accept the sample as there isthe amount of drug in the sample (99.75) is more than the Lower Limit (99.64).

Problem to solve

- A computer leasing company firm has stated that the average monthly cost for a certain model is $4,800 with a population SD of $900. The firm has sampled 40 of its customers who lease their models and found that average monthly cost is $4,500. At 5 percent significance level, is the firm overestimating the cost for the model?
- Given: μ_{Ho} = 4,800, σ=900, n=40 and x=4,500
- $SE = \sigma/\sqrt{n}$ = ?
- UL= x+ z*SE = ?
- See if hypothesised value is more than this limit. Then you can conclude that the company is overestimating

Two tailed tests for proportions

- The HR director tells the president that 80 % of the employees are promotable. A committee conducts a study taking a sample of 150 employees and finds that 70 % of the sample are promotable.
- Given: p_{Ho} = 0.8, q_{Ho} = 0.2, n=150 p=.7 and q=.3
- SE= $\sqrt{p_{Ho} * q_{Ho} / n}$ = $\sqrt{0.8*0.2/150}$ = 0.0327
- UL= p_{Ho} + z*SE = 0.8 + 1.96 * 0.0327 = 0.8641
- LL= p_{Ho} - z*SE = 0.8 - 1.96 * 0.0327 = 0.7359
- Figures between UL and LL are acceptable. We can see that our samplemeandoes not lie between these figures. Thus, we conclude that there is a significant difference between the hypothesis of HR director and observed proportion.

Problem to solve

- Grant Inc., a manufacturer of men's shirts knows that its brand is carried in 15 percent of the men's clothing stores in USA. Grant recently sampled 75 men's clothing stores on the west coast and found that 18.7 % of the stored sampled carried the brand. At 0.05 level of significance, is there evidence that Grant has better distribution on the west coast than nationally? (Ans. Accept Ho).
- Given: $p_{Ho} = 0.15, q_{Ho} = 0.85$, n=75 p=.187 and q=.813
- $SE = \sqrt{p_{Ho} * q_{Ho} / n} = \sqrt{0.15*0.85/75}$ = ?
- $UL = p_{Ho} + z*SE$ = ?
- $LL = p_{Ho} - z*SE$ = ?
- Does the sampleproportion fallbetween these limits? If yes, accept null hypothesis, if not reject it.

One tailed test of proportions

°Members of the public interest groups asserts that less that 60 % of the industrial plants in the area comply with air pollution standards. Environmental Protection Agency believes that 60 % of the industries comply with the standards. Sampling the record of 60 plants one finds that 33 are complying with the standards. Test the assertion of the public interest group at 0.02 significance level.

°Ho: p = 0.6, H1: p<0.6α= 0.02

°p_{Ho}= 0.6,q_{Ho}=0.4, n = 60 p = 33/60 = .55 and q = 27/60 = .45

°SE= √$p_{Ho*}q_{Ho/}$n = √0.6*0.4/60 = 0.0632

°LL=p_{Ho}- z*SE = 0.6 – 2.05 * 0.0632 = 0.47

°The acceptance region as per sample survey is 0.45 to 0.55 and the calculated value is .47 which falls between them. Hence the claim of the EPA official can be accepted.

Problem to solve

- A ketchup manufacturer is in the process of deciding whether to produce a new extra spicy brand. The marketing department of the company used a national telephone survey of 5000 housewives and found that 235 said that they would purchase extra spicy brand. A similar study 2 years ago showed percentage of housewives who said they would purchase the brand was 4 percent. At the 2 percent significance level should the company conclude that there is an increased interest in the extra spicy flavour?
- Ho: p =0.04, H1: p>0.04α= 0.02
- p_{Ho}= 0.04,q_{Ho}=0.96,n=5000,p=235/5000= 0.047, q=0.953
- SE= $\sqrt{p_{Ho} * q_{Ho} / n}$ = $\sqrt{0.04*0.96/5000}$= ?
- UL=p_{Ho}+ z*SE = ?
- Does the upper limit fall within the acceptance region. If yes, accept null hypothesis, if not reject it. Answer to the question is reject it.

