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Risk Management of Supply and Cash Flows in Supply Chains

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Abstract

This book focuses on several key issues of risk management in supply chains. Initially, the authors studied the supplier selection problem with supply risk. Specifically, the optimal sourcing strategy was identified in a one-retailer two-suppliers supply chain with random yields. The optimal sourcing strategy of a retailer and the optimal pricing strategies of two suppliers were investigated under an environment of supply disruption. Then, the authors studied the dynamic inventory control problems with cash flow constraints, financing decisions, as well as delayed cash payment. Finally, the authors created a model for the bargaining process, of an annual international iron ore price negotiation, to deal with the risk of wholesale price in the game analysis context.

Preface

Risk management has become an essential issue in supply chain management, from the modeling of the decision maker's risk preference, and the studies on uncertain elements such as demand, supply, price, lead time, etc., to the consideration of more practical background including cash flow constraints, inventory financing and delayed cash payment. Theoretically, the book provides a framework to study the interaction of various factors related to risk and their influence on supply chain management.

The core of this book is to analyze risk management of supply and cash flows in supply chains. The book consists of eight chapters. The contents of the book are outlined in the following.

Chapter 1 surveys the applications of risk management to supply chains and reviews the existing literature. The numerous literature in this field is classified into three categories, i.e., risk analysis of supply chain models, disruption management, and financial risk measurement. Throughout this chapter, some representative models are selected and their relationships and distinctions are analyzed.

The sourcing strategy of a retailer who procures from two unreliable suppliers is investigated in Chap. 2. "Unreliable" means that the suppliers may default on their obligations to deliver order quantities at the end of a given production period. The retailer facing stochastic demand needs to determine whether to choose single sourcing from one supplier or dual sourcing from two suppliers, and further how much to order. A two-period model is developed, and for each period, the authors identify the conditions under which the retailer will choose different sourcing strategies. It should be mentioned that more structural results can be found under the setting of deterministic demand.

Chapter 3 investigates not only the sourcing strategy of a retailer but also the pricing strategies of two suppliers under a supply disruption environment. The sourcing strategies of the retailer are characterized in a centralized system and a decentralized system respectively. Based on the assumption of a uniform demand distribution, the explicit form of the solutions is obtained when the suppliers are competitive. Finally, a coordination mechanism is devised to maximize the profits of both the suppliers.

In Chap. 4, a dynamic inventory control problem of a self-financing retailer is investigated. The retailer can periodically replenish his stock from a supplier and sell it to the market. The replenishment decisions of the retailer are constrained by cash flows, which is updated periodically following the purchasing and the sales in each period. Excess demand in each period is lost when insufficient inventory is available. The retailer's objective is to maximize its expected terminal wealth at the end of the planning horizon. The optimal inventory control policy is characterized. A simple algorithm is designed for computing the optimal policies in each period. Conditions are identified under which the optimal control policies are identical across periods. Finally, comparative static results on the optimal control policy are also presented.

Based on the model introduced in Chaps. 4 and 5 studies the dynamic inventory control problem with the assumption that asset-based financing is allowed for the retailer, when being short of cash flow. Excess demand in each period is lost when insufficient inventory is available. The retailer's objective is to maximize its expected terminal wealth at the end of the planning horizon. The optimal inventory control policy and its dependence on the wealth level are explored. Conditions are identified under which the retailer will choose either to borrow or to deposit in each period. The bankruptcy probability is also studied.

Furthermore in Chap. 6, a framework is proposed for incorporating financial considerations including delayed cash payment and receivable into dynamic inventory models. The financial constraint is updated periodically according to production activities. The dynamic financial constraint and the optimal operational policy are explored. The optimal operational policy's dependence on the financial state is also studied. It demonstrates the importance of firms considering delayed cash payment.

Chapter 7 seeks to provide insights for an annual international iron ore price negotiation by establishing mathematical and economical models and especially extending the Nash bargaining framework. Specifically, a one-supplier two-manufacturer supply chain is studied. The Nash game is first analyzed between the two manufacturers and then the bargaining process between the supplier and each manufacturer is modeled by a sequential Nash bargaining. The results demonstrate the importance of steel manufacturers in increasing the investment on iron ore.

Chapter 8 concludes the book and suggests some topics for future research.

Within the perspectives of risk management in supply chains, analysis on the risk management of supply and cash flows are still in its infancy, and more efforts are needed from academia. Hence the ambition and innovation of this book is to contribute on risk management in supply chains in following ways:

(1) Characterizing the explicit sourcing strategy (i.e., single sourcing or dual sourcing) to deal with supply risk
(2) Introducing the concepts of financial risk measurement by incorporating cash flow constraints, inventory financing and delayed cash payment into inventory management models

(3) Providing insights for the iron ore price negotiation to help the steel manufacturers to handle the risk of price increase

This book is intended for researchers interested in conducting in-depth studies on supply chain risk management. The book is also intended for business practitioners seeking to understand the nature and law governing the working of supply chain risk management and looking for guidance and decision support for the implementation of supply chain risk management. Therefore, the book can be useful not only for researchers but also for practitioners and graduate students in operations management, management science, and business administration.

We would like to thank many friends and colleagues for their help and support rendered to us in preparing this monograph. First, we thank Prof. Fangruo Chen of Columbia University, Prof. Xiuli Chao of The University of Michigan, Prof. Jeannette Song of Duke University, Prof. Gang Yu of The University of Texas at Austin, Prof. T.C. Edwin Cheng of The Hong Kong Polytechnic University and Prof. Kin Kleung Lai of City University of Hong Kong, Prof. Hanqin Zhang, Prof. Ke Liu and Dr Jingan Li of the Academy of Mathematics and Systems Science of the Chinese Academy of Sciences for their helpful discussions, suggestions and valuable comments on our research in this area. We also thank the National Natural Science Foundation of China, the Academy of Mathematics and Systems Science of the Chinese Academy of Sciences, and Beijing University of Chemical Technology for their financial support to our research.

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Chapter 1
Introduction

In recent years, the adopting of some supply chain practice such as outsourcing and lean production helps in smoothing the operations, but it also results in little buffer inventory in a supply chain which may lead to increased vulnerability of the chains.[1] At the same time, the business environment has evolved to be an increasingly complex scenario characterized by high uncertainty and rapid and frequent changes. For example, supply chains are subject to many potential external sources of disruption, e.g., natural disasters, terrorist attacks, and industrial actions, etc. The disruption in one firm can rapidly result in a significant adversary impact on the entire chain. Due to such changes, firm managers not only concern profit maximization but also pay much attention to risk containment or loss minimization for their firms. Motivated by the requirements of real world practice, supply chain risk management attracts more and more attention from academia (Chen et al. 2007; Shi 2004; Tang 2006a; Wu and Wang 2004a,b; Wu et al. 2006a; Zhou et al. 2006).

So far there is no generally agreed definition of supply chain risk management. It is even not clear to distinguish risk and uncertainty in supply chain operations (Tang and Nurmaya Musa 2010; Hua et al. 2010b, 2010c). Based on the review of existing literature, we think a comprehensive definition of supply chain risk management should refer to agents in the supply chain, collaboratively with their partners or on their own, to apply risk management process tools to deal with risks and uncertainties in the supply chain so as to ensure profitability and continuity.

The flows in a supply chain mainly include the forms of material, finance and information. Thus, supply chain risks can also be classified into three types: material flow risks, financial flow risks and information flow risks. For example, organizations increasingly rely on information technology such as enterprise resource planning solutions and internet to improve the supply chain process. Vast assistance

[1]The following discussion in this chapter is largely based on the ideas and results presented in Wu et al. (2011).

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from these IT systems has, however, exposed to another consequence, namely information disruption (Tang and Nurmaya Musa 2010). Tang and Nurmaya Musa (2010) also points out that quantitative methods are missing in information flow risk analysis. This book mainly focus on material flow risks and financial flow risks.

Demand fluctuation and supply disruption are two kinds of primary uncertainties in supply chain material flow and various specific examples of supply chain risks, e.g., uncertainties of purchasing costs, selling prices, and contract parameters, etc., can be ultimately attributed to the variations of supply and demand. Hence, most of the supply chain material flow risks can be classified into two categories known as demand risk and supply risk. Besides, the risks of the cash flow and possibility of bankruptcy need to be incorporated into supply chain management process.

In terms of influence and frequency, risks can also be classified into two types, one is normal risk and the other is abnormal risk. Generally speaking, normal risk mainly includes risks that occur frequently and are easy to track and control. These risks, e.g., demand uncertainty, price fluctuation, and supply variation etc., attract more and more attention to measure and to model from the academia. On the other hand, there is broad category of risk, known as abnormal risk, which arises from great disruptions to normal activities. Such risks are usually caused by unexpected events such as natural disasters, strikes, economic disruptions, and acts of purposeful agents (including terrorists) etc. The famous example is the event so-called “911” in USA. It happened without any premonition and the tangible loss is estimated at more than one hundred billion! Hence, there is special requirement to incorporate abnormal risk into supply chain management. Still abnormal risk management, also known as disruption management, is relatively new and often ignored since it is difficult to quantify, predict and manage.

To the best of our knowledge, most of the existing literature in relation to risk management in supply chains mainly focuses on modeling of the decision maker’s aversion to risks, consisting of various risk measurement approaches, such as utility function theory, mean–variance trade-off, and value-at-risk (VaR), etc. In the following chapter, we focus on the literature dealing with the decision maker who wants to take risk into consideration. Thus, it is the ambition of this chapter to bring together the research results from these fields of study in order to contribute to the study of risk management in supply chains. This chapter is not meant to be comprehensive or inclusive, but reviews through representative papers of the various issues studied in the risk management literature on supply chains. This chapter seeks answers to two key questions: What are the underlying supply chain risk management problems? and how have they been addressed in the current literature? We hope that it will be helpful for those who are interested in supply chain risk management.

In the following, Sect. 1.1 attempts to establish a framework of supply chain risk management problems from modeling of decision maker’s aversion to risks and supply chain management models. Then some literature on disruption management is reviewed according to the classical risk management paradigm in Sect. 1.2. Finally, Sect. 1.3 proposes some financial and economical instruments which are incorporated into supply chain management models.

1.1 Risk Analysis of Supply Chain Models

Traditional supply chain management mainly focuses on maximizing the expected profit or minimizing the expected cost (Li et al. 2006, 2007, 2008; Gong et al. 2009; He et al. 2010). Seldom of them consider the decision maker's risk preference towards the risk. It is well-known that decision makers are classified into three types based on their preference towards the risk, they are risk-averse, risk-neutral and risk-taking. In supply chain management, risk-averse usually reflects the real risk preference of an agent, either a retailer or a supplier. Hence, the modeling of the decision maker's preference towards the risk mainly focuses on the risk measurement of decision maker's aversion to risks. Almost all methods of risk measurement in supply chain risk management models are originated from economics and finance. From theory of utility function (von Neumann and Morgenstern 1944) and mean–variance methodology (Markowitz 1959) in midterm of the last century, to value-at-risk model and conditional value-at-risk (CVaR) model in recent years, we incorporate all of them into the framework of supply chain management.

In the following, the literature is categorized by risk management tools, and then several representative papers are surveyed with basic models and main results.

1.1.1 Risk Management Tools

1.1.1.1 Utility Maximization

Utility function was first presented by von Neumann and Morgenstern in 1944 with the objective aiming to maximizing the decision maker's expected utility. The literature adopting utility function to study supply chain models includes Bouakiz and Sobel (1992), Eeckhoudt et al. (1995), Agrawal and Seshadri (2000a), Chen et al. (2003), Keren and Pliskin (2006), Wang and Webster (2007, 2009). In the following, several representative papers are reviewed.

Bouakiz and Sobel (1992) studied the inventory replenishment strategy by minimizing the expected utility of the present value of costs over a finite planning horizon and an infinite horizon. Based on an exponential utility function, they had shown that the optimal ordering policy is given by a sequence of critical numbers if the ordering costs are linear and the penalty and holding costs are convex. The infinite-horizon policy is ultimately stationary and approaches the risk-neutral policy as the period gets larger.

Eeckhoudt et al. (1995) studied the effects of risk aversion in the newsvendor model by using expected utility functions. The optimal ordering quantity is given by maximizing the expected utility of a profit function. The decision is based on a subjective utility function of the decision maker. For certain utility functions the solution within this framework is larger or smaller than the solution in the risk neutral case; also the fraction of losses may be reduced. In particular, they presented a comparatively static effects of changes in the various price and cost parameters in the risk aversion setting. Although many of the comparative effects generally are ambiguous, some fairly simple restrictions on preferences and/or risks increases are shown to lead to qualitatively deterministic results.

Agrawal and Seshadri (2000a) considered how a risk-averse retailer, whose utility function is assumed to be increasing and concave in wealth, chooses the ordering quantity and the selling price in a single-period inventory model. They showed that in comparison to a risk-neutral retailer, a risk-averse retailer would charge a higher price and order less if a change in price affects the scale of demand; whereas, a risk-averse retailer would charge a lower price if a change in price only affects the location of the demand distribution.

Chen et al. (2003) studied a joint optimization problem on both ordering quantity and price. In the paper, a framework is proposed for incorporating risk aversion in multi-period inventory models as well as multi-period models that coordinate inventory and pricing strategies. In each case, the authors characterized the optimal policy for various risk measurements. In particular, they showed that the structure of the optimal policy for a decision maker with exponential utility functions is almost identical to the structure of the optimal risk-neutral inventory (and pricing) policies. These structural results are extended to models in which the decision maker has access to a (partially) complete financial market and can hedge its operational risk through trading financial securities. Computational results demonstrate the importance of this approach not only to risk-averse decision makers but also to risk-neutral decision makers with limited information on the demand distribution.

Wang and Webster (2007) analyzed a supply chain composed of a risk-neutral manufacturer selling a perishable product to a loss-averse retailer. They found that the independence between parameters and market demand breaks down in a buyback contract when the retailer is loss averse. Their results indicate that coordinating contracts based on the assumption of risk neutrality may result in markedly lower supply chain profit when retailers are loss averse. Manufacturers should consider the impact of loss aversion in contract design along with mitigating provisions such as a gain/loss (G/L)-sharing clause, especially when dealing with small retailers for which the assumption of risk neutrality is less likely to hold.

Wang and Webster (2009) used a kind of loss aversion utility function to model a manager's decision-making behavior in the single-period newsvendor problem. They found that if shortage cost is not negligible, then a loss-averse newsvendor may order more than a risk-neutral newsvendor. They also found that the loss-averse newsvendor's optimal ordering quantity may increase in wholesale price and decrease in retail price, which can never occur in the risk-neutral newsvendor model.

Although the utility theory is widely applied in the area of risk management in supply chain risk management, the approach itself can be criticized, since it relies on an independence axiom which may be violated (Kischka and Puppe 1992). Moreover, from a more pragmatic point of view the application of the expected utility is more difficult than expectation since the decision maker has to specify a utility function, which needs further uncertain procedure including choosing different utility function and specifying different parameters, which makes the decision-making is doubtful and inconvenient.

1.1.1.2 Mean–Variance Trade-Off

Besides the utility theory, another widely used risk measurement approach is mean–variance methodology (Xie et al. 2008). This methodlogy was firstly introduced by Markowitz (1959). Actually, the mean–variance methodology is a special form of utility function theory, noting that mean–variance objective maximization is equivalent to expected utility maximization when the utility function is quadratic as shown in Mossin (1973) and Wang and Xia (2002).

Chung (1994) and Sobel (1994) used the mean–variance methodology to study undiscounted Markov decision process (MDP). They introduced the mean–variance methodology into MDP model, and laid the foundation for further research of using the mean–variance methodology to study supply chain risk management.

Another classical and fundamental work is done by Chen and Federgruen (2000). They revisited a number of basic inventory models by using the mean–variance methodology. They showed how a systematic mean–variance trade-off analysis can be carried out efficiently, and how the resulting strategies differ from those obtained in the standard analysis. Specifically, in the infinite horizon models, they focused on the mean–variance trade-off of customer waiting time as well as the mean–variance trade-off of inventory levels. Based on these classical works, some researchers studied more complicated supply chain models by using the mean–variance methodology.

Martinez-de-Albéniz and Simchi-Levi (2006) studied the mean–variance trade-offs faced by a manufacturer in signing a portfolio of option contracts with its suppliers and having access to a spot market.

Lau and Lau (1999) studied a supply chain consisting of a monopolistic supplier and a retailer. The supplier and the retailer employ a return policy, and each of them has a mean–variance objective function. They found the optimal wholesale price and return credit for the supplier to maximize the utility.

Buzacott et al. (2003) studied a class of commitment–option supply contracts within a mean–variance framework. They showed that a mean–variance trade-off analysis with advanced reservation can be carried out efficiently. They got results demonstrating that the mean–variance objective is convex with respect to both the contract commitment portion and the option potion. Moreover, a monotonicity result with respect to the quality of information revision is also obtained. Their numerical experiments demonstrate that, in particular, the mean–variance analysis is efficient when the quality of information revision is low to medium. Also, a number of their results remain to be true for a general utility function formulation as well. They also pointed out to model this type of volume contract with down side risk would be a good direction.

Agrawal and Seshadri (2000b) considered the issue of coordination of the channel. Their aim is to increase the channel's ordering quantity to the optimal level in the risk-neutral case. They designed a two-part tariff contract, in which a mean–variance risk-averse retailer receives a side payment from a risk-neutral distributor with the remainder of the channel profit going to the distributor. The channel risk in their contract is assumed only by the distributor, whereas it is shared by all in their risk-sharing contract.

