Time Value of Money

The chapter essentially deals with the concept and significance of money worth today compared to any future date. It talks about the present value and future value of money. It discusses the importance of opportunity cost and the importance to know the process of computing the time value of money, so that we can differentiate between the worth of investments that offer you returns at different times.

Concept of Time Value of Money

If you are offered the choice between having Rs.10,000 today and having Rs.10,000 at a future date, you will usually prefer to have Rs. 10,000 now. Similarly, if the choice is between paying Rs.10,000 now or paying the same Rs. 10,000 at a future date, you will usually prefer to pay Rs.10,000 later. It is simple common sense that in the first case by accepting Rs.10,000 early, you can simply put the money in the bank and earn some interest. Similarly in the second case by deferring the payment, you can earn interest by keeping the money in the bank, therefore the time gap allowed helps us to make some money. This incremental gain is time value of money.

Reasons why money in the Future is worth less than similar Money Today. There are three reasons why money can be more valuable today than in the future:

1. **Preference for Present Consumption:** Individuals have a preference for current consumption in comparison to future consumption. In order to sacrifice the present consumption for a future one, they need a strong incentive. Say for example, if the individual's present preference is very strong then he has to be offered a very high incentive to sacrifice it like a higher rate of interest and vice-versa.
2. **Inflation:** Inflation means when prices of things rise faster than they actually should. When there is inflation, the value of currency decreases over time. If the inflation is more, then the gap between the value of money today to the value of money in future is more. So, greater the inflation, greater is the gap and vice-versa.
3. **Risk:** Risk of uncertainty in the future lowers the value of money. Say for example, non-receipt of payment, uncertainty of investor's life or any other contingency which may result in non-payment or reduction in payment.

Simple Interest and Compound Interest

Simple Interest: Simple Interest may be defined as Interest that is calculated as a simple percentage of the original principal amount. The formula for calculating simple interest is:

SI = P0 (i)(n)

Where,
SI = simple interest in rupees
P0 = original principal
i = interest rate per time period (in decimals)
n = number of time periods

If we add principal to the interest, we will get the total future value (FV). (Future value is also known as Terminal Value). For any simple interest rate, the future value of an account at the end of n period is:-

FVn = P0+ SI = P0 + P0(i)(n)

Consider that Rs. 2,000 (P0) is deposited in a bank for two (n) years at simple interest of 6% (i). How much will be the balance at the end of 2 years?

Solution:- Required balance is given by
$FV_n = P_0 + P_0(i)(n) = 2{,}000 + 2{,}000(0.06)(2) = 2{,}000 + 240 =$ **Rs. 2,240.**

Compound Interest: If interest is calculated on original principal amount it is simple interest. When interest is calculated on total of previously earned interest and the original principal it compound interest. Naturally, the amount calculated on the basis of compound interest rate is higher than when calculated using simple interest formula. Consider the above example used in simple interest computation, the process involved in compounding is described as under –
Step 1:- Balance at the end of 1st Year would be:-
$FV_1 = P_1 + P_1(i)(n) = 2{,}000 + 2000\ (0.06)(1) = 2{,}000 + 120 =$ Rs.2,120
Step 2:- Balance at the end of 2nd Year would be:-
$FV_2 = P_2 + P_2(i)(n) = 2{,}120 + 2{,}120\ (0.06)(1) = 2{,}000 + 128 =$ Rs.2,248 (Rounded off)

In our above example, we chose the compounding period to be annually. **This time interval between successive additions of interests is known as conversion (or payment) period.** The accrued amount FV_n on a principal P after n payment periods at i (in decimal) rate of interest per payment period is given by: $\mathbf{FV_n = P_0 (1+i)^n}$
Where,
i = Annual rate of interest / number of payment periods per year = r / k
Hence, $FV = P_0 (1 + r/k)^n$, when compounding is done k times a year at an annual interest rate r.
Alternatively, $FV = P_0 (FVIF_{i,n})$

Where, $(FVIF_{i,n})$ is the future value interest factor at I% for n periods equal $(1+ I)^n$