Tests of means using t distribution

- The chairman reports the average score on aptitude test as 90. Review of 20 cases by the management reveals that the average score is 84 with SD as 11. Should the management accept the claim of the chairman at 0.10 significance level? Note: t value at 10 percent SL and for 19 DF is 1.729.
- Given: μ_{Ho} = 90, s = 11, n = 20 and x = 84
- SE= s/√n = 11/4.47 = 2.46
- UL= μ_{Ho} + t*SE = 90 + 1.729*2.46= 94.25
- LL= μ_{Ho} - t*SE = 90 - 1.729*2.46 = 85.75
- The acceptance limit is 85.75 to 94.25. The sample mean however, lies outside these limits. Hence the claim of the chairman should be rejected

Problem to solve

- A television documentary on overeating claimed that Americans are 16 pounds overweight on an average. To test this claim, 9 randomly selected individuals were examined. The average excess weight was found to be 18 pounds with SD of 4 pounds. At the 0.05 percent level of significance, is there a reason to believe the claim of 16 pounds to be in error? (t value at 5 percent SL and at 8 DF is 2.306).
- Given: μ_{Ho} = 16, s = 4, n = 09 and x = 18
- SE = s/√n = 4/3 = ?
- UL = μ_{Ho} + 2.306 * SE = ?
- LL = μ_{Ho} – 2.306* SE = ?
- Ans. acceptHo, the claim is not an error.

Two tailed tests for difference between means

- A manpower development statistician is asked to determine whether the hourly wages of semiskilled workers are the same in two cities. He took a sample of 200 workers from city A and 175 workers from city B and found that the mean hourly earnings were $6.95 and 7.10 with SDs $.40 and .60 respectively. He then wants to test the hypothesis that at 0.05 significance level there is no difference between hourly wages.
- SE = $\sqrt{\sigma_1^2/n_1+\sigma_2^2/n_2}$
- = $\sqrt{(.4)^2/200 +(.6)^2/175}$
- = 0.053 (estimated standard error)

Continued

- UL = D + 1.96 SE = 0.1039
- LL = D – 1.96 SE = - 0.1039
- Here D is the hypothesised difference between the two population means. It is zero as it is hypothesised that there is no difference.
- Difference of means from the data
- $x_1 - x_2 = 6.95 - 7.10 = -0.15$
- Observed mean does not lie between the upper and lower limit calculated above. Hence the null hypothesis is rejected. It means there is a significant difference in the average wages of semiskilled workers in two cities.

Problem to solve

- A supervisor of a typing pool is interested in knowing whether typists required to correct their own errors have the same error rate as typists not required to correct their own errors. Of the 40 typists required to correct their own errors, the average number of errors per day is 20.2 with standard deviation of 2.5. Of the 56 typists not required to correct their own errors the average number of errors per day is 21.0 with a standard deviation of 3.1. At significance level of 10 percent, is there a significant difference in number of errors made by two kinds of typists? (Ans. Accept Ho)

One tailed tests for difference between means

- A company has been investigating two education programmes for increasing the sensitivity of its managers to the needs of Spanish speaking employees. The original programme consisted of several informal question-answer sessions with leaders of Spanish speaking community. Over the past few years a programme involving formal classroom contact with professional psychologists and sociologists has been developed. The new programme is considerably more expensive and the president wants to know at the 0.05 level of significance whether this expenditure has resulted in greater sensitivity. The data collected are given in the following table.

continued

Progsampled	Mean sensitivity	Number of managers	Estimated SD
Formal	92 %	12	15 %
Informal	84 %	15	19 %

- $SE = s_p \sqrt{1/n_1 + 1/n_2}$
- Where $s_p = \sqrt{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2 / n_1 + n_2 - 2}$
- Substituting the values we get $s_p = 17.35$
- and SE = 6.721
- UL = D + t SE
- = 0 + 1.708 * 6.721 = 11.48 %
- The difference between observed sample sensitivities is 92-84 = 8 %. It is smaller than the calculated upper limit. Hence the null hypothesis is true. It means there is no difference in the sensitivities before and after the new programme.