Chen and Seshadri (2006) revisited the problem proposed in Agrawal and Seshadri (2000b) and reconstructed their results when the number of retailers is infinite and their coefficient of risk aversion is drawn from a continuous distribution. They showed that this distribution uniquely determines the channel structure.

Choi and Chow (2008), Choi et al. (2008a, 2008b, 2008c), Wei and Choi (2010), and Zhao et al. (2010a) studied some related problems in inventory and supply chain within the mean–variance framework systematically. The topics involve the newsvendor problem, return policy, and channel coordination, etc. Several significant insights are presented from the comparisons with the traditional performance evaluation by the expectation maximization.

Wu et al. (2009) studied the risk-averse newsvendor model with a mean–variance objective function. They showed that the stockout cost has a significant impact on the newsvendor's optimal ordering decisions. In particular, with the stockout cost, the risk-averse newsvendor does not necessarily order less than the risk-neutral newsvendor. They illustrated this finding analytically for the case where the demand follows the power distribution.

Xing et al. (2010) investigated the strategies of a manufacturer and a retailer in a decentralized supply chain with a fully liquid B2B online exchange by using mean–variance theory. They showed that for the retailer, the B2B electronic market can serves as a speculation market or a second procurement source. Correspondingly, by using the pricing strategy, the manufacturer can achieve bully or risk-sharing intentions.

The approach of mean–variance trade-off is widely used into the area of supply chain risk management. However, as a utility function, the mean–variance methodology also suffers two drawbacks. Firstly, it equally penalizes desirable upside and undesirable downside outcomes. While in reality, a decision maker usually cares only the downside loss while maximizing profit. Hence, opposite to mean–variance, semi-variance or downside risk measurement may be applied in that case. Secondly, it is well known that if the investors hold diversified portfolios of financial assets, the relevant risk of an investment by a value-maximizing firm cannot be appropriately measured by the total variance of the return from that investment. Under such circumstances, the proper measure of the project's risk aversion of the decision maker may be different from the return-risk trade-off imposed by shareholders, and thus this criterion may imply the existence of the agency problems.

1.1.1.3 Value-at-Risk and Conditional Value-at-Risk

In the last several years, value-at-risk (VaR) has increasingly been used by financial managers as a powerful tool to measure and manage risk exposure and at the same time hedge trading and other financial policies. VaR is defined as the expected loss arising from an adverse market movement with specified probability over a period of time. Please see Duffie and Pan (1997) for an exposition of this subject. Although VaR is currently broadly used, it has some critiques. For example, Artzner et al. (1999), showed that VaR should not be used as the sole measure of risk exposure since it does not satisfy all the properties needed from a risk measure. Therefore, research of supply chain management incorporating VaR approach is rare.

A rare exception is the work by Tapiero (2005). The author solved a specific problem based on VaR risk exposure in inventory manager's concern for their "money and risk" preferences. He also showed that single-period, multi-period, and multi-products inventory problems as well as inventory with price and demand uncertainty can be considered by using the standard VaR approach. Although the mathematical problems arising from such applications of the VaR approach are difficult to solve analytically, solutions can be found by applying standard computational and simulation techniques.

Based on VaR, another criterion used in inventory management is the CVaR (Chen et al. 2003 and the literature cited there). The CVaR is the conditional expected profit under the condition that the profit is below the γ-quantile. Thus, CVaR encompasses the amount of loss. Moreover, CVaR has an advantage over VaR of being a coherent risk measure (Artzner et al. 1999). The recent works include Jammernegg and Kischka (2004), Wu et al. (2006a, 2006b), Xu et al. (2006), Gotoh and Takano (2007), Zhou et al. (2008), Chen et al. (2009), Yang et al. (2009), Goh and Meng (2009), Wu et al. (2010), Xu (2010), and Carneiro et al. (2010).

Jammernegg and Kischka (2004) work was a certain extent related to the CVaR approach. They used the well-known newsvendor model to determine the optimal performance measures for an objective function with two risk parameters, one is for the convex combination of two conditional expected values of the profit, and the other discriminates the low profits and high profits by being used as the α-quantile of the profit distribution. In contrast to CVaR models, not only the CVaR is taken into account for low profits but also the high profits are taken into consideration. The authors then provided a complete characterization for this approach with respect to the performance measures which expected profit and service level, and showed that a risk-averse inventory manager can not dominate a risk neutral or a risk seeking inventory manager. Furthermore, they provided a managerial guidelines for selecting the appropriate risk parameters of the objective function. The CVaR criterion is two-faced, as it describes risk aversion but neglects a large part of the profit distribution, or encompasses a large part of the profit distribution but approaches risk neutrality. The method proposed in their work avoids this disadvantage.

Wu et al. (2006a, 2006b) applied CVaR approach to study a pay-to-delay contract first proposed in Brown and Lee (1997), and then analyzed by

Buzacott et al. (2003). The results show advantages of using the CVaR approach over the mean–variance approach in some aspects. It avoids the disadvantage of the mean–variance methodology which equally penalizes the desirable upside and the undesirable downside outcomes. The CVaR approach also provides an explicit solution, which has better computational characteristics.

Zhou et al. (2008) proposed an optimal-order model for multi-product with CVaR constraints which is formulated as a linear programming problem. The model is simulated for the case of a newsvendor to analyze to what degree it succeeds. The comparison of return-CVaR model and the classical model shows that return-CVaR model is more flexible.

Chen et al. (2009) considered a risk-averse newsvendor with a stochastic price-dependent demand using CVaR. They investigated the optimal pricing and inventory decisions for both additive and multiplicative demand models and provided sufficient conditions for the uniqueness and the existence.

Yang et al. (2009) studied the coordination of supply chains with a risk-neutral supplier and a risk-averse retailer in a CVaR framework. They showed that the supply chain can be coordinated with the revenue-sharing, buy-back, two-part tariff and quantity flexibility contracts. Furthermore the revenue-sharing contracts are still equivalent to the buy-back contracts when the retail price is fixed. The risk-averse retailer of the coordinated supply chain can increase its profit by raising its risk-averse degree under mild conditions.

Goh and Meng (2009) established a stochastic programming formulation for supply chain risk management by using CVaR and introduced the sample average approximation method for solving the underlying stochastic model.

Wu et al. (2010) introduced the concept of CVaR as the evaluation criterion in a supply contract model. They derived the manufacturer's optimal decisions and analyzed the impact of risk aversion on the manufacturer's decisions. They obtained results that characterize the explicit relationship between the manufacturer's risk attitude and the optimal decisions. They also showed the dependence of the decision variables on the price and cost parameters, which is seldom given in the literature.

Xu (2010) considered a newsvendor model in which a risk-averse manager faces a stochastic price-dependent demand in either an additive or a multiplicative form. By adopting CVaR as the decision criterion, he investigated the optimal pricing and ordering decisions and the effects of parameter changes in such a setting.

1.1.1.4 Downside Risk

Mean–variance equally treats upside and downside outcomes and hence is inappropriate for use. Another risk measurement approach, downside risk measure, solves the problem well (Zhu et al. 2009; Yu et al. 2010). The development of downside risk measures started with Roy (1952), whose aim is to develop a practical method for determining risk-return trade-offs. He stated that an investor would prefer safety of the principal first and set some minimum acceptable returns. A general definition of downside risk is introduced by Fishburn (1977) in the form of lower partial moments (LPM) operationalized as the probability-weighted functions of deviations below some target return.

As for the issue of supply chain management, the recent works include Gan et al. (2004, 2005) and Yang et al. (2007). Gan et al. (2004, 2005) considered downside risk measure which is defined as the probability that the return is below a target level. Gan et al. (2004) took up the issue of Pareto-optimality and began with defining a coordinating contract as one that results in a Pareto-optimal solution acceptable to each agent. Their definition generalizes the standard one in the risk-neutral case and then developed coordinating contracts in three specific cases. In each case, they showed how to find the set of Pareto-optimal solutions, and then designed a contract to achieve the solutions. The authors also exhibited a case in which they obtained Pareto-optimal sharing rules explicitly, and outlined a procedure to obtain Pareto-optimal solutions. Gan et al. (2005) investigated how a supply chain involving a risk-neutral supplier and a downside-risk-averse retailer can be coordinated with a supply chain contract. They showed that the standard buy-back or revenue-sharing contracts may not coordinate such a channel. Using their earlier definition of supply chain coordination with risk-averse agent, they designed a risk-sharing contract that achieves coordination. Li et al. (2009) analyzed loan limit indicator of seasonal inventory financing in supply chain financial innovation. They analyzed loan limit consistency to risk tolerance level of downside-risk-averse banks in seasonal inventory financing. The results show that downside-risk limits can control the risk of seasonal inventory financing and make the loan consistent to risk tolerance level of banks.

1.1.1.5 Prospect Theory

Prospect theory arose from behavior finance and psychology, and was proposed in Kahneman and Tversky (1979). In the prospect theory, opposite to that in traditional utility function theory, a decision maker may have reference dependent preferences and is risk-averse over the domain of gains and risk seeking over the domain of losses. The definition of reference point is decided subjectively and circumstantially. An obvious example is that, in the context of the newsvendor problem, it is natural to assume the reference point equals current wealth. With the reference point chosen, the convex/concave utility function then predicts risk-seeking behavior in the domain of losses and risk-averse behavior in the domain of gains. Although the prospect theory has been broadly used in economics and finance, to the best of our knowledge, there is little literature introducing the prospect theory into supply chain risk management.

Schweitzer and Cachon (2000) referred to the theory, but they mainly focussed on the experimental evidence for the explanations to that subjects that always order too few of high-profit products and too many of low-profit products. In fact, the prospect theory could explain the asymmetry if subjects are relatively more risk-seeking in losses than risk averse in gains, and more specifically, could handle the case of demand disruptions by putting a large weight on the small-probability events. Therefore, the prospect theory may have a good prospect of being applied in disruption risk management for supply chains.

A rare exception is Hua et al. (2010a). They examined the loss-averse newsvendor problem with the prospect theory. The newsvendor can order goods at a regular purchase price before the selling season, and can also order at an emergency purchase price, if any, before the selling season ends. The optimal order policy and profit in this scenario are obtained and are compared with those in the classical model and the loss aversion version.

1.1.1.6 Other Risk Measurements

As shown above, CVaR is a coherent risk measurement while value-at-risk is not. The general notion of a coherent risk measure arises from an axiomatic approach for quantifying the risk of a financial position. As Artzner et al. (1999) showed, a function is said to be coherent risk measure if it satisfies some axioms. The reader is recommended to read the paper for more details. Ahmed et al. (2005) analyzed the classical newsvendor and multi-period inventory models where the objective function is a coherent risk measure. By using a dual representation, a one-to-one correspondence is established between the risk aversion formulation and the min–max type formulations. The results showed that the structure of the optimal solution of the risk aversion model is similar to that of the classical expected value problem, for both single period newsvendor problem and finite horizon dynamic inventory model. Furthermore, the authors analyzed the monotonicity properties of the optimal ordering quantity with respect to the degree of risk aversion for certain risk measures. Another risk measurement approach is suggested by Lau (1980), where the risk measurement is defined as the probability of not achieving a certain level of profit. The author analyzed the classical newsvendor model under two different objective functions. In the first objective, the focus is on maximizing the decision maker’s expected utility of total profit which has been discussed above. The other objective function is the maximization of the probability of achieving a certain level of profit. For this purpose, Lau (1980) presented formulas for the moments of the profit for a general demand distribution. In particular, the author considered a special demand distribution called Schmeiser–Deutsh and demonstrates some of its advantages. Although this criterion avoids the definition of risk altogether, it is not at all clear how a value-maximizing firm should specify the cut-off rate of profit.

1.1.2 Newsvendor Model

As a fundamental problem in stochastic inventory control and supply chain management, the newsvendor problem has been extensively studied for a long time and applied in a broad array of business settings with the objective of expectation performance. This section focuses on the risk analysis of newsvendor model from a risk management perspective.

1.1.2.1 Model Formulation

In the newsvendor problem, a manager is assumed to sell a single product during a short selling season facing stochastic demand. He has only one ordering opportunity before the selling season, and no further replenishments are allowed. If too much is ordered, he will incur an overage cost, whereas if too little is ordered, sales are lost with underage cost. Therefore, he must balance the shortage costs of ordering too little against the overage costs of ordering too much to obtain his maximal profit.

Let Q be the newsvendor's ordering quantity. Let D be the future stochastic demand during the selling season. Let F be the cumulative distribution function and f the probability density function of demand, respectively. It is assumed that F is a continuous and strictly increasing function and f is a nonnegative function. The purchasing cost of the product is c per unit, the selling price is r per unit, the salvage value of any unsold product is s per unit, and the stockout cost of unsatisfied demand is p per unit. To avoid unrealistic and trivial cases, it is assumed that $0 < s < c < r$ and $0 < p$. Throughout the book, we use the following notation: for any number a and b, $a^- = \min\{a, 0\}$, $a^+ = \max\{a, 0\}$, $a \vee b = \max\{a, b\}$ and $a \wedge b = \min\{a, b\}$.

1.1.2.2 Risk-Neutral Case with Expectation Maximization

This section considers a risk-neutral newsvendor with expectation maximization as its objective function, which is always treated as a benchmark. The objective function is $\max_{Q \geq 0}\{E[\Pi(Q)]\}$.

The optimal ordering quantity Q^* is given by $F(Q^*) = \frac{r+p-c}{r+p-s}$, which is the well known newsvendor solution.

1.1.2.3 Risk-Averse Case with Mean–Variance Trade-Off

A newsvendor model within a mean–variance framework was proposed by Chen and Federgruen (2000). A newsvendor needs to balance between the mean and the variance of his random profit. The objective function with mean–variance trade-off is $\max_{Q \geq 0}\{E[\Pi(Q)] - \alpha \text{Var}[\Pi(Q)]\}$ with α $(\alpha \geq 0)$ as the risk parameter to balance between the return and the risk.

Theorem 1.1.1. *The mean function is a concave function of Q and the variance function is a non-decreasing function of Q. Moreover, the optimal order quantity*

Q^*_{MV} that balance the mean and variance (it is equal to maximize the expected utility under certain condition) is less than or equal to the newsvendor solution Q^*.

For details of the above result, please see Chen and Federgruen (2000).

1.1.2.4 Mean–Variance Trade-Off with Stockout Cost

Wu et al. (2009) incorporated the stockout cost in the above newsvendor model and finds out that the properties of the variance function and the mean–variance trade-off may be very different from those of the model without stockout cost. Moreover, some results obtained in the previous literature may no longer be valid.

When the demand faced by the newsvendor follows a power distribution (for example, $F_D(x) = x^k, 0 \leq x \leq 1, k > 0$), there is the following result.

Theorem 1.1.2. *For power distributed demand, there exists one unique minimizer Q^0_P for $\mathrm{Var}[\pi(Q)]$ on $(0, 1)$, where $\mathrm{Var}[\pi(Q)]$ is decreasing in $[0, Q^0_P]$ and increasing in $[Q^0_P, 1]$. Moreover, there exists a critical value k^* with $0 < k^* < 1$ and the newsvendor's optimal order quantity is distinguished by three cases as follows:*

(1) If $0 < k < k^*$, then $Q^* < Q^0_P$ and the optimal order quantity is in the interval $[Q^*, Q^0_P]$.
(2) If $k = k^*$, then $Q^* = Q^0_P$ and the optimal order quantity is exactly Q^*.
(3) If $k > k^*$, then $Q^* > Q^0_P$ and the optimal order quantity is in the interval $[Q^0_P, Q^*]$.

Theorem 1.1.2 leads to the insightful result that the risk-averse newsvendor may order less than the risk-neutral newsvendor, if the stockout cost is positive, which will never happen if the stockout cost is zero. This result disproves a claim by Lau and Lau (1999).

For details of the above result, please see Wu et al. (2009).

1.1.2.5 Risk-Averse Case with Utility Maximization

A risk-averse newsvendor, who would like to select maximizing his expected utility function, is considered in Keren and Pliskin (2006). The objective function with utility maximization is given by $\max_{Q \geq 0}\{E[U(\Pi(Q))]\}$, which can be simplified as $\max_{Q \geq 0}\{E[U(r(Q \wedge D) + s(Q - D)^+ - p(D - Q)^+ - cQ)]\}$.

With the concavity property, the first order condition can be given by

$$\frac{r + p - c}{c - s} = \frac{\int_0^Q u'[(r - s)x + (s - c)Q]f(x)\mathrm{d}x}{\int_Q^\infty u'[(r + p - c)Q - px]f(x)\mathrm{d}x}. \tag{1.1}$$

Under certain assumptions of utility function and/or demand distribution, some results may be obtained.

Theorem 1.1.3. *If the demand for newsvendor is uniformly distributed over [A, B], a risk-averse newsvendor with a uniformly more concave utility function sets his optimal ordering quantity Q^*_{CU} to a lower value than a less risk-averse newsvendor. We note here that a more concave utility function is more risk-averse according to Pratt's (1964) measure of risk $r(x) = -\frac{u''(x)}{u'(x)}$.*

Theorem 1.1.4. *A risk-averse newsvendor with a concave utility function sets Q^*_{CU} less than a newsvendor who is risk-neutral.*

For details of the above results, please see Keren and Pliskin (2006).

A multi-period (then an infinite horizon) stochastic inventory model with respect to risk-averse criterion by using utility function is studied in Bouakiz and Sobel (1992). The utility function is assumed to be an exponential one given by $u(x) = \mathrm{e}^{-\mu x}$ where $\mu > 0$. The objective function of M period planning horizon is $\max E[\mathrm{e}^{\mu(B(M))}]$, where $B(M)$ is the present value of costs incurred during an M-period planning horizon.