Present Value (PV) and Future Value (FV)

"Present Value" is the current value of a Future Amount". It can also be defined as the amount to be invested today (Present Value) at a given rate over specified period to equal the future amount. If we reverse the flow by saying that we expect a fixed amount after n number of years, and we also know the current prevailing interest rate, then by discounting the future amount, at the given interest rate, we will get the present value of investment to be made. Discounting future amount converts it into present value amount. compounding converts present value amount into future value amount. Hence, the present value of a sum of money to be received at a future date is determined by discounting the future value at the interest rate that the money could earn over the period. This process is known as Discounting.

The present value interest rate or the future value interest rate is known as the discount rate. This discount rate is the rate with which the present value or the future value is traded off. A higher discount rate will result in a lower value for the amount in the future. This rate also represents the opportunity cost as it captures the returns that an individual would have made on the next best alternative. Present value can be expressed as under –

$$\mathbf{PV = \frac{FV_n}{(1 + i)^n}}$$

FV_n = Future value n years hence,
i= Rate of interest per annum
n= No. of years for which discounting is done.

Annuity

An annuity is a stream of regular periodic payment made or received for a specified period of time. In an ordinary annuity, payments or receipts occur at the end of each period.

Future Value of an Annuity (FVA): Expressed algebraically, FVAn is defined as future (compound) value of an annuity, R the periodic receipt (or payment), and n the length of the annuity, the formula for FVA n is:-

$FVA_n = R(I + i)^{n-1} + R(1+i)^{n-2} + \ldots + R(I + i)^1 + R(1+i)^0$

FVAn is simply sum of the future value interest factors at i percent for time periods 0 to n-1. If R be the periodic payments, the amount FVAn of the annuity is given by--

$FVA_n = R\,[(1+i)^n - 1/ I]$ or $FVA_n = R\,(FVIFA_{i,n})$

Where **$FVIFA_{i,n}$** stands for the future interest factor of an annuity at i% for n periods. Table for FVAn at different rates of interest may be used conveniently, if available, to workout the numericals; $[(1+i)^n - 1/ I]$ or $FVIFA_{i,n}$ can easily be found through financial tables.

Present Value of an Annuity: Sometimes instead of a single cash flow the cash flows of the same amount is received for a number of years. The present value of an annuity may be expressed as follows--

$PVA_n = R\,(PVIF_{i,1} + PVIF_{i,2} + PVIF_{i,3} + \ldots\ldots + PVIF_{i,n})$
$\qquad = R\,(PVIF_{i,n})$

Where,

PVA_n = Present value of annuity which has duration of n years

R = Constant periodic flow

n = Present value interest factor of an (ordinary) annuity at I percent for n periods.

I= Discount rate

Problem 1: Determine the compound interest for an investment of Rs. 7,500 at 6 % compounded half-yearly. Given that $(1+i)^n$ for i = 0.03 and n = 12 is 1.42576.

Problem 2: What is the present value of Re. 1 to be received after 2 years compounded annually at 10%?

Problem 3: Find the present value of ` 10,000 to be required after 5 years if the interest rate be 9%. Given that $(1.09)^5 = 1.5386$.

Problem 4: Mr. X has made real estate investment for ` 12,000 which he expects will have a maturity value equivalent to interest at 12% compounded monthly for 5 years. If most savings institutions currently pay 8% compounded quarterly on a 5 year term, what is the least amount for which Mr. X should sell his property? Given that $(1 + i)^n = 1.81669670$ for i = 1% and n = 60 and that $(1 + i)^{-n} = 0.67297133$ for i = 2% and n = 20.

Problem 5: A person is required to pay four equal annual payments of ` 5,000 each in his deposit account that pays 8% interest per year. Find out the future value of annuity at the end of 4 years.

Problem 6: Y bought a TV costing Rs.13,000 by making a down payment of ` 3,000 and agreeing to make equal annual payment for 4 years. How much would be each payment if the interest on unpaid amount be 14% compounded annually?