Problem to Solve

- A consumer research organization routinely selects several car models a year and tests their claims regarding safety, mileage, and comfort. In one study of two similar subcompact models manufactured by two different automakers, the average gas mileage for 7 cars of make A was 21 miles per gallon with a standard deviation of 5.8. For 9 cars of make B, the average gas mileage was 26 miles per gallon with a standard deviation of 5.3 miles per gallon. Test the hypothesis that the average gas mileage for cars of make B is greater than the average gas mileage for cars of make A. Use the .05 level of significance.

Two tailed tests for difference between proportions

- A pharmaceutical manufacturing company is testing two new compounds intended to reduce blood pressure. The compounds are administered to two different sets of laboratory animals. In group one 71 of 100 animals tested respond to drug with lower blood pressure levels. In group two 58 of 90 animals tested respond to drug with lower blood pressure levels. The company wants to test at the 0.05 level whether there is a difference between the efficacies of these two drugs.
- Given: $p_1 = .71$, $q_1 = .29$, $n_1 = 100$.
- $p_2 = .644$, $q_2 = .356$, $n_2 = 90$

Continued

- We know that the standard error of proportion (SE_p) is givenby$SE_p = \sqrt{p*q/n}$ and standard error of the difference between two proportions is given by $SE_{p1-p2} = \sqrt{p_1*q_1/n_1 + p_2*q_2/n_2}$
- Overall proportion of success (p) is given by
- $p = n_1 * p_1 * + n_2 * p_2 / n_1 + n_2 = 100 * .71 + 90 * .644 / 190 = .6789$
- P = .6789, Hence q = 1-.6789 = .3211. Using these values
- $SE_{p1-p2} = \sqrt{(.6789) * (.3211)/100 + (.6789) * (.3211)/90}$
- =.0678
- UL = D + 1.96 SE = 0.1329
- LL = D – 1.96 SE = - 0.1329
- Calculate difference between sample proportions = .71 - .644 = .066. This value lies between UL and LL. Hence null hypothesis is accepted. We, thus, conclude that effect produced by two substances on BP is not different.

One tailed tests for difference between proportions

- A city govt. has been using two different methods of listing the property. The first requires the property owner to appear in person before ataxlisterbutthe second requires the property owner to mail in a tax form. The city manager thinks the personal-appearance method produces far few mistakes than the mail-in method. To test this hypothesis 50 papers from method 1 and 75 papers from method 2 were checked and it was found that there are 10 percent and 13.3 percent mistakes in them. The city manager wants to test at the 0.15 level of significance the hypothesis that the personal appearance method produces a lower proportion of errors. What should she do?
- Given: $p_1 = .10$, $q_1 = .90$, $n_1 = 50$.
- $p_2 = .133$, $q_2 = .867$, $n_2 = 75$

Continued

- Overall proportion of success (p) is given by
- $p = n_1 * p_1 * + n_2 * p_2 / n_1 + n_2 = 50 * .10 + 75 * .133 / 125 = .12$
- p = .12, Hence q = 1-.12 = .88. Using these values
- $SE_{p1-p2} = \sqrt{(.12) * (.88)/50 + (.12) * (.88)/75} = .0593$
- LL = D – 1.04 SE = 0 – 1.04* .0593 = -.0617
- Calculated difference between sample proportions = .10 - .133 = -.033. This value lies to the right of LL.

Hence null hypothesis is accepted. We, thus, conclude that there is no difference between two methods. City govt. should go for mail-in method as it is less expensive.

Problem to solve

- A coal-fired power plant is comparing two different systems for pollution abatement. The first system has reduced the emission of pollutants to acceptable levels 63 percent of the time, as determined from 200 air samples. The second (and more expensive) system has reduced the emission of pollutants to acceptable levels 79 percent of the time, as determined from 300 air samples. At the 0.10 level of significance, can management conclude that the more expensive system is not significantly more effective than theinexpensive system?(Ans. Reject H_0)

Problem to solve

- Smoothy peanut butter company has interviewed 8 randomly chosen customers to gauge the success of its latest advertising campaign. The customers were asked how many ounces of Smoothy they purchased per week, both before and after the advertising campaign began. The answers are given in the table below. At 0.05 level of significance do these data indicate that the advertising campaign has been successful in increasing the customer's demand for Smoothy? (Ans. Accept Ho)
- Before 64 32 16 32 24 48 64 64
- After 72 32 48 16 16 64 64 56

Thanks, Thinkof various hypotheses associated with this structure