In period m, let x_m and y_m be the respective inventory levels before and after additional goods are ordered (and delivered). It is assumed that: (1) the demand in period m, D_m is unknown when y_m is selected, (2) $D_1, D_2, \ldots, D_m$ are independent and identically distributed nonnegative random variables.

The present value of costs during M period planning horizon is given by

$$B(M) = \sum_{i=1}^{M} \beta^{i-1}[c(y_i - x_i) + g(y_i, D_i)] - c\beta^M v(y_m, D_m), (0 \leq \beta < 1). \quad (1.2)$$

It is proved in Bouakiz and Sobel (1992) that the M period problem for each M has an optimal base stock policy. Specifically, there is a sequence of functions $y_i(\cdot)$, such that the optimal base stock level in period i, is $y_i(\beta^{i-1}\mu)$, $i = 1, 2, \ldots, M$. It is also proved that the infinite period problem has an optimal base stock policy. Specifically, there is a sequence of functions $y(\mu)$, $y(\beta\mu)$, $y(\beta^2\mu)$, ..., to characterize the optimal policy. Moreover, for sufficient large m, $y(\beta^{n-1}\mu)$ is an optimal risk-neutral base stock level.

For details of the above result, please see Bouakiz and Sobel (1992).

1.1.2.6 Risk-Averse Case with Conditional Value-at-Risk

A risk-averse newsvendor with CVaR approach was studied by Chen et al. (2003). A close-form solution by using a definition presented in Chen et al. (2003) is obtained. The objective function is $\max_{Q \geq 0} C_\gamma \pi(x, Q)$, where $C_\gamma \pi(x, Q) = \max_{v \in R}\{v + \frac{1}{\gamma} E[(\pi(x, Q) - v)^-]\}$, v is a real number, E is the expectation taken on the random demand D, Q is the ordering quantity, and γ $(0 < \gamma < 1)$ is the risk aversion parameter.

Theorem 1.1.5. *A risk-averse newsvendor with CVaR approach orders* $F^{-1}\left(\frac{\gamma(r+p-c)}{r+p-s}\right)$*, which is less than a risk-neutral newsvendor. Moreover, the more averse the newsvendor is, the less quantity he will order.*

For details of the above result, please see Chen et al. (2003).

1.1.3 Supply Chain Contract: A Pay-to-Delay Capacity Reservation Contract

There is numerous literature devoted to supply chain contracts that coordinate a supply chain with risk-neutral agents. For these literature, the reader is referred to Tayur et al. (1999), and the survey by Cachon (2003), and the references therein. In the following text, we focus on the literature dealing with supply chain contract with risk-averse agents. Specifically, a pay-to-delay capacity reservation contract is analyzed in both risk-neutral case and risk-averse case as an example to derive the impact of risk aversion on the optimal supply chain decisions.

1.1.3.1 Model Formulation

A pay-to-delay capacity reservation contract in which capacity may be reserved in the form of options was given by Brown and Lee (1997). It is an agreement between a downstream manufacturer and an upstream supplier. With such a contract, the manufacturer can make two procurement decisions in the whole time horizon. At t_1, long before the selling season begins, the manufacturer should decide to buy the commitment capacity y with a cost c_f per unit, and reserve the option capacity $z-y$ with a cost c_o per unit. At t_2, with the demand forecasting, the manufacturer makes his final decision whether to use the option or not and how many options he should use. Let w be the manufacturer's final ordering quantity, where $w \geq y$. For any option exercised, the manufacturer pays c_e per unit. Besides purchasing by contract in the second stage, the manufacturer can also purchase from a spot market with a price c_p for each unit. At the end of the selling season, the manufacturer gains revenue p per unit and sells the remaining inventory for a salvage value s for each unit. To ensure realistic decisions, it is assumed that $s < c_f < c_e + c_o < c_p < p$ and $s < c_e$.

1.1.3.2 Risk-Neutral Case with Expectation Maximization

With the expected profit maximization as the criterion, the objective function is $\max_{0\leq y\leq z}\{E_{D_1}[\pi_1(y,z)]\}$. The main results are given as follows:

Theorem 1.1.6. *For any given commitments portion y and option portion z, the optimal ordering quantity $w^*(y, z, D_1)$ in the second stage is given by*

$$w^*(y,z,D_1) = \begin{cases} z \vee w_2, & \text{if } z < w_1, \\ w_1, & \text{if } y \le w_1 \le z, \\ y, & \text{otherwise,} \end{cases}$$

where w_1 and w_2 are two order-up-to levels given by $w_2(D_1) = H^{-1}\left(\frac{p-c_{\mathrm{p}}}{p-s}|D_1\right)$, and $w_1(D_1) = H^{-1}\left(\frac{p-c_{\mathrm{e}}}{p-s}|D_1\right)$.

Theorem 1.1.7. *The optimal commitments portion y^* and the optimal option reservation z^* are given by*

$$(y^*, z^*) = \begin{cases} (y_1, z_1), & \text{if } y_1 < z_1, \\ (y_2, y_2), & \text{else,} \end{cases}$$

where y_1, z_1, and y_2 are the optimal solutions satisfying the first order conditions, respectively.

For details of the above results, please see Brown and Lee (1997).

1.1.3.3 Risk-Averse Case with Mean–Variance Trade-Off

Buzacott et al. (2003) studied the above pay-to-delay capacity reservation contract within the mean–variance framework. It is assumed that the decision maker in stage 1 is risk-neutral and the decision maker in stage 2 is risk-averse with mean–variance criterion.

The objective function at stage 2 is to choose optimal w to maximize

$$\Pi_2(y,z,w,D_1) = E[\pi_2(y,z,w|D_1)] - \alpha \mathrm{Var}[\pi_2(y,z,w|D_1)]. \tag{1.3}$$

The objective function at stage 1 is to choose optimal y and z to maximize

$$\max_{0 \le y \le z} \Pi_1(y,z) \tag{1.4}$$

where

$$\Pi_1(y,z) = E[E[\pi_2(y,z,w^*,D_2|D_1)]] - c_{\mathrm{o}}(z-y) - c_{\mathrm{f}}y. \tag{1.5}$$

The main results are given as follows:

Theorem 1.1.8. *For any given commitments portion y and option portion z, the optimal ordering quantity $w^*(y,z,D_1)$ in the second stage is given by*

$$w^*(y,z,D_1) = \begin{cases} z \vee w_{\mathrm{p}}(D_1), & \text{if } y < z < w_{\mathrm{e}}(D_1), \\ w_{\mathrm{e}}(D_1), & \text{if } y \le w_{\mathrm{e}}(D_1) \le z, \\ y, & \text{otherwise}, \end{cases} \tag{1.6}$$

where $w_{\mathrm{e}}(D_1)$ and $w_{\mathrm{p}}(D_1)$ are two order-up-to levels given as follows:

$$[1 - H(w_{\mathrm{p}}(D_1)|D_1)]\left[1 - 2\alpha(p-s)\int_0^{w_{\mathrm{p}}(D_1)} H(x|D_1)\mathrm{d}x\right] = \frac{c_{\mathrm{p}} - s}{p - s},$$

$$[1 - H(w_{\mathrm{e}}(D_1)|D_1)]\left[1 - 2\alpha(p-s)\int_0^{w_{\mathrm{e}}(D_1)} H(x|D_1)\mathrm{d}x\right] = \frac{c_{\mathrm{e}} - s}{p - s}. \tag{1.7}$$

Theorem 1.1.9. *Assume that the updating demand $D_2|D_1$ is stochastic increasing in the observed information D_1. For any $0 \le y \le z$,*

$$L(y) \le U_1(z) \le U_2(z) \tag{1.8}$$

and $L(\cdot)$, $U_1(\cdot)$ and $U_2(\cdot)$ are non-decreasing functions. Moreover, the optimal purchase quantity w^* at stage 2 can be written as

$$w^*(y,z,D_1) = \begin{cases} w_{\mathrm{p}}(D_1), & \text{if } U_2(z) < D_1 < \infty, \\ z, & \text{if } U_1(z) < D_1 \le U_2(z), \\ w_{\mathrm{e}}(D_1), & \text{if } L(y) < D_1 \le U_1(z), \\ y, & \text{if } D_1 \le L(y). \end{cases} \tag{1.9}$$

Theorem 1.1.10. *The optimal commitments portion y^* and the optimal option reservation z^* are given by*

$$(y^*, z^*) = \begin{cases} (\hat{y}, \hat{z}), & \text{if } \hat{y} < \hat{z}, \\ (\bar{y}, \bar{y}), & \text{if } \hat{y} \ge \hat{z}. \end{cases} \tag{1.10}$$

For details of the above results, please see Buzacott et al. (2003).

1.1.3.4 Risk-Averse Case with Conditional Value-at-Risk

Motivated by the work and suggestions presented in Buzacott et al. (2003), Wu et al. (2006a, 2006b, 2010) used the CVaR approach to analyze the pay-to-delay capacity

Table 1.1 Property comparisons of the three different criteria

	w^*	y^*	z^*
Expectation	Explicit solution	Implicit solution	Implicit solution
Mean–variance	Implicit solution	Implicit solution	Implicit solution
CVaR	Explicit solution	Implicit solution	Implicit solution

reservation contract model as a measure of downside risk. The ordering quantity in the second stage is mainly used to cover the demand uncertainty, while, the ordering quantity in the first stage is the main source to meet most of the expected demand. The main results are given as follows:

Theorem 1.1.11. *For any given commitments portion y and option portion z, the optimal ordering quantity $w^*(y, z, D_1)$ in the second stage is given by*

$$w^*(y,z,D_1) = \begin{cases} z \vee w_{\rm p}(D_1), & \text{if } y < z < w_{\rm e}(D_1), \\ w_{\rm e}(D_1), & \text{if } y \le w_{\rm e}(D_1) \le z, \\ y, & \text{otherwise}, \end{cases}$$

where $w_{\rm e}(D_1)$ and $w_{\rm p}(D_1)$ are two order-up-to levels given by

$$\begin{cases} w_{\rm p}(D_1) = H^{-1}\left(\gamma \dfrac{p-c_{\rm p}}{p-s} | D_1\right), & \text{if } y < z < w, \\ w_{\rm e}(D_1) = H^{-1}\left(\gamma \dfrac{p-c_{\rm e}}{p-s} | D_1\right), & \text{if } y \le w \le z. \end{cases}$$

The solution structures in the two stages are summarized in Table 1.1.

When the manufacturer is allowed to use options after the demand is realized, there is the following result.

Theorem 1.1.12. *The manufacturer's optimal ordering strategy is dependent on his risk attitude. If $\frac{c_{\rm e}+c_{\rm o}-c_{\rm r}}{c_{\rm e}} \le \frac{p-(c_{\rm e}+c_{\rm o})}{p-c_{\rm e}}$, then the optimal strategy is $y^* = F^{-1}\left(\gamma \frac{c_{\rm e}+c_{\rm o}-c_{\rm r}}{c_{\rm e}}\right)$ and $z^* = F^{-1}\left(\gamma \frac{p-(c_{\rm e}+c_{\rm o})}{p-c_{\rm e}}\right)$. If $\frac{c_{\rm e}+c_{\rm o}-c_{\rm r}}{c_{\rm e}} > \frac{p-(c_{\rm e}+c_{\rm o})}{p-c_{\rm e}}$, then the optimal strategy is $y^* = z^* = F^{-1}(\gamma \frac{p-c_{\rm r}}{p})$.*

For details of the above results, please see Wu et al. (2006a, 2006b, 2010).

1.1.4 Supply Chain Coordination

There are numerous works on supply chain coordination with assumption that the agents in the supply chain are risk neutral (Shu et al. 2010; Zhao et al. 2010b). However, seldom literature deals with supply chain coordination with risk-averse agents. In this section, the main results of Gan et al. (2004) and Wang and Webster (2007) are recalled.

1.1.4.1 Pareto-Optimality Criterion

Gan et al. (2004) used the Pareto-optimality criterion, derived from the group decision theory, to evaluate the supply chain's performance.

Definition 1.1.13. Supply Chain Coordination. A contract agreed upon by the agents of a supply chain is said to coordinate the supply chain if the optimizing actions of the agents under the contract:

1. Satisfy each agent's reservation payoff constraint.
2. Lead to an action pair $(s^*, \theta^*(s^*))$, that is, Pareto-optimal.

Moreover, three specific cases in supply chain management are considered.

Case 1: One risk-neutral supplier and one retailer averse to downside risk.

Theorem 1.1.14. *If the supplier is risk neutral and the retailer maximizes his expected profit subject to a downside risk constraint, then a feasible action pair $(s, \theta(s))$ is Pareto-optimal if and only if the supply chain's expected profit is maximized over the feasible set.*

Case 2: Risk-averse supplier and retailer both with mean–variance trade-off.

Theorem 1.1.15. *An action pair (s^*, θ^*) is Pareto-optimal if and only if $s^* = \arg\max_{s\in S}\left[E\Pi(s) - \left(\frac{1}{\sum_j \frac{1}{\lambda_j}}\right)V(\Pi(s))\right]$ and almost surely $\Pi_i(s, \theta^*(s)) = \frac{\frac{1}{\lambda_i}}{\left(\sum_j \frac{1}{\lambda_j}\right)}\Pi(s) + \Pi_i$, $i = 1, 2, \ldots, N$.*

Case 3: Risk-averse supplier and retailer both with concavity utility.

Theorem 1.1.16. *An action pair $(s^*, \theta^*(s^*))$ is Pareto-optimal if and only if $s^* = \arg\max_{s\in S} E\exp\left(-\frac{\lambda_r\lambda_s\Pi(s)}{\lambda_r+\lambda_s}\right)$, and, almost surely,*

$$\Pi_r(s^*, \theta(s^*)) = \frac{\lambda_s}{\lambda_r + \lambda_s}\Pi(s^*) - \lambda \ln\frac{\alpha_s\lambda_s}{\alpha_r\lambda_r},$$

and

$$\Pi_s(s^*, \theta(s^*)) = \frac{\lambda_r}{\lambda_r + \lambda_s}\Pi(s^*) + \lambda \ln\frac{\alpha_s\lambda_s}{\alpha_r\lambda_r},$$

where $\alpha_r, \alpha_s > 0$, $\alpha_r + \alpha_s = 1$.

For details of the above results, please see Gan et al. (2004).

1.1.4.2 Loss Aversion Function

Wang and Webster (2007) considered a channel coordination of a supply chain with a risk-neutral manufacturer and a loss-averse retailer.

Table 1.2 Representative literature categorized by risk tools

Mean–variance	Utility function	Var and CVar	Other measurements
Chen and Federgruen, Chung (1994), Sobel (1994), Lau and Lau (1999), Agrawal and Seshadri (2000b), Martinez-de-Albéniz and Simchi-Levi (2006), Buzacott et al. (2003), Wu et al. (2009), Chen and Seshadri (2006), Choi and Chow (2008), Choi et al. (2008a, 2008b, 2008c), Wei and Choi (2010)	Bouakiz and Sobel (1992), Eeckhoudt et al. (1995), Agrawal and Seshadri (2000a), Chen et al. (2003), Wang and Webster (2007, 2009), Keren and Pliskin (2006)	Carneiro et al. (2010), Tapiero (2005), Chen et al. (2003), Jammernegg and Kischka (2004), Xu et al. (2006), Gotoh and Takano (2007), Wu et al. (2006b, 2010), Zhou et al. (2008), Chen et al. (2009), Yang et al. (2009), Goh and Meng (2009), Xu (2010)	Lau (1980), Schweitzer and Cachon (2000), Ahmed et al. (2005), Gan et al. (2004, 2005), Yang et al. (2007)

The retailer's loss aversion utility function is

$$U(W) = \begin{cases} W - W_0, & W \geq W_0, \\ \lambda(W - W_0), & w < W_0, \end{cases}$$

where $\lambda \geq 1$ is the loss-aversion level, W is the retailer's initial wealth, and W_0 is the retailer's final wealth. If $\lambda = 1$, then the retailer is risk neutral. The objective function is to maximize the retailer's expected utility function given by $\max\limits_{Q>0} EU[\pi(Q, W)]$.

For details of the above result, please see Wang and Webster (2007).

1.1.5 Concluding Remarks

As we have discussed above, a large amount of the literatures has been devoted to the study of risk analysis of supply chain models. The existing literature has been categorized into three kinds of models including newsvendor, supply chain contract, and supply chain coordination. In the following, the literature are summarized according to risk management tools in Table 1.2 for a general view.

1.2 Disruption Management in Supply Chains

1.2.1 Introduction

Uncertainties mentioned in Sect. 1.1 can be represented by probability distributions. However, these distributions lack attributes to represent rare and extreme events, known as disruption risk. Supply chain disruptions are unplanned and unanticipated events that disrupt the normal flow of goods and materials within a supply chain (Hendricks and Singhal 2003; Kleindorfer and Saad 2005) and, as a consequence, expose firms within the supply chain to operational and financial risks (Stauffer 2003). For example, Ford Motor Company was forced to intermittently idle production at five of its assembly plants due to delays at US land borders after the September 11 terrorist attacks (Rice and Caniato 2003). The 2002 longshoreman union strike at a US West Coast port interrupted transshipments and deliveries to many US-based firms, with port operations and schedules not returning to normal until 6 months after the strike had ended. The one-month-long brutal winter weather caused by heavy snowfalls that occurred in large tracts of China in January 2008 is a few recent reminders of the potential for significant disruptions (Wang et al. 2010). It caused transport chaos and disrupted supplies of energy and food. The delivery dates of the goods on most delivery trucks were way overdue. It can be catastrophic for a short-life product if the disruption coincides with the selling season.

In most of the cases, the business impact associated disruption risks is much greater than that of the operational risks (Tang 2006a). Chopra et al. (2007) showed that bundling the two sources of uncertainty results in higher inventory and supply chain costs than optimal. These errors get exaggerated as the probability of disruption grows. Hence, it is important to decouple operational risks and disruption risk when planning appropriate mitigation strategies.

In the last few years, supply chain disruptions have received increasing attention from academics and practitioners. The reason for this is twofold basically. Firstly, the real world is increasingly more uncertain and vulnerable. According to many studies, the historical data indicates that the total number of natural and man-made disasters has risen dramatically over the last 10 years. Secondly, the vulnerability of supply chains disruption has increased. Many supply chain managers strive to seek efficiency improvements through “lean” solutions. These “lean” solutions have created longer and more complex global supply chains in which the domino effects of disruptions have been exacerbated (Christopher and Lee 2004).

It is not surprising that there has been a large number of literature on supply chain disruption risks since supply chain disruption risk is unavoidable and can potentially be so harmful and costly. In the following, some literature are reviewed to trace approaches for exploring supply chain disruption risks according to the classical risk management paradigm, i.e., risk identifying, risk assessing, and risk mitigating.

1.2.2 Disruption Risk Identifying

Disruption risk identifying is to classify supply chain disruption risks into different categories and to identify drivers of these different categories. It is the fundamental step in managing disruption risk in supply chains. However, most of the current studies on disruption risk identifying focus on the different categories of disruption risks and not on the drivers of them. The study on disruption risk identifying was conducted mainly by employing case study methodology and interview methodology.

Table 1.3 Failure modes

Failure mode	Description
Disruption in supply	Delay or unavailability of materials from suppliers, leading to a shortage of inputs that could paralyze the activity of the firm
Disruption in Transportation	Delay or unavailability of the transportation infrastructure, leading to the impossibility to move goods, either inbound or outbound
Disruption at Facilities	Delay or unavailability of plants, warehouses and office buildings, hampering the ability to continue operations
Freight breaches	Violation of the integrity of cargoes and products, leading to the loss or adulteration of goods (can be due either to theft or tampering with criminal purpose, e.g., smuggling weapons inside containers)
Disruption in communications	Delay or unavailability of the information and communication infrastructure, either within or outside the firm, leading to the inability to coordinate operations and execute transactions
Disruption in demand	Delay or disruption downstream can lead to the loss of demand, temporarily or permanently, thus affecting all the companies upstream

Source: MIT research group on "Supply Chain Response to Global Terrorism", Sheffi et al. (2003)

After the attack on 9/11/2001, firms are starting to realize that the disruption risk from terrorism is affecting their ability to successfully manage their supply chain. MIT research group on "Supply Chain Response to Global Terrorism" have shown that firms usually focus on the type of disruption, i.e., the limited ways in which the disruption affects the supply-chain, and not its source. The group distinguishes 6 different types of failure modes from the perspective of a single firm as listed in Table 1.3 (Sheffi 2001; Rice et al. 2003).

Christopher and Peck (2004) presented a comprehensive framework in which disruption risks are divided into three categories, namely disruption risks internal to the firm, disruption risks external to the firm but internal to the supply chain network (SCN), and disruption risks external to the network. Furthermore, the first category is sub-divided to process disruption risk and control disruption risk. The second category is sub-divided to demand disruption risk and supply disruption risk. The final category indicates those events which may of course directly impact upon some or all agents of the supply chains, or indeed on the marketplace itself. They may be the result of sociopolitical, economic or technological events.

From the above literature review, it can be found that most of the studies focus on classifying supply chain disruption risks into different categories and not on identifying drivers of them. However, it is not adequate for supply chain managers to plan for disruptions. For example, although the effect on the supply chain of a terrorist attack can be very similar to those of a natural disaster, the expected duration and the occurrence likelihood of it may be different. Terrorism is equally likely to happen at any time of year. However, any particular natural disaster, e.g., storm, is more likely to happen in some parts of the year and less likely in others (Ross et al. 2008). Hence, the strategies taken by mangers for mitigating them should also be different if the disruption source is different.

To summarize, not only classifying of disruption risk categories but also identifying of disruption risk sources are important research issues. Thus, processes and tools need to be developed that will help managers to identify drivers of disruptions in their supply chains in order to mitigating the disruption risk by more appropriate strategies.

1.2.3 Disruption Risk Assessing

Disruption risk assessing is to estimate the likelihood of each type of major disruption to occur and to assess potential loss due to a major disruption. Both the probability and the magnitude of supply disruption are important to overall perceptions of supply disruption risk (Ellis et al. 2010). This is a critical step in managing the supply chain disruption risk. Most companies recognize the importance of risk assessment programs and use different methods, ranging from formal quantitative models to informal qualitative plans, to assess supply chain disruption risks (Rice and Caniato 2003; Zsidisin et al. 2004). However, it is difficult to obtain good estimates of the probability of the occurrence of any particular disruption. Some of the current studies focus on the measure of potential impact of each disaster by empirical analysis. The following three papers which make empirical analysis of the negative economic impact of supply chain disruptions maybe contributed to this issue.

Hendricks and Singhal (2003) estimated the short-term effects of supply chain disruptions such as production or shipment delays on the shareholder wealth. Their research is based on a sample of 519 disruptions announcements made by firms during 1989–2000. They showed that the mean decrease in firm market value is 10.28% over the two-day period after the public announcement of a supply chain disruption.

Hendricks and Singhal (2005) investigated the long-run effects and risk effects due to supply chain disruptions. Based on a sample of 827 disruptions announced by publicly traded firms during 1989–2000. They found that companies suffering from supply chain disruptions experienced 33–40% lower stock returns relative to their industry benchmarks over a 3-year time period that starts 1 year before and ends 2 years after the disruption announcement date.

Kleindorfer and Saad (2005) considered empirical results from a rich data set covering the period 1995–2000 on accidents in the US Chemical Industry. Based on these results, they developed a conceptual framework that trades off risk mitigating investments against potential losses caused by supply disruption.

The results obtained from the above empirical analysis show that supply disruption can have grave financial consequences for firms relying on suppliers for crucial items. The detrimental effects of various major disruptions may motivate firms to examine ways to identify supply chain strategies that are efficient and yet resilient to major disruptions. However, disruptions are low-probability events whose non-stationary probabilities may be difficult to estimate. In the absence of accurate measures of the probability of an occurrence of a major disruption, many firms invested little time or resources in managing supply chain disruption risks even though they learned the potential detrimental impact of a disruption (Rice and Caniato 2003; Zsidisin et al. 2004). Surveys confirm this perplexing dichotomy. For example, according to a study conducted by Computer Sciences Corporation in 2003, 43% of 142 companies reported that their supply chains are vulnerable to disruptions, and 55% of these companies have no documented contingency plans (Poirier and Quinn 2003).

From the above literature review, it can be found that disruption risk is often ignored in practices because it is difficult to predict an occurrence of a major disruption. Thus, the issue of estimating the likelihood of disruption to occur should be addressed by future research. Effective tools should be developed to estimate the likelihood and duration of disruption. Obviously this issue relates to the issue of identifying disruption source properly. Correctly identifying disruption source is the foundation of good predicting of the occurrence.

1.2.4 Disruption Risk Mitigating

Disruption risk mitigating is to mitigate the uncertainties identified from the various disruption risk sources by undertaking some strategic moves deliberately (Miller 1992). It relates properly to the execution period of disruption risk management in supply chains.

There are many strategies for mitigating disruption risks. For example, Oke and Gopalakrishnana (2009) suggested some kinds of measures to mitigate supply risks, such as better planning and co-ordination of supply and demand, flexible capacity, identifying supply chain vulnerability points and having contingency plans, and multiple sourcing strategy, etc. In general, strategies for mitigating disruption risk can be classified into four types: contingency planning, robust optimization, stochastic models, and real-time disruption management.[2]

1.2.4.1 Contingency Planning

Contingency planning uses a pre-allocated set of resources and a well-documented recipe to cope with each scenario identified during the planning stage. This approach is completely scenario-based and usually includes the following elements: identifying the threshold for action (the trigger event); identifying the specific event that would cause a disruption to supply chain; identifying the key personnel in the contingency plan; identifies contingency options.

The key step for contingency plan making is to identify each possible scenario in supply chain. It is also the most difficult step for a complex supply chain in which the number of the future scenarios is vast. Most of the current analysis on contingency planning are qualitative.

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[2]The "real-time disruption management" strategy is also referred as "disruption management" named by Clausen et al. (2001), and sometime as "real-time operations control and recovery".

Table 1.4 Robust supply chain strategies

Robust Supply Chain Strategy	Main objective	Benefit(s) under normal circumstances	Benefit(s) after a major disruption
Postponement	Increases product flexibility	Improves capability to manage supply	Enables a firm to change the configurations of different products quickly
Strategic Stock	Increases product availability	Improves capability to manage supply	Enables a firm to respond to market demand quickly during a major disruption
Flexible Supply Base	Increases supply flexibility	Improves capability to manage supply	Enables a firm to shift production among suppliers promptly
Make-and-Buy	Increases supply flexibility	Improves capability to manage supply	Enables a firm to shift production between in-house production facility and suppliers rapidly
Economic Supply Incentives	Increases product availability	Improves capability to manage supply	Enables a firm to adjust order quantities quickly
Flexible Transportation	Increases flexibility in transportation	Improves capability to manage supply	Enables a firm to change the mode of transportation rapidly
Revenue Management	Increases control of product demand	Improves capability to manage demand	Enables a firm to influence the customer product selection dynamically
Dynamic Assortment Planning	Increases control of product demand	Improves capability to manage demand	Enables a firm to influence the demands of different products quickly
Silent Product Rollover	Increases control of product exposure to customers	Improves capability to manage supply and demand	Enables a firm to manage the demands of different products swiftly

Source: "Robust Strategies for Mitigating Supply Chain Disruptions", Tang (2006b)

1.2.4.2 Robust Optimization

Robust optimization is another approach to handle uncertainty in the planning stage. The philosophy of robust optimization is to help firms to reduce cost and/or improve customer satisfaction under normal circumstances and to sustain their operations during and after a major disruption. In robust optimization, future uncertainties are modeled by a set of scenarios. A typical robust planning process usually includes the following elements: identifying the potential disruptive scenarios; choosing a robustness criterion appropriate for the decision maker; incorporating information and measure in planning to generate a robust plan; carrying out the plan (Yu and Qi 2004).

Tang (2006b) presented some "robust" strategies as listed in Table 1.4.

Klibi et al. (2010) developed a comprehensive SCN design methodology under uncertainty. Through an analysis of supply chains the authors reviewed key random environmental factors, uncertainty sources and risk exposures, and discussed the nature of major disruptive events threatening SCN. They argued for the assessment of SCN robustness as a necessary condition to ensure sustainable value creation. Several definitions of robustness, responsiveness and resilience are reviewed, and the importance of these concepts for SCN design is discussed. This work contributes framing of the foundations for a robust SCN design methodology.

The advantages of robust strategies are that they can guarantee the performance of a supply chain regardless of the occurrence of major disruptions. However, a robust plan will sacrifice average performance, especially when the probability of some disruptive events may be very small (Yu and Qi 2004). So managers must carefully analyze the trade-offs between higher costs for implementing these robust strategies and negative economic consequences associated with disruptions. Till now, most of the current analysis on robust optimization are qualitative.

1.2.4.3 Stochastic Models

Stochastic model is a typical method of generating an operational plan within an uncertain environment when the precise probability distribution of future uncertainty is known in advance. An operational plan or policy based on stochastic models usually contains the following steps: building stochastic models to describe the future uncertainty; finding the optimal policy so that the future output is optimized in terms of the average output; executing the plan by taking the obtained policy for each scenario that occurs (Yu and Qi 2004).

The most common type of stochastic disruption appearing in the literature is that of supply disruption. In the existing supply-disruption models, the uncertain source of supply disruption is from the state of supplier. The supplier is either up or down. If the supplier is up, the order will be delivered on time. If the supplier is down, no order can be supplied. The inter-failure time and/or the repair time are uncertain. In what follows, these studies are classified into two categories, based on the number of supplier: singular supplier models and multi supplier models.

With no alternative source available for single-supplier systems, inventory mitigation is the only disruption management strategy under consideration in these papers. The focus of these papers is to identify the optimal inventory policy or the optimal parameters for particular inventory policy when there is supply disruption risk.

The singular supplier models are further sub-divided to deterministic demand models and stochastic demand models based on the types of demand. Recent singular supplier models with deterministic demand include, but are not limited to, Parlar and Berkin (1991), Parlar and Perry (1995), Moinzadeh and Aggarwal (1997), Arreola-Risa and DeCroix (1998), and Abboud (2001). In these models, the demand faced by system is assumed to be deterministic and the supply source is subject to random failure. The uncertainty of supply is characterized by exponentially distributed up and down periods, constant failure rate, and general randomly distributed repair times, or geometrically distributed inter-failure time and repair time. The policies identified in the models include, EOQ, EPQ, (Q, r, T), and (s, S) policy. Their analysis yields the optimal values of the policy parameters, and provides insight into the optimal inventory strategy. There are also a few of literature to consider stochastic demand in addition to stochastic supply, e.g., Gupta; Parlar; Özekici and Parlar; Burke et al.; Lewis et al.; Li et al. (1996, 1997, 1999, 2004, 2006, 2007), and Ross et al. (2008). These models assume not only the unreliable supplier is up or down for random durations, but also the unit demands are also stochastic. For example, Lewis et al. (2006) considered a periodic-review inventory model in which the lead time probability distribution is dependent on the state of a completely observed, exogenous Markov chain. They also analyzed the effect of a possible major supply chain disruption (e.g., a border closure) on a firm's long run average cost. Ross et al. (2008) considered a firm that faces random demand and random supply. The probability of supply disruption, as well as the demand intensity, can be time dependent. They modeled this problem as a two-dimensional non-homogeneous continuous-time Markov chain (CTMC). The model is solved numerically to obtain the total cost under various ordering policies. They proposed several such policies, some of which are time dependent while others are not. They found that non-stationary policies can provide an effective balance of optimality (low cost) and robustness (low sensitivity to errors).

Recent work dealing with multiple suppliers include Parlar and Perry (1996), Gürler and Parlar (1997), Tomlin (2005, 2006), Yu et al. (2009), Li et al. (2010), Sarkar and Mohapatra (2009), and Yan and Liu (2009). In these models, it is assumed that retailer sources from two or more suppliers. The inter-failure time and the repair time are scholastic for all suppliers. The disruption management strategies include sourcing mitigation, contingent rerouting, dual sourcing, emergency sourcing, and demand management, etc. For example, Tomlin (2005) went beyond the existing literature by explicitly modeling the trade-offs and limitations inherent in mitigation and contingency strategies. Yu et al. (2009) evaluated the impacts of supply disruption risks on the choice between the famous single and dual sourcing methods in a two-stage supply chain with an on-stationary and price-sensitive demand. The expected profit functions of the two sourcing modes in the presence of supply chain disruption risks are first obtained, and then compared so that the critical values of the key factors affecting the final choice are identified. It is found that either single or dual sourcing can be effective depending on the magnitude of the disruption probability. Li et al. (2010) studied a supply chain consisting of one retailer and two suppliers with unreliable supply. They investigated not only the sourcing strategy of a retailer but also the pricing strategies of two suppliers in a supply chain under an environment of supply disruption. Sarkar and Mohapatra (2009) formulated a model in a decision tree-like structure to determine the optimal size of supply base with considering risks of supply disruption due to occurrence of super, semi-super, and unique events. They also forwarded a tabular method of solution that overcomes the problem of dimensionality. Yan and Liu (2009) considered the problem of joint replenishment and pricing for a single product with two suppliers and supply disruption. They not only obtained that the form of the optimal policy has a $(s, S, p, \sigma, \Sigma)$-type, but also analyzed how supply disruption affects the profit function and the optimal policy.

In the above literature, the probability distribution of an occurrence of a major disruption is assumed to be known. Thus, the estimating the likelihood of disruption to occur is the foundation of executing stochastic plan. However, as mentioned above, it is difficult to measure the probability, which could undermine the analysis.

1.2.4.4 Real-Time Disruption Management

Real-time disruption management refers to the real time dynamic revision of an operational plan when disruptions occur. This concept can be formally stated as follows: At the beginning of a business cycle, an optimal or near-optimal operational plan is obtained by using certain optimization models and solution schemes. When such an operational plan is executed, disruptions may occur from time to time and is caused by internal and external uncertain factors. As a result, the original operational plan may not remain optimal, or even feasible. Consequently, it is necessary to dynamically revise the original plan and to obtain a new one that reflects the constraints and objectives of the evolved environment while minimizing the negative impact of the disruption. This process is referred to as real-time disruption management (Yu and Qi 2004).

There are many papers on real-time disruption management in supply chains. Most of them focus on the algorithm of obtaining the optimal solution and the changes of the optimal solution when some disruptions occur. Disruptions of demand and supply are the source of risk. In the following text, only a few notable works are listed.

Yang et al. (2001) considered possible disruptions in a finite production and inventory model with a deterministic demand. They also gave a steepest decent method to obtain the optimal solution of the problem. Golany et al. (2002) proposed a general approach based on a three-level lexicographical goal programming formulation, to address various types of disruptions. Qi et al. (2004) considered the coordination of a supply chain with one supplier and one retailer under demand disruptions. They modeled the cost of deviating from the original production plan for several scenarios, and showed that under certain wholesale quantity discount policy, the supply chain can be coordinated. Xia et al. (2004) considered real-time disruption management for a two-stage production and inventory system. They presented a general disruption management approach for this system and introduce the concept of a disruption recovery time window. Xiao and Yu (2005) studied the effect of the supply chain disruptions including the raw material supply and demand disruptions on the retailers' strategies by employing an indirect evolutionary game model. Xu et al. (2006) studied a supply chain coordination problem under production cost disruptions. They designed coordination schemes under disruptions. Xiao et al. (2007) investigated the coordination mechanism for a supply chain with one manufacturer and two competing retailers when the demands are disrupted. Xiao and Qi (2008) studied the coordination of a supply chain with one manufacturer and two competing retailers after the production cost of the manufacturer was disrupted. The model is also extended to the case with both cost and demand disruptions. Chen and Xiao (2009) developed two coordination models of a supply chain to investigate how to coordinate the supply chain after demand disruption. They considered two coordination schedules, linear quantity discount schedule and Grove's wholesale price schedule. They found that linear quantity discount schedule is better for the manufacturer when the increased amount of demand is very large and production cost is sufficiently low. However, Grove's wholesale price schedule is always better when the production cost is sufficiently large. Cauvin et al. (2009) presented a general framework for disruption management aiming at supporting decision-making in a disrupted and distributed environment. They proposed an approach to minimize the impact of disrupting events on distributed industrial systems. It is based on an analysis of disrupting events and the characterization of the recovery process, and on a cooperative repair method for the whole systems.

The outstanding contribution of the above works is the introduction of deviation costs and disruption management time window. Firms incur deviation costs associated with the transition from the original plan to a new plan. The deviation cost can be a real dollar cost caused by raw material waste, or using on-call or reserved personnel; it can also mean the loss of the customers' goodwill for waiting and delay. One of the roles of introducing the deviation costs is to force the revised plan to stay close to the original plan. Disruption management time window is a time point by which the system should restore to its normal operation after a disruption occurs. By setting the time window, the impact of a disruption can be contained within a limited time period (Yu and Qi 2004).

The supply chain real time disruption management is a meaningful and interesting field. There are still many questions that need to be studied and analyzed. For example, most of the above models are extensions of the simple models in supply chain management such as EOQ model, EPQ model, and supply chain model with one supplier and one retailer, etc. Besides, even though the demand or the supply is varying, they are assumed deterministic, i.e., these models are deterministic. Hence, the model can be extended to the case in which there are multiple retailers, multiple periods, and longer supply chains, to the case in which demand is stochastic.

1.2.5 Concluding Remarks

This section seeks to investigate disruption risk management in supply chains. As we have discussed above, a large amount of the literature has been devoted to the study of disruption risk management in supply chains. We first review the literature to trace approaches for exploring supply chain disruption risks according to the risk management paradigm, i.e., disruption risk identifying, disruption risk assessing, and disruption risk mitigating. Some feasible solutions, such as contingency planning, robust optimization, stochastic models, and real-time disruption management, have been advised to mitigate disruption risk in supply chain. Obviously, these measures are not free. Many firms find it difficult to justify certain costly strategies for mitigating supply chain disruptions that may not occur. Thus, it is an interesting and important issue to justify why resources should be devoted to proactively manage such risks.

1.3 Financial Risk Measurement in Supply Chains

1.3.1 Introduction

Most of the above risk measurement approaches are incorporated into the risk management for a single decision maker or some decision makers respectively. As shown above, these attempts are to incorporate the riskiness of inventory investment decisions that fail to measure properly the project's risk and to use a market-based trade-off between risk and return. On the other hand, most applied inventory control policies, e.g., the (s, S) or (Q, r) policy, are derived from stationary infinite horizon models discounted at a fixed interest rate. In practice, the discounted model is often approximated by an average-cost model where the effect of discounting is taken into account by an opportunity holding cost term. In the mean while, how to determine the size of the interest rate is still not clear.

It is well known that in a world where investors hold diversified portfolios of financial assets, the relevant risk of an investment by a value-maximizing firm can not be appropriately measured by the total variance of the return from the investment (Fama and Miller 1972). Under such circumstances, the proper measure of the project's risk is its systematic, i.e., non-diversifiable risk.

It is also known from the financial economics literature that the size of a discount rate depends on the systematic (business-cycle-related) risk of the costs that are to be discounted. According to the Capital Asset Pricing Model (CAPM, see for example, Sharpe 1964; Lintner 1965; Merton 1973), the expected return of the firm can be measured by merging risk free interest rate and a risk premium, which is decided by the correlation between the expected market return and the systematic risk. The formulation of CAPM is given as follows:

$$E(R) = R_f + \beta[E(R_m) - R_f], \tag{1.11}$$

where $E(R)$ and $E(R_m)$ are the expected returns of the firm and the market respectively, and R_f is the risk-free interest. Furthermore, the systematic risk coefficient $\beta = \mathrm{Cov}(R, R_m)/\mathrm{Var}(R_m)$. With this relationship defined, adjustments for risk of the firm can be incorporated into discount rates on future returns. Finally the firm or project could be evaluated based on discounted expected cash flows.

Recently, there is a growing interest in hedging operational risk using these financial instruments. This can be traced back to Anvari (1987), who analyzed the one-period newsvendor problem by using the CAPM and defining the risk as the covariance of random liquidating dividend and value of market portfolio. To obtain useful results, he assumed that the joint distribution of random demand and value of market portfolio is bivariate normal. The resulting optimal policy is characterized and is compared with the classical expected benefit maximization framework. It is shown that when the relevant risk of the inventory investment is considered, results are dramatically different.

The work by Anvari (1987) also indicates that measuring the riskiness of the inventory investment project by its systematic risk, however, may not be appropriate under certain circumstances. The correctness of this treatment is based upon the ability of investors to diversify, at zero cost, their portfolios of risky assets, and thus eliminate the non-systematic risk of the project. The particular firm analyzing the inventory investment decision cannot assume that its claimholders can do so, e.g., small, closely held companies, to the extent that the CAPM will not be the appropriate valuation framework. Furthermore, in circumstances where the magnitude of the investment is large enough to affect the chances of bankruptcy, the total variance or the downside risk of the project may have to be taken into account, as introduced above.

The existing stochastic inventory models incorporate the risk of holding inventory by specifying the opportunity cost of capital, and consider the effect of inventory decisions on the risk of cash flows. Besides Anvari (1987), Singhal (1988), and Kim and Chung (1989) independently showed that the level of inventory determines the risk, and therefore, the opportunity cost of capital. These authors used the CAPM to value the cash flows for the newsvendor problem. They got two key conclusions. Firstly, the opportunity cost of capital for investments in inventory is an increasing function of the inventory level. Secondly, the opportunity cost of capital is higher for a firm facing more risky demand.

Singhal et al. (1994) addressed settings where the demand risk is measured by the covariance of demand with the market return, and the objective of optimization is minimizing the present value of total cost. The CAPM is used to value the uncertain cash flows from inventory decisions. The paper analyzes the effect of the demand risk on the lot size and reorder point decisions of a firm in the standard (Q, r) inventory model. By numerical analysis the authors found that the influence of demand risk on inventory decisions depends on the scale of replenishment lead time, and the average inventory is decreasing in the demand risk. Recently, Caldentey and Haugh (2006) showed that different information assumptions lead to different types of solution techniques, and Gaur and Seshadri (2005) investigated the impact of financial hedging on the operations decision.

Thorstenson (1988), Chung (1990), and Birge and Zhang (1999) derived an optimal policy for the newsvendor problem by applying the option valuation model in Black and Scholes (1973). Neither, they did not estimate the size of the systematic demand risks to be expected, nor they did evaluate the general importance of these financial risks on inventory control. In Berling and Rosling (2005), the authors analyzed the effects of financial risks on (Q, r) inventory policies in a real options framework, with the assumption that stochastic price and demand follow a Wiener process. The objective to maximize is net present values (NPV), which are assumed determined by the Consumption Capital Asset Pricing Model (C-CAPM, Breeden 1979). A single-period model of the newsvendor type and an infinite-horizon model with a fixed set-up cost are studied. The systematic risk of stochastic demand is proved to have a negligible effect on the optimal value of r and Q, while the systematic risk of the purchase price has a significant effect on r and Q.

There is another important issue that manufacturing and service operations decisions depend critically on capacity and resource limits. These limits directly affect the risk inherent in those decisions. While risk consideration is well developed in finance through the efficient market theory and the CAPM, operations management models do not generally adopt these principles. One reason for this apparent inconsistency may be that analysis of an operational model does not reveal the level of risk until the model is solved. Birge (2000) used some results from the option pricing theory, and showed that this inconsistency can be avoided in a wide range of planning models. By assuming the availability of market hedges, they show that risk can be incorporated into planning models by adjusting capacity and resource levels. The result resolves some possible inconsistencies between finance and operations and provides a financial basis for many planning problems. The author illustrated the proposed approach using a capacity-planning example.

So far we have introduced the incorporation of systematic risk measurement into the operational decisions context. From another point of view, financial and operational decisions of the firm are usually studied separately. It may be due to the fact that production managers in large firms cannot influence financial policy and financial officers are typically detached from production decisions. Furthermore, for more than 40 years it has been known that a firm's capital structure does not affect its market value if the capital markets in which the firm operates are perfect and complete (Miller and Modigliani 1961). However, real capital markets are imperfect because the information is asymmetric and there are taxes, transaction costs, etc. Also, for a small firm, say a start-up, the responsibilities of a chief operational officer and a chief financial officer are often delegated to a single person or a small group of people who are obliged to be involved actively in all types of decisions. In many cases, growing firms are capital constrained and cannot implement the operational decisions that would be optimal if financial considerations were ignored.

In addition, firms may prefer more debt rather than equity due to the tax advantage of debt. However, this advantage can be nullified by direct and indirect bankruptcy costs and the risk of bankruptcy. Direct costs include legal and administrative costs of liquidation or reorganization a business, and indirect costs include damaged relationships with customers and suppliers, and sometimes "fire sale" liquidation of the firm's assets below their market value. The financial economic research literature studied optimal capital structure by trades off between tax advantages and bankruptcy risk (Kraus and Litzenberger 1973; Scott 1977; Brennan and Schwartz 1978; Kim 1978; Turnbull 1979; DeAngelo and Masulis 1980).

Until a decade ago, financial considerations were conspicuously absent in the extensive literature on models of inventory and production processes. However, recent studies (Archibald et al. 2002; Buzacott and Zhang 2004; Xu and Birge 2006, 2004a, 2004b; Li et al. 1997; Sobel and Zhang 2003; Babich and Sobel 2004; Hu and Sobel 2005; Chao et al. 2008) in operations have recognized the importance of the interplay between financial and operational decisions.

Hu and Sobel (2005) studied a dynamic newsvendor model with the criterion of maximizing the expected present value of dividends, and examine the interdependence of a firm's capital structure and its short-term operating decisions concerning inventories, dividends, and liquidity. They obtained interesting results on the interaction between firm's capital structure and operational decisions.

Buzacott and Zhang (2004) analyzed a Stackelberg game between the bank and the retailer in a newsvendor inventory model. They considered a single period inventory management problem where the bank's decisions include the interest rate to charge and the loan limit, and the retailer needs to decide the amount to borrow within the loan limit and the amount of inventory to order from suppliers. Both the bank and retailer maximize their expected returns.

Chao et al. (2008) considered a classic dynamic inventory control problem of a self-financing retailer who periodically replenishes its stock from a supplier and sells it to the market. The replenishment decisions of the retailer are constrained by cash flows, which is updated periodically following purchasing and sales in each period. The retailer's objective is to maximize its expected terminal wealth at the end of the planning horizon. They characterized the optimal inventory control policy and present a simple algorithm for computing the optimal policies for each period. They also study the dependencies of the optimal control policy on the system parameters.

In the following, a consideration of systematic risk measurement is taken into supply chain management in Sect. 1.3.2 by applying CAPM model or C-CAPM model. In Sect. 1.3.3, other methods are employed to study the risk of cash flow as well as possible bankruptcy of firms. Conclusions are given in Sect. 1.3.4.

1.3.2 A Systematic Risk Analysis

The systematic risk is usually measured by the correlation between the returns of the firm and the market as the CAPM model shows. Anvari (1987) introduced the methodology into the analysis of inventory control problem. In the following we first present his model. Notice that only the main model and its results are introduced, and for the details, please see Anvari (1987).

1.3.2.1 Optimality Criteria with CAPM Model

According to Anvari (1987), consider the one-period newsvendor problem with no set-up costs. Suppose the all-equity firm is established at $t = 0$, with total capital

C to be invested in two independent projects. At $t = 0$, an amount equal to cQ is invested in the inventory project, and $C - cQ$ is invested in other projects. The latter investment generates returns at the random rate of μ, where μ incorporates the terminal value of all physical assets involved in other projects. The investment in inventory is used to purchase Q units of a commodity at the net price of c per unit. They are sold at $t = 1$ at a price p per unit, where p is the price net of storage charges. The demand for the commodity, D, is a random variable characterized by a probability density function $f(\cdot)$ and a cumulative distribution function $F(\cdot)$. Surplus units are disposed of at a net price of $s < p$.

The firm is liquidated at $t = 1$, which proceeds accruing to the shareholders. Common-share trading takes place at $t = 0$, at which time prices are established based on investors' homogeneous expectations concerning pay-offs at $t = 1$. Assuming the one-period CAPM applies, the value of the firm's common shares at $t = 0$ is given by:

$$S(Q) = \frac{E(v(Q)) - \Omega \mathrm{Cov}(v(Q), M)}{1 + r_{\mathrm{f}}}, \tag{1.12}$$

where $v(Q)$ is the random liquidating dividend, Ω is the market price per unit of risk, M is the value of market portfolio at $t = 1$, and r_{f} is the risk-free rate.

The objective of the value-maximizing firm is to select Q such that the current shareholders' wealth, $S(Q) - C$, is maximized.

Notice that $v(Q)$ can be written as $v(Q) = (1 + \mu)(C - cQ) + p \min D, Q + s(Q - D)^{+}$. Let $E(D)$ and σ denote the mean and standard deviation of D. To characterize the optimal policy, further let $Q_{\mathrm{c}} = E(D) + \sigma^2 / \Omega \mathrm{Cov}(D, M)$.

Finally, the optimal ordering quantity, Q^* can be found as follows:

Case 1: $\mathrm{Cov}(D, M) > 0$

$$S(Q^*) = \max\{S(D_{\max}), S(Q_{\mathrm{c}}), S(Q_{\max})\}, \tag{1.13}$$

where $D_{\max}$ is the maximum possible value of D and $Q_{\max}$ is the value of Q that maximizes the function in the concave region determined from the first order condition.

Case 2: $\mathrm{Cov}(D, M) < 0$

$$S(Q^*) = \max\{S(0), S(Q_{\mathrm{c}}), S(Q_{\max})\}. \tag{1.14}$$

Case 3: $\mathrm{Cov}(D, M) = 0$

$$S(Q^*) = S(Q_{\max}). \tag{1.15}$$

Notice that the NPV of the overall production plan, i.e., $S(Q^*) - C$, may be negative, in which case the firm should be dissolved at time $t = 0$.

The CAPM model is applied to analyze the single-period newsvendor problem. The newsvendor problem is a useful starting point to illustrate the fundamental idea that inventory decisions affect the risk of cash flows. However, the focus is on the newsvendor problem that has two limitations. Firstly, it limits the generalization of the results to just the safety stock component of a firm's inventory. Cycle stock issues – the result of the trade-off between setup, holding, and backorder cost – are ignored. There exists a high degree of interrelationship between the cycle stock and safety stock decisions and these two should not be separated. Secondly, the newsvendor problem is a single period model whereas inventory decisions are made in a multi-period framework.

In the following we will present the CAPM-based multi-period (Q, r) inventory model, which shows how risk considerations affect the cycle stock (order quantity) and safety stock (recorder point) when both these decisions are made simultaneously in a multi-period setting. The details of the model can be found in Singhal et al. (1994).

1.3.2.2 The (Q, r) Inventory Model Applying CAPM

Consider the traditional (Q, r) inventory model with setup, inventory, and backorder costs. This is a widely known model where the inventory control policy is to place an order of size Q, the lot size, whenever the inventory level drops to r, the reorder point. The objective is to minimize the present value of total cost, using the CAPM to value the uncertain cash flows.

Similar to (1.12), in the single period CAPM context, the risk-adjusted present value $V_0(X)$ of the risky cash flow X, realized at the end of the year, is given by the single period CAPM as

$$V_0(X) = \frac{E(X) - \Omega \mathrm{Cov}(X, M)}{(1 + R_\mathrm{f})}, \tag{1.16}$$

where $E(X)$ is the expected cash flow and $\mathrm{Cov}(X, M)$ is the covariance of the cash flow with the market return, the relevant measure of risk in valuing a risky cash flow in the CAPM framework.

Furthermore, using the version of the multi-period CAPM developed by Fama (1977), consider a firm that will have an uncertain cash flow of X at the end of year t and no cash flows at any other time. Assume that the distribution of X is stationary. The risk-free rate and the market price per unit of risk in future periods are non-stochastic. It is reasonable and necessary for tractability to assume the values of the risk-free rate and the market price per unit of risk are the same in each period. Given these conditions, Fama (1977) showed that when the values in each period are determined according to the CAPM, the market value of the firm at time 0 is

$$V_0(X) = \frac{E(X) - \Omega \mathrm{Cov}(X, M)}{(1 + R_\mathrm{f})^t}. \tag{1.17}$$

Equation (1.17) is the appropriate formula to value the cash flows from the (Q, r) model.

Next, the certainty equivalent of the costs incurred during any period t is determined. Since an order is placed every cycle, a fixed setup cost S is incurred every cycle. The backorder costs per cycle are uncertain and depend on the demand during the lead time. Using the results in the Appendix of Anvari (1987) with the assumption that the demand and the market return are jointly normal, the certainty equivalent of backorder costs per cycle, BC, can be written as

$$BC = b\left\{B(r) - \Omega\mathrm{Cov}(D_l, M_l)(1 - F(r))\right\}, \tag{1.18}$$

where b is the per unit backorder cost, D_l and M_l are respectively the uncertain demand and the market return over the lead time l, and $B(r) = E(D - r)^+$ is the expected number of backorders per cycle. Notice that $F(\cdot)$ here denotes the cumulative distribution of D_l.

Using the economic interpretation of the certainty equivalent of an uncertain cash flow, the valuation impact of the uncertain backorder costs per cycle can be replicated by substituting the uncertain cash flow with a certain 'fixed' backorder cost per cycle equal to the certainty equivalent of the backorder costs per cycle. Thus in every cycle, the valuation impact of setup and ordering costs is like incurring a fixed cost equal to $S + BC$. The inventory system goes through one cycle every Q units of demand realized. By allocating the setup and backordering costs equally over Q units of demand, $(S+BC)/Q$ is the setup and backordering costs per unit of demand realized. Given the assumption that all costs during a year results in a single cash flow at the end of the year, an uncertain cash outflow equal to $D(S + BC)/Q$ will be incurred at the end of any year t. Thus, the certainty equivalent of this uncertain cash flow, Z, in any year t is

$$Z = \frac{(D - \Omega\mathrm{Cov}(D, M))(S + BC)}{Q}, \tag{1.19}$$

where D and M are the demand and market return of each period, and $D - \omega\mathrm{Cov}(D, M)$ is the certainty equivalent of demand.

Similarly, the certainty equivalent of holding cost, H, in any period t is

$$H = hc\left[\frac{1}{2}Q + r - (D_l - \Omega\mathrm{Cov}(D_l, M_l))\right], \tag{1.20}$$

where h is the per unit holding cost. The present value of the cash flow of the investment in inventory is

$$I = c\left[\frac{1}{2}Q + r - (D_l - \Omega\mathrm{Cov}(D_l, M_l))\right]. \tag{1.21}$$

Finally the present value of total costs, PV_{TC}, over an infinite number of periods can be written as

$$PV_{\mathrm{TC}} = I + \sum_{t=1}^{t=\infty} \frac{H+Z}{(1+r_{\mathrm{f}})^t} = I + \frac{H+Z}{r_{\mathrm{f}}}. \tag{1.22}$$

Maximizing the present value of total costs, we find the first order optimality conditions as

$$Q^* = \sqrt{\frac{2(d-\Omega\mathrm{Cov}(D,M))(S+b(B(r^*)-\Omega\mathrm{Cov}(D_l,M_l)(1-F(r^*))))}{c(h+r_{\mathrm{f}})}}, \tag{1.23}$$

and

$$1-F(r^*) = \frac{Q^*c(h+r_{\mathrm{f}})}{b(d-\lambda\mathrm{Cov}(D,M))} + \Omega\mathrm{Cov}(D_l,M_l)f(r^*). \tag{1.24}$$

Further note that the first order conditions for the traditional model are given by

$$Q^* = \sqrt{\frac{2d(S+bB(r^*))}{c(h+\beta)}}, \tag{1.25}$$

and

$$1-F(r^*) = \frac{Q^*c(h+\beta)}{bd}. \tag{1.26}$$

Comparing (1.23) and (1.24) with (1.25) and (1.26) indicates that using the CAPM to adjust for the risk of the cash flows results in three differences. Firstly, the expected demand and expected backorder cost per cycle are replaced by their respective certainty equivalents. Secondly, an additional term is introduced in the right hand side of (1.24), similar to the adjustment made in the optimality condition by Anvari (1987) and others, in their analysis of the newsvendor problem. Thirdly, in (1.25) and (1.26) the fixed opportunity cost of capital, k, accounts for both the timing and the risk of the cash flows. In our model, the risk-free rate, r_{f}, accounts for the timing of cash flows whereas the various covariance terms account for the risk of cash flows. Furthermore, in contrast to (1.25) and (1.26), the risk adjustments depend on the inventory decisions themselves. Also note that when $\mathrm{Cov}(D,M)$ is zero, (1.23) and (1.24) reduce to (1.25) and (1.26) with k replaced by R_{f}, the risk-free rate of return.

In the following, Sect. 1.3.2.3 will propose the incorporation of financial risk by adopting the real option framework and C-CAPM. Also the impact of both the stochastic demand and price are studied. For details of the model, please see Berling and Rosling (2005).

1.3.2.3 The Effect of Stochastic Demand and Price

Consider a single-period model in which there is a period of length t. Stock bought at the beginning of the period costs $c(0)$ per unit and what is left over at time t is sold back at price $c(t)$. Demand evolves from an initial intensity $D(0)$, or D, to $D(t)$, which is the total period demand. Shortages at the end of the period are satisfied externally at the price $c(t)$, but in addition the customer is compensated by a discount of β per unit.

All the decisions are made to maximize the firm's market value. Let $X(t)$ represent a monetary value at time t. $X(t)$ are assumed to follow logarithmic Wiener processes, i.e., the growth rates (or logarithms) of the variables obey regular Wiener processes. Thus, $\ln[X(t)]$ is normally distributed with $E[\ln(X(t))] = \ln[X(0)] + \delta t$ and $VAR[\ln(X(t))] = \sigma^2 t$ for all $t \geq 0$, where δ, the drift, and $\sigma \geq 0$ are constants. Consequently $X(t)$ is lognormally distributed with $E[X(t)] = X(0)\mathrm{e}^{t(\delta+\sigma^2/2)}$ and $VAR[(X(t))] = E^2[X(t)](\mathrm{e}^{t\sigma^2} - 1)$ for all $t \geq 0$.

Assume that the stochastic variables are stationary in the sense that $E[X(t)] = X(0)$ for all t, so that $\delta = -\sigma^2/2$.

By adopting the real option valuation framework, the present value of $X(t)$ in the financial market depends on the expected return that the market requires for outcomes as risky as X. This relationship is assumed to follow the C-CAPM. The aggregate per capital consumption (of all goods and services), $G(t)$ – which is understood as a proxy for the business cycles then supposed to be jointly lognormally distributed with $X(t)$. The market's required expected return per period, R, on the investment in X satisfies

$$R = r_{\mathrm{f}} + \Omega \mathrm{Cov}(G, X).$$

The present market value of $X(t)$ is found as

$$PV[X(t)] = E[X(t)]\mathrm{e}^{-Rt}.$$

Consider a monetary value, $g(X(t))$, that is a general function of $g(t)$. According to the Risk Neutral Valuation Principle (or Martingale Property) the present market value of $g(X(t))$ can be found as

$$PV[g(X(t))] = E[g(Y(t))]\mathrm{e}^{-r_{\mathrm{f}}t}, \tag{1.27}$$

where

$$Y(t) = X(t)\mathrm{e}^{-\Omega \mathrm{Cov}(G,X)t}. \tag{1.28}$$

Thus, $X(t)$ is replaced by its risk-adjusted value, $X(t)\mathrm{e}^{-\Omega \mathrm{Cov}(G,X)t}$. The expected value of $g(Y(t))$ is then discounted by the risk-free rate, r_{f}.

Next to study the effect of stochastic demand, in the single-period model the inventory level is set to r, the order point. By (1.27) and (1.28) the present value of the expected cost is

$$\begin{aligned} TC(r) &= cr + e^{-r_f t}\beta E[\max(0, D'-r)] - e^{-r_f t} cE[r - D'] \\ &= c(1-e^{-r_f t})(r - E[D']) + cE[D'] + \beta e^{-r_f t} E[\max(0, D'-r)], \end{aligned}$$

where $D' = D(t)e^{-\Omega \mathrm{Cov}(G,D)t}$. As D' is lognormally distributed, the optimal solution satisfies

$$\begin{aligned} \frac{\beta e^{-r_f t} - c(1-e^{-r_f t})}{b e^{-r_f t}} &= \Phi\left(\frac{\ln(r^*) - E[\ln(D')]}{\sqrt{VAR[\ln(D')]}}\right) \\ &= \Phi\left(\frac{\ln(r^*) - E[\ln(D(t)) - \Omega \mathrm{Cov}(G,D)t]}{\sigma\sqrt{t}}\right), \end{aligned}$$

where Φ denotes the standard normal distribution function and $\sigma^2 t$ is the variance.

Hence, systematic demand risk generally seems to have little influence on the optimal order point. This conclusion is extended to the related infinite-horizon problem in Berling and Rosling (2005). Then myopic policies are not optimal due to the dependent increments of the lognormal process.

Furthermore, consider the single-period model which is realized with both the end-of-period price, $c(t)$, and the demand, $D(t)$, stochastic. Assume the random variables are independent and there is a risk premium, $\Omega \mathrm{Cov}(G,c)t$, associated with $c(t)$ only. In the risk neutral formulation, $c(t)$ is replaced by $c(t)e^{-\Omega \mathrm{Cov}(G,c)t}$. Thus,

$$\begin{aligned} TC(R) &= cr + e^{-r_f t}\{\beta E[\max(0, D(t)-r)] - E[c(t)e^{-\Omega \mathrm{Cov}(G,c)t}]E[r - D(t)]\} \\ &= c(1 - e^{-(r_f + \Omega \mathrm{Cov}(G,c))t}(r - E[D(t)]) + cE[D(t)] \\ &\quad + \beta e^{-r_f t} E[\max(0, D(t) - r)]. \end{aligned}$$

The optimal solution, r^*, satisfies

$$\frac{\beta e^{-r_f t} - c(1 - e^{-(r_f + \Omega \mathrm{Cov}(G,c))t})}{\beta e^{-r_f t}} = \Phi\left(\frac{\ln(r^*) - E[\ln(D(t))]}{\sigma\sqrt{t}}\right),$$

where $\sigma^2 t$ denote the variance of $\ln[D(t)]$.

Thus, the modification rule for systematic risk now reads that the opportunity cost of holding inventory should include the purchase-price risk premium. Note that the model is insensitive to the detailed assumptions about the discount shortage cost, β.

Sometimes very substantial relative losses may be incurred by not adjusting the formulas for the risk premium. It can be concluded that the effect of a systematic purchase-price risk may be of considerable importance when determining the order point.

1.3.3 *The Risk of Capital and Bankruptcy*

In real world, the operational decisions of the firm are usually constrained by cash flows. Many firms become bankrupt due to the breakdown of capital chain. Thus, beyond supply chain management, the firms also need to establish appropriate capital structure and manage the capital. Next, we first introduce the model which focuses on the impact of capital structure on the firms' operational decisions. The detail can be found in Hu and Sobel (2005).

1.3.3.1 The Importance of Capital Structure

Assume that the firm has initial levels of debt and equity and, thereafter, these levels remain constant. Let η be the amount of equity. Specifically, the bonds have an infinite maturity date and the proceeds are mB which entails a periodic coupon payment of B. Let $X_n(n = 1, 2, \ldots)$ be the firm's internal capital at the beginning of period n, so X_1 is the firm's initial working capital. In order to reflect the tax advantage attraction of debt-financing, let $1 - \tau$ be the firm's marginal income tax rate.

At the beginning of each period $n(n = 1, 2, \ldots)$, the firm knows X_n and the size of its physical goods inventory, denoted x_n, and makes three decisions: b_n, the amount of a short-term loan; z_n, the physical goods replenishment quantity; and v_n, the amount of dividends to issue. The repayment of b_n is due at the end of period n. The constraints on the decision variables are $b_n \geq 0$ and $z_n \geq 0$, but dividends are not constrained $v_n \geq 0$ for the following reason. Capital subscriptions occur frequently in entrepreneurial firms. If $v_n < 0$, $|v_n|$ is interpreted as a capital subscription. Later in the section we comment on the effects of imposing the constraint $v_n \geq 0$.

Although a broad array of inventory replenishment models would be consistent with the model, it is assumed for specificity and simplicity that ordered goods are provided by a singlestage source without delay, excess demand is backlogged, and successive periods' demands are independent and identically distributed non-negative random variables D_1, D_2, ... Let $F(\cdot)$ denote the distribution function of D. The model can be generalized in numerous ways including positive lead times, excess demand being lost, a multi-stage source for the ordered goods, and autocorrelated demands. Under the stated assumptions,

$$y_n = x_n + z_n. \tag{1.29}$$

Equation (1.29) is the total amount of the total amount of goods available to satisfy demand in period n. The constraint $z_n \geq 0$ corresponds to $y_n \geq x_n$.

The interest on the short-term loan b_n in period n is modeled as a random variable $\lambda_n(b_n, y_n)$ whose distribution depends on the amount borrowed and on the total supply of goods. This representation can reflect the dependence of the interest rate on the firm's current risk of bankruptcy, and it includes borrowing limits contingent on the firm's current condition. It is assumed that $\lambda_1(b, y)$, $\lambda_2(b, y)$, ... are independent and identically distributed random variables (for each pair (b, y) with $b \geq 0$).

The firm is declared insolvent at the end of a period if it has insufficient funds to pay the bond coupon B and repay the short-term loan (if one was made at the beginning of the period). Insolvency at the end of period n leads to reorganization of the firm accompanied in period $n+1$ by the resumption of operations after payment of a bankruptcy penalty $p(X_{n+1})$. The costs of reorganization bankruptcy are both direct, such as fees paid to lawyers and accountants, and indirect, such as lost sales and damaged supplier and customer relationships.

Represent the sales revenue net of inventory costs in period n as a function $g(y_n, D_n)$ of total supply and demand. Let $L(z_n)$ be the cost incurred in period n to replenish the quantity $z_n = y_n - x_n$. Assume that the firm is subject to a liquidity constraint that obliges it to fund its expenditures early in the period:

$$X_n + b_n \geq v_n + \tau[L(z_n) + \lambda_n(b_n, y_n) + p(X_n)].$$

The left side is the sum of retained earnings and the short-term loan. The right side is the sum of dividends and, net of tax credits, inventory replenishment cost, short-term interest, and bankruptcy penalty.

Define $s_n = X_n - v_n - \tau[L(z_n) + \lambda_n(b_n, y_n) + p(X_n)]$ as the residual internal capital after making expenditure early in the period, then the liquidity constraint is

$$b_n + s_n \geq 0. \tag{1.30}$$

It can be easily shown that an optimal policy specifies $b_n = -s_n$.

The single period discount factor in period n is modeled as a random variable $\beta_n(s_n, y_n)$ whose distribution depends on the residual internal capital and the total supply of goods. Assume that $\beta_1(s, y)$, $\beta_2(s, y)$, ... are independent and identically distributed random variables (for each pair (s, y)), and that $(D_1, D_2, \ldots)$, $(\lambda_1(b, y), \lambda_2(b, y), \ldots)$, and $(\beta_1(s, y), \beta_2(s, y), \ldots)$ are mutually independent sequences (for each (b, s, y) with $b \geq 0$).

The dynamics reflect the backlogging of excess demand and the balancing of cash flow:

$$x_{n+1} = y_n - D_n, \tag{1.31}$$

$$X_{n+1} = s_n + \tau[g(y_n, D_n) - B]. \tag{1.32}$$

The value of the firm is the maximal value of $E(\Pi)$ where the random variable Π denotes the present value of dividends. Specifically,

$$\begin{aligned}\Pi &= \sum_{n=1}^{\infty}\left[\Pi_{j=1}^{n-1}\beta_j(s_j, y_j)\right] v_n \\ &= \sum_{n=1}^{\infty}\left[\Pi_{j=1}^{n-1}\beta_j(s_j, y_j)\right]\{X_n - s_n - \tau[L(z_n) + \lambda_n(b_n, y_n) + p(X_n)]\} \\ &= \sum_{n=1}^{\infty}\Pi_{j=1}^{n-1}\beta_j(s_j, y_j)\mathcal{K}_n(b_n, s_n, x_n, y_n; B) + X_1 - p(X_1), \end{aligned} \tag{1.33}$$

with the definition

$$\begin{aligned}&\mathcal{K}_n(b, s, x, y; Q) \\ &\quad = -[1 - \beta_n(s, y)]s + \tau\beta_n(s, y)[g(y, D_n) - p\{s + \tau g(y, D_n) - \tau B\} - B] \\ &\quad\quad -\tau[L(y - x) - \lambda_n(b, y)], \end{aligned} \tag{1.34}$$

where $\mathcal{K}_n$ does not depend on w_n, and $\mathcal{K}_1(b, s, x, y; B)$, $\mathcal{K}_2(b, s, x, y; B)$, ... are independent and identically distributed random variables.

Therefore,

$$\begin{aligned}E(\Pi) &= E\left\{\sum_{n=1}^{\infty}\left[\Pi_{j=1}^{n-1}\beta_j(s_j, y_j)\right] v_n\right\} \\ &= E\left[\sum_{n=1}^{\infty}\Pi_{j=1}^{n-1}\beta_j(s_j, y_j)\mathcal{K}(b_n, s_n, x_n, y_n; B)\right] + X_1 - p(X_1). \end{aligned} \tag{1.35}$$

Next the optimal policies and impacts of long-term debt are identified. Without loss of generality, the following linearity assumptions are given:

$$\begin{aligned}&L(z) = cz, \\ &\lambda_n(b, \cdot) = \rho b, \\ &\beta_n(s, y) = \beta \in [0, 1), \\ &p(a) = \theta(-a)^+, \\ &g(y, d) = p\min\{y, D\} - h(y - D)^+ = py - (p + h)(y - D)^+. \end{aligned}$$

Where c is the unit cost of acquisition, $\theta > 0$ is a unit default penalty, and ρ is a scalar interest rate.

The following result characterizes the parameter sets yielding an optimal $s < 0$ (so $b > 0$).

Proposition 1.3.1. *The firm borrows short-term only when*

$$\tau\beta\theta F\left(\frac{h\hat{y} + B}{h + p}\right) < 1 - \beta - \tau\rho, \tag{1.36}$$

where $\hat{y}$ satisfies

$$(p+h)F(\hat{y}) + \tau\theta h F\left(\frac{h\hat{y}+B}{h+p}\right) = \frac{p-c(1-\beta)}{\beta}. \tag{1.37}$$

Equation (1.36) implicitly restricts $1-\beta > \tau\rho$, i.e., the gain from receiving a dollar of dividend now rather than next period must be greater than the interest payment for 9 one-dollar short-term loan. Otherwise, borrowing would never be optimal. In addition, $F[(h\hat{y}+B)/(h+p)]$ is next period's probability of bankruptcy when $s=0$, so the left side of (1.36) represents next period's expected bankruptcy cost; the right is the benefit of distributing a dollar dividend now while borrowing the dollar short-term to maintain solvency. Hence, (1.36) states that the firm should borrow only if the incremental benefit of receiving a dollar of dividend now rather than next period, net of interest payment, is greater than next period's expected default cost.

The following result characterizes y^* and s^* including comparative statics with respect to B.

Proposition 1.3.2. *(a) The optimal base-stock level y^**

(i) Is invariant with respect to the long-term debt level
(ii) Is nondecreasing in τ and ρ if $s^* < 0$
(iii) Depends only on inventory related parameters if $s^* \geq 0$
(iv) Is at least as high when $s^* < 0$ as when $s^* \geq 0$

(b) The optimal capital level s^*

(i) Is nondecreasing as the long-term debt level increases
(ii) Is nondecreasing in θ, ρ, and τ, and depends on the same inventory-related parameters as y^*

The linearity assumption yields two influences of capital structure on operational policies. Firstly, the optimal physical goods base-stock level does not depend on the capital structure, but the optimal residual internal capital increases as the long-term debt level increases. Secondly, if the firm borrows short-term, financial parameters (except for the tax rate and short-term interest rate) do not affect the optimal physical goods base-stock level, whereas the optimal residual internal capital depends on both financial and inventory-related parameters. The managerial insight from these results is that maximization of the firm's value is consistent with (i) ignoring financial parameters (other than the tax rate and short-term interest rate) when making short-term operational decisions, and (ii) taking operational parameters into account when making short-term financial decisions. Note that nonlinearity of the default penalty function would invalidate the conclusions that the optimal physical goods base-stock level does not depend on the capital structure, and financial parameters do not affect the physical goods base-stock level.

1.3.3.2 Asset-Based Financing

This section proposes a framework of inventory management with asset-based financing. The details of the model can be found in Buzacott and Zhang (2004). Consider a single-period newsvendor problem. The decision maker can get loan from the bank or deposit residuals in the bank. The sequence of events is as follows.

At time 0, the retailer has initial cash of X_0 but no other assets. The retailer places an order of size q at a cost of c per unit from her suppliers without knowing the actual demand, only the probability distribution of demand D given by $F(\xi) = Pr\{D \leq \xi\}$. Let $\bar{F}(\xi) = 1 - F(\xi)$ and $f(\xi) = F'(\xi)$. It is assumed that the retailer and the bank have the same belief about the distribution of demand. Full payment of cq is required when the order is placed, so the retailer borrows w from the bank. This means that there are no accounts payable and the retailer will have a cash balance of $X' = X_0 + w - cq$ after payment. The amount borrowed by the retailer has to be such that $X' \geq 0$ or $w \geq \max\{0, cq - X_0\}$, otherwise full payment for the order could not be made. Also, $w \leq \psi$, the asset-based loan limit of $\psi = \gamma_C x' + \gamma cq$ once the order is placed. Suppose that the bank funds the loan using money received from depositors to whom it promises to pay interest at the rate of α'.

Over time $(0, T)$, the retailer receives orders for D units of the product.

At time T, available product is shipped to customers who pay immediately p per unit and unsold inventory is disposed at a price of c', $c' < c$, per unit. So the retailer receives a total payment of $p \min\{D, q\} + c' \max\{q - D, 0\}$ and there are no accounts receivable. The retailer then has to pay the bank $w + \alpha w$, the required loan repayment plus the required loan interest, although it will receive a credit of $\alpha' X'$ for the interest due on the retailer's cash balance. Again, it will be assumed that $\alpha > \alpha'$. Because of our assumption that the retailer had no other assets but X_0 at time 0, the retailer will be bankrupt if they are unable to repay the loan and interest due on it. Assuming that the retailer is set up as a corporation with limited liability, the liability of the retailer's equity owners is limited to the amount invested in the ownership shares so the potential loss would be at most X_0. Therefore, the retailer's final cash position $X_T(D)$ after repaying the loan will be given by

$$X_T(D) = X' + p \min\{D, q\} + c' \max\{q - D, 0\} - w(1 + \alpha) + \alpha' X'$$

and the retailer is bankrupt if $X_T(D) < 0$. Because $\alpha' < \alpha$, it can be easily shown that it is optimal for the retailer to ensure that $X' \times w = 0$. That is, the retailer should use up all the cash before considering borrowing money from the bank, and so the value of γ_C is irrelevant to decision making. There are some simple properties that follow immediately.

Lemma 1.3.3. *The retailer borrows with no bankruptcy risk if the order quantity is such that $x_0/c < q \leq \hat{q}$, where $\hat{q}$ is given by*

$$\hat{q} = \frac{x_0(1 + \alpha)}{c(1 + \alpha) - c'} = \frac{x_0}{c(1 - \gamma')}, \tag{1.38}$$

where $\gamma' = c'/c(1 + \alpha)$.

Lemma 1.3.4. *If the order quantity q is such that $q > \hat{q}$, then bankruptcy will occur if demand ξ is less than $d(q)$, where*

$$d(q) = \frac{(c(1+\alpha) - c')(q - \hat{q})}{p - c'}.$$

The retailer's ending cash will then be given by

$$X_T(D) = \begin{cases} (p - c')(\min\{D, q\} - d(q)), & \textit{if } D \geq d(q), \\ 0, & \textit{if } D < d(q), \end{cases} \tag{1.39}$$

and the bank's economic return from its loan to the retailer is

$$\Pi(D) = (cq - X_0)(\alpha - \alpha') - \begin{cases} 0, & \textit{if } D \geq d(q), \\ (p - c')[d(q) - D], & \textit{if } D < d(q). \end{cases} \tag{1.40}$$

By using Lemmas 1.3.3 and 1.3.4, the expected returns to the retailer and bank for any given q can be derived. There are three possible situations that can arise:

(1) The retailer has enough initial capital that it does not borrow ($q \leq X_0/c$, no borrowing)
(2) The retailer borrows, but not sufficient to create any risk of bankruptcy ($X_0/c < q \leq \hat{q}$, borrowing without bankruptcy risk)
(3) The retailer borrows and there is the risk of bankruptcy ($q > \hat{q}$, borrowing with bankruptcy risk)

The probability of retailer bankruptcy is $Pr\{D < d(q)\}$ and the retailer's expected cash position, $E[X_T(D)]$, is given by

$$E[X_T(D)] = \begin{cases} (p - c')\int_0^q \bar{F}(\xi)\mathrm{d}\xi - [c(1+\alpha') - c']q + X_0(1+\alpha'), & \textit{if } q \leq \frac{X_0}{c}, \\ (p - c')\int_0^q \bar{F}(\xi)\mathrm{d}\xi - [c(1+\alpha) - c']q + X_0(1+\alpha), & \textit{if } \frac{X_0}{c} < q \leq \hat{q}, \\ (p - c')\int_{d(q)}^q \bar{F}(\xi)\mathrm{d}\xi, & \textit{if } q > \hat{q}. \end{cases} \tag{1.41}$$

The bank's expected return by lending to the retailer is

$$E[\Pi(D)] = \begin{cases} 0, & \textit{if } q \leq \frac{X_0}{c}, \\ (cq - X_0)(\alpha - \alpha'), & \textit{if } \frac{X_0}{c} < q \leq \hat{q}, \\ (cq - X_0)(\alpha - \alpha') - (p - c')\left[d(q) - \int_0^{d(q)} \bar{F}(\xi)\mathrm{d}\xi\right], & \textit{if } q > \hat{q}. \end{cases} \tag{1.42}$$

For a given bank interest rate α, no retailer will borrow if $p \le c(1+\alpha)$. So only the case where $p > c(1+\alpha)$ is considered. The following theorem identifies the order quantity a retailer with initial capital X_0 will choose.

Theorem 1.3.5. *For increasing failure rate (IFR) distributions of demand, the order quantity a retailer with initial capital of X_0 will choose, q^{R*}, for given α is as follows:*

$$q^{R*} = \begin{cases} q^{\mathrm{NB}}, & \text{if } X_0 > cq^{\mathrm{NB}}, \\ X_0/c, & \text{if } cq^{\mathrm{BWO}} \le X_0 \le cq^{\mathrm{NB}}, \\ q^{\mathrm{BWO}}, & \text{if } cq^{\mathrm{BWO}}(1-\gamma') \le X_0 < cq^{\mathrm{BWO}}, \\ q(X_0), & \text{if } 0 \le X_0 < cq^{\mathrm{BWO}}(1-\gamma'). \end{cases} \tag{1.43}$$

where q^{NB}, q^{BWO}, and $q(X_0)$ are determined by

$$\bar{F}(q^{\mathrm{NB}}) = \frac{c(1+\alpha') - c'}{p - c'}, \tag{1.44}$$

$$\bar{F}(q^{\mathrm{BWO}}) = \frac{c(1+\alpha) - c'}{p - c'}, \tag{1.45}$$

$$\bar{F}(q(X_0)) = \frac{c(1+\alpha) - c'}{p - c'} \bar{F}(d(q(X_0))). \tag{1.46}$$

Furthermore, $q(X_0)$ is decreasing in X_0 for $0 \le X_0 < cq^{\mathrm{BWO}}(1-\gamma')$ with $q(cq^{\mathrm{BWO}}(1-\gamma')) = q^{\mathrm{BWO}}$.

From the above theorem, the optimal retailer order quantity decreases as X_0 increases from $X_0 = 0$ until $X_0 = cq^{\mathrm{BWO}}(1-\gamma')$, then it is constant until $X_0 = cq^{\mathrm{BWO}}$ and eventually it increases once $X_0 > cq^{\mathrm{BWO}}$. Because the bank faces a set of retailers who differ only by their initial cash X_0, without loan limits, the retailers will respond in the following ways for a given α.

(i) Retailers with cash level $x_0 > cq^{\mathrm{NB}}$ will have more than enough cash for their operations and will not borrow.
(ii) Retailers with $cq^{\mathrm{BWO}} \le X_0 \le cq^{\mathrm{NB}}$ will use all the cash they have to finance their inventory and will not borrow.
(iii) Retailers with $cq^{\mathrm{BWO}}(1-\gamma') \le X_0 < cq^{\mathrm{BWO}}$ will borrow, but not sufficient to create any risk of bankruptcy.
(iv) Retailers with the least initial cash (i.e., $X_0 < cq^{\mathrm{BWO}}(1-\gamma')$) will be the big borrowers. As a matter of fact, the less wealthy the retailers are, the more the retailers will borrow and the more inventory the retailers will stock.

Furthermore, we will first determine the retailers' order quantities that achieve the optimal bank returns and then compare these quantities with the quantities that the retailers will order for a given α. As long as there is no risk of bankruptcy by lending to a retailer, the bank's return, $E[\Pi(D)]$, increases with increasing w and, hence, for a retailer with initial capital X_0, increases with increasing q. However, once there is bankruptcy risk, the maximum bank return occurs when $q = q_B$, where

$$\bar{F}(d(q_B)) = \frac{c(1+\alpha') - c'}{c(1+\alpha) - c'}, \tag{1.47}$$

or

$$q_B = \frac{X_0}{c(1-\gamma')} + \frac{p - c'}{c(1+\alpha) - c'}\bar{F}^{-1}\left(\frac{c(1+\alpha') - c'}{c(1+\alpha) - c'}\right). \tag{1.48}$$

With the option of asset-based financing, Theorem 1.3.5 and (1.47) propose the optimal quantities for the retailer and bank respectively. Here the study is for single-period newsvendor problem. In Chap. 4, the multi-period model with cash flow constraints will be investigated.

1.3.4 Concluding Remarks

This section proposes many supply chain management models related to financial risk, i.e., systematic risk or the risk of cash flow and bankruptcy. These models adopt financial theories (e.g., CAPM and Modigliani-Miller (M&M) Theorem, etc.) to study various financial risks.

Firstly, the CAPM model is applied to measure the systematic risk of the inventory control process. The CAPM-based inventory model is formulated and the optimal inventory control policy is proposed. Furthermore, the multi-period (Q, r) inventory model incorporated with CAPM is studied and the optimal ordering quantity and reorder point are proposed. We also present the analysis of the effect of the stochastic demand and price on operations.

Then the risks of cash flow and bankruptcy in supply chain management are studied. First the impact of capital structure on inventory control strategy is proposed, by establishing the inventory model to maximize the present value of dividends of the firm. Then a general asset-based financing inventory model is established, with the assumption that a newsvendor can carry out both operational decisions and financing decisions.

There are also other studies which coordinate the theory and practice supply chain management and finance. The innovation of financial theory and practice will also drive the development of supply chain management.

1.4 Organization and Main Conclusions of the Book

The first part of the book surveys the applications of risk management to supply chains and review the existing literature categorized by modeling of decision maker's risk preference, supply disruption management, and financial risk measurement in supply chains. Some representative works are selected for demonstrating the application of various risk management tools in supply chains.

The second part of the book focuses on the studies on supply uncertainty. Specifically, the sourcing strategy of a retailer who procures from two supplier with random yield is investigated in Chap. 2. The single-period and two-period problems with stochastic demand is analyzed. In each of the case, the condition whether the retailer will choose single soucring or dual sourcing is identified. Furthermore, an explicit form of sourcing threshold could be found for the special case of deterministic demand. Chapter 3 investigates not only the sourcing strategy of a retailer but also the pricing strategies of two suppliers under an environment of supply disruption. A coordination mechanism is devised to maximize the profits of both suppliers.

The third part of the book focuses on the financial risk measurement in supply chains from the perspective of cash flow constraints, financing decisions, and delayed cash payment.

Chapter 4 considers a classic dynamic inventory control problem of a self-financing retailer who periodically replenishes its stock from a supplier and sells it to the market. The replenishment decisions of the retailer are constrained by cash flows, which is updated periodically following the purchasing and the sales in each period. Excess demand in each period is lost when insufficient inventory is available. The retailer's objective is to maximize its expected terminal wealth at the end of the planning horizon. We characterize the optimal inventory control policy and present a simple algorithm for computing the optimal policies for each period. Conditions are identified under which the optimal control policies are identical across periods. We also present comparatively static results on the optimal control policy.

Chapter 5 presents a classic dynamic inventory control problem of a retailer who periodically replenishes its stock from a supplier and sells it to the market. Asset-based financing is allowed for the retailer, when being short of cash flow. Excess demand in each period is lost when insufficient inventory is available. The retailer's objective is to maximize its expected terminal wealth at the end of the planning horizon. The optimal inventory control policy is characterized. The dependence of the optimal policy on the wealth level is studied. Conditions are identified under which the retailer will choose to borrow or deposit in each period. The bankruptcy probability is also studied.

In Chap. 6, a framework is proposed for incorporating financial considerations including delayed cash payment and receivable into multi-period inventory models. Specifically, we characterize the dynamic financial constraint that is updated periodically according to production activities. The optimal operational policy and its dependence on the financial state are studied. It also demonstrates the importance of firms considering delayed cash payment.

The last part of the book studies on wholesale price negotiation. Specifically, a one-supplier two-manufacturers supply chain is studied in Chap. 7. The Nash game is first analyzed between the two manufacturers and then the bargaining process between the supplier and each manufacturer is modeled by a sequential Nash bargaining. The results demonstrate the importance of steel manufacturers increasing the investment on iron ore.

Chapter 2
Dynamic Suppliers Selection: Single or Dual Sourcing?

2.1 Introduction

Supply chain disruptions are unplanned and unanticipated events that disrupt the normal flow of goods and materials within a supply chain (Hendricks and Singhal 2003; Kleindorfer and Saad 2005) and, as a consequence, expose firms within the supply chain to operational and financial risks (Stauffer 2003).

Generally speaking, most supply chain disruptions can be broadly classified into three categories, namely supply-related, demand-related, and miscellaneous risks (Oke and Gopalakrishnana 2009). Supply disruption occurs when suppliers are unable to fill the orders placed with them. These risks could potentially affect or disrupt the supply of products or services that the supply chain offers its customers. Demand disruption may be due to a sudden drop or a sudden rise in customer orders. Demand-related risks could potentially affect or disrupt the operations of the retailer and affect its ability to make products available to its customers. Miscellaneous risks are risks that could potentially affect the costs of doing business, such as unexpected changes to purchasing costs, interest rates, currency exchange rates, safety regulations by government agencies, etc. This book mainly focuses on supply disruption.

A supplier may be unable to fill an order for a variety of reasons, including equipment failures, damaged facilities, problems in procuring the necessary raw materials, or rationing its supply among its customers.

This chapter addresses the problems faced by a retailer who deals with two unreliable suppliers who may default on their obligations to deliver order quantities at the end of a given production period. Using a simple two-periods model of a supply chain with one retailer and two unreliable suppliers, this chapter studies questions of supplier selection and ordering policies among firms.

Our work is related to the research on random yields. An excellent review of the random yield literature is offered by Yano and Lee (1995). Examples of model constructions appearing in these literature include, among others, the case of all-or-nothing delivery (Anupindi and Akella 1993; Gerchak 1996),

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the case of random capacity (Ciarallo et al. 1994), the case of binomial yield (Chen et al. 2001; Xie et al. 2010), the case of stochastic proportional yield (Henig and Gerchak 1990), and combinations thereof (Wang and Gerchak 1996). Specifically, Anupindi and Akella (1993) studied one- and multi- period discrete-time problems of a retailer who can order from one or two suppliers whose failure processes are uncorrelated. The authors derived optimal ordering policies under various stochastic yield assumptions including all-or-nothing, partial recovery, and delayed delivery.

One can also interpret the problem considered in this chapter as a multi-supplier sourcing problem. Recent survey articles by Elmaghraby (2000) and Minner (2003) described a variety of models proposed in a multi-supplier supply chain management literature. When there are multi suppliers, quite different disruption management strategies including dual sourcing, emergency sourcing, etc. could be employed. The focus of multi suppliers model is how to evaluate these different strategies and find a trade-off between the strategies cost and the disruption negative consequences. These studies can be found in Parlar and Perry (1996), Swaminathan and Shanthikumar (1999), Dada et al. (2003), Tomlin and Wang (2005), Babich et al. (2007a, 2007b), and Tomlin (2006).

Babich et al. (2007a, 2007b) studied a supply chain where one retailer deals with competing risky suppliers who may default during their production lead-times. The suppliers, who compete for business with the retailer by establishing wholesale prices, are leaders in a Stackelberg game with the retailer. The retailer, facing uncertain future demands, chooses order quantities while weighing the benefits of procuring from the cheapest supplier against the advantages of order diversification. For the model with two suppliers they show that low supplier default correlations dampens competition among the suppliers, increasing the equilibrium on wholesale prices.

Tomlin and Wang (2005) investigated a single-period, yield-uncertainty problem in which the firm faces trade-offs between mix flexibility and dual sourcing. They assumed that the firm is risk neural or risk averse, respectively. Loss-averse objective and CVaR measure are used to quantify the firm's downside risk tolerance in this system. Their results indicate that the appropriate levels of diversification and flexibility are very sensitive to the firm's downside risk tolerance.

Tomlin (2006) considered a model in which the firm may order from a cheap but unreliable supplier and/or an expensive but reliable supplier. Tomlin examined the conditions under which the firm's optimal strategy is to manage disruptions by holding extra inventory, by dual sourcing, by emergency sourcing, or by taking no action, and simply accepting the disruption risk. The author investigated the influence of the firm's attitude to risk on mitigation and contingency strategies for managing supply disruption risks. Risk is measured by using a mean–variance approach. The authors proved that a mixed mitigation strategy (partial sourcing from the reliable supplier and carrying inventory) can be optimal if the firm is risk-averse or if the unreliable supplier has finite capacity.

In this section, the retailer facing stochastic demand needs to determine whether to choose single sourcing from one supplier or dual sourcing from two suppliers, and further how much to order. For each period, we identify the conditions under which the retailer will choose different sourcing strategies, and find that the supplier selection process is the trade-off between the ordering cost and the randomicity of the yield rate. It is further pointed out that more structural results can be found under the setting of deterministic demand.

Another contribution of our model is the corresponding insights that it yields. In particular, it is found that the supplier selection process is the trade-off among the cost, the average yield rate, and the variance of yield rate. Moreover, these results are complemented with useful comparative statistics.

The rest of this chapter is organized as follows. Section 2.2 introduces the proposed problem and some assumptions. Sections 2.3 and 2.4 present the model and results for both single-period problem and two-period problem. Section 2.5 considers a special case that the demand is deterministic. Some numerical studies are included in Sect. 2.6. The chapter concludes in Sect. 2.7 with some remarks and some possible extensions.

2.2 The Problem and Assumptions

Consider a single-product supply chain with one retailer who can procure from two suppliers. The yield rates of both suppliers are random but independent of each other. The yield is random in the sense that, if an order for q_i units is placed by the retailer with supplier i, a quantity $Y_i q_i$ will be delivered to the retailer. It is assumed that the yield rate Y_i, $0 \leq Y_i \leq 1$, is a normal random variable with mean θ_i and variance σ_i^2, $i = 1, 2$. G_i and g_i denote the distribution and density function of the supplier i's yield rate, respectively. Assume the procurement leadtime from suppliers is zero.

Denote p as the selling price of the product and w_i as the unit ordering cost from supplier i. The retailer needs to determine ordering quantities from each supplier for two planning periods. If the retailer orders from both suppliers, then it is said that he uses dual sourcing; otherwise, he uses single-sourcing. The ending inventory of the first period is carried over to the second one. The customer demands occur only at the retailer and are i.i.d random variables in different periods. Unsatisfied demand of period one is fully backlogged, while is lost in the period two. The objective of the retailer is to maximize her total expected profit over two planning periods.

Some other notation is summarized in the following. For $t = 1, 2$,
$q_{t,i}$ = the order quantity from supplier i in period t, $i = 1, 2$.
D = the generic one-period demand with mean d.

To guarantee $0 \leq Y_i \leq 1$, it is assumed that $0 < \mu_i \pm 3\sigma_i < 1$, which makes the probability of Y_i falling into the interval $[0, 1]$ exceed 99%. Without loss of generality, it is assumed that $p\mu_i > w_i$, otherwise, the retailer will not order at all. We also assume $pG_i(0) < w_i$ to avoid trivial case.

Suppose the intial inventory at the retailer is 0. The decision problem for the retailer is

$$\max_{q_{11}\geq 0, q_{12}\geq 0} \Pi_1(q_{11}, q_{12})$$
$$= \max_{q_{11}\geq 0, q_{12}\geq 0} \{pd_1 - w_1 q_{11} - w_2 q_{12} + E[V_2(Y_1 q_{11} + Y_2 q_{12} - D_1)]\}, \quad (2.1)$$

where

$$V_2(x) = \max_{q_{21}\geq 0, q_{22}\geq 0} pE[\min\{D_2, x + Y_1 q_{21} + Y_2 q_{22}\}] - w_1 q_{21} - w_2 q_{22}. \quad (2.2)$$

2.3 Single Period Analysis

We first consider the problem of period two. At this period, the retailer's objective is

$$V_2(x) = \max_{q_{21}\geq 0, q_{22}\geq 0} \Pi_2(q_{21}, q_{22})$$
$$= \max_{q_{21}\geq 0, q_{22}\geq 0} pE[\min\{D_2, x + Y_1 q_{21} + Y_2 q_{22}\}] - w_1 q_{21} - w_2 q_{22}. \quad (2.3)$$

Firstly the following lemma ensures that there are optimal solutions for the optimization problem. The proof is straightforward by definition of joint concavity and submodularity. So we skip it here.

Lemma 2.3.1. *(a)* $\Pi_2(q_{21}, q_{22})$ *is jointly concave, and submodular in* q_{21} *and* q_{22}. *(b)* $V_2(x)$ *is concave in* x.

For notational convenience, define $Y = Y_1 q_{21} + Y_2 q_{22}$. Since Y_1 and Y_2 both follow normal distribution and are independent, thus $Y \sim \mathcal{N}[\mu_1 q_{21} + \mu_2 q_{22}, \sigma_1^2 q_{21}^2 + \sigma_2^2 q_{22}^2]$. Let $g(y, q_{21}, q_{22})$ and $G(y, q_{21}, q_{22})$ be the density function and cumulative distribution function for Y, respectively. Notice that if $p\mu_i \leq w_i$, then the retailer will order nothing. Next taking partial derivative of $\Pi_2(q_{21}, q_{22})$ on q_{21} yields the first order condition

$$(\Pi_2(q_{21}, q_{22}))'_{q_{21}}$$
$$= pE\{Y_1 \mathbf{1}_{\{Y_1 q_{21} + Y_2 q_{22} \leq D_2 - x\}}\} - w_1$$
$$= pE\left[\int_{-\infty}^{+\infty} \int_{-\infty}^{\frac{D_2 - x - q_{22} y_2}{q_{21}}} y_1 g_1(y_1) \mathrm{d}y_1 g_2(y_2) \mathrm{d}y_2\right] - w_1$$
$$= pE\left[\int_{-\infty}^{+\infty} \int_{-\infty}^{\frac{D_2 - x - q_{22} y_2}{q_{21}}} [\mu_1 g_1(y_1) - \sigma_1^2 g_1'(y_1)] g_2(y_2) \mathrm{d}y_1 \mathrm{d}y_2\right] - w_1$$

$$
\begin{aligned}
&= p\mu_1 E\left[\int_{-\infty}^{+\infty} G_1\left(\frac{D_2 - x - q_{22}y_2}{q_{21}}\right) g_2(y_2)\mathrm{d}y_2\right] \\
&\quad - p\sigma_1^2 E\left[\int_{-\infty}^{+\infty} g_1\left(\frac{D_2 - x - q_{22}y_2}{q_{21}}\right) g_2(y_2)\mathrm{d}y_2\right] - w_1 \\
&= p\mu_1 E[G(D_2 - x, q_{21}, q_{22})] - p\sigma_1^2 q_{21} E[g(D_2 - x, q_{21}, q_{22})] - w_1 = 0, \quad (2.4)
\end{aligned}
$$

where the third equality is due to the fact that $g_1(y_1)$ follows normal distribution and hence $g_1'(y_1) = -(y_1 - \mu_1)g_1(y_1)/\sigma_1^2$. And similarly the first order condition on q_{22} is given by

$$
\begin{aligned}
&(\Pi_2(q_{21}, q_{22}))'_{q_{22}} \\
&\quad = p\mu_2 E[G(D_2 - x, q_{21}, q_{22})] - p\sigma_2^2 q_{22} E[g(D_2 - x, q_{21}, q_{22})] - w_2 = 0. \quad (2.5)
\end{aligned}
$$

Notice that if $x \geq F^{-1}\left(1 - \frac{w_1}{p\mu_1}\right)$, then

$$
\begin{aligned}
&(\Pi_2(q_{21}, q_{22}))'_{q_{21}} \\
&\quad = p\left[\int_{x}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{\frac{z-x-q_{22}y_2}{q_{21}}} y_1 g_1(y_1) g_2(y_2)\mathrm{d}y_1\mathrm{d}y_2 f(z)\mathrm{d}z\right] - w_1 \\
&\quad \leq p\mu_1(1 - F(x)) - w_1 \\
&\quad \leq 0.
\end{aligned}
$$

Where the equality follows from $Pr(Y_1 q_{21} + Y_2 q_{22} \leq D_2 - x \leq 0) \approx 0$. Thus the retailer will not order from supplier 1 if $x \geq F^{-1}\left(1 - \frac{w_1}{p\mu_1}\right)$.

Similarly if $x \geq F^{-1}\left(1 - \frac{w_2}{p\mu_2}\right)$, then $(\Pi_2(q_{21}, q_{22}))'_{q_{22}} \leq 0$. It is found that the retailer will not order from supplier 2 if $x \geq F^{-1}\left(1 - \frac{w_2}{p\mu_2}\right)$.

Therefore, the retailer will order nothing if $x \geq \max\left\{F^{-1}\left(1 - \frac{w_1}{p\mu_1}\right), F^{-1}\left(1 - \frac{w_2}{p\mu_2}\right)\right\}$, and choose single sourcing from supplier i if $F^{-1}\left(1 - \frac{w_{3-i}}{p\mu_{3-i}}\right) \leq x < F^{-1}\left(1 - \frac{w_i}{p\mu_i}\right)$. Next we discuss the optimal sourcing strategy of the retailer when $x < \min\left\{F^{-1}\left(1 - \frac{w_1}{p\mu_1}\right), F^{-1}\left(1 - \frac{w_2}{p\mu_2}\right)\right\}$.

Let $q_{21}(q_{22})$ and $q_{22}(q_{21})$ be the optimal solutions of (2.4) and (2.5), respectively. Because $\Pi_2(q_{21}, q_{22})$ is concave, there exists at least one optimal solution (q_{21}^*, q_{22}^*). If both $q_{21}^* > 0$ and $q_{22}^* > 0$, then the retailer adopts dual sourcing; otherwise, she uses single sourcing. We discuss the optimal sourcing strategy of the retailer. In what follows, let ϕ and Φ be the pdf and cdf of standard normal distribution.

Theorem 2.3.2. *In the second period, when* $x < \min\left\{F^{-1}\left(1-\frac{w_1}{p\mu_1}\right),\ F^{-1}\left(1-\frac{w_2}{p\mu_2}\right)\right\}$, *the retailer's optimal sourcing strategy is*

(i) Single sourcing from supplier 1 *if*

$$\mu_1 w_2 - \mu_2 w_1 \geq p\sigma_1\mu_2 E\left[\phi\left(\frac{D_2 - x - \mu_1\hat{q}_1(x)}{\sigma_1\hat{q}_1(x)}\right)\right], \tag{2.6}$$

(ii) Single sourcing from supplier 2 *if*

$$\mu_2 w_1 - \mu_1 w_2 \geq p\sigma_2\mu_1 E\left[\phi\left(\frac{D_2 - x - \mu_2\hat{q}_2(x)}{\sigma_2\hat{q}_2(x)}\right)\right], \tag{2.7}$$

(iii) Dual sourcing from both suppliers if

$$\mu_1 w_2 - \mu_2 w_1 < p\sigma_1\mu_2 E\left[\phi\left(\frac{D_2 - x - \mu_1\hat{q}_1(x)}{\sigma_1\hat{q}_1(x)}\right)\right], \tag{2.8}$$

and

$$\mu_2 w_1 - \mu_1 w_2 < p\sigma_2\mu_1 E\left[\phi\left(\frac{D_2 - x - \mu_2\hat{q}_2(x)}{\sigma_2\hat{q}_2(x)}\right)\right], \tag{2.9}$$

where $\hat{q}_1(x)$ *and* $\hat{q}_2(x)$ *satisfy*

$$p\mu_2 E\left[\Phi\left(\frac{D_2 - x - \mu_1\hat{q}_1(x)}{\sigma_1\hat{q}_1(x)}\right)\right] - w_2 = 0, \tag{2.10}$$

$$p\mu_1 E\left[\Phi\left(\frac{D_2 - x - \mu_2\hat{q}_2(x)}{\sigma_2\hat{q}_2(x)}\right)\right] - w_1 = 0. \tag{2.11}$$

Proof. We discuss the condition for the retailer to determine whether to choose single sourcing from supplier 1. The case that single sourcing from supplier 2 is parallel. Let $q_{22} = 0$, the optimal solution $q_{21}(0)$ satisfies the first order condition (2.4). Transform Y into standard normal,

$$\frac{\partial\Pi_2(q_{21}(0),0)}{\partial q_{21}} = p\mu_1 E\Phi\left(\frac{D_2 - x - \mu_1 q_{21}(0)}{\sigma_1 q_{21}(0)}\right) - p\sigma_1 E\phi\left(\frac{D_2 - x - \mu_1 q_{21}(0)}{\sigma_1 q_{21}(0)}\right) - w_1 = 0.$$

Given $\hat{q}_1(x)$ from (2.10), then

$$\left.\frac{\partial\Pi_2(q_{21},0)}{\partial q_{21}}\right|_{q_{21}=\hat{q}_1} = \frac{1}{\mu_2}\left[\mu_1 w_2 - \mu_2 w_1 - p\sigma_1\mu_2 E\phi\left(\frac{D_2 - x - \mu_1\hat{q}_1}{\sigma_1\hat{q}_1}\right)\right].$$

If $\mu_1 w_2 - \mu_2 w_1 \geq p\sigma_1\mu_2 E\phi\left(\frac{D_2 - x - \mu_1\hat{q}_1}{\sigma_1\hat{q}_1}\right)$, then we find $\frac{\partial\Pi_2(\hat{q}_1,0)}{\partial q_{21}} \geq 0$. Since $\Pi_2(q_{21}, q_{22})$ is concave in q_{21}, we find that $q_{21}(0) \geq \hat{q}_1$. Moreover, given $q_{21} = q_{21}(q_{22})$ and $q_{22} = 0$, the partial derivative on q_{22} is

$$\left.\frac{\partial\Pi_2(q_{21}(q_{22}), q_{22})}{\partial q_{22}}\right|_{q_{22}=0} = p\mu_2 E\Phi\left(\frac{D_2 - x - \mu_1 q_{21}^*(0)}{\sigma_1 q_{21}^*}\right) - w_2 \leq 0,$$

where the inequality follows from that if $D_2 - x > 0$,

$$p\mu_2 E\Phi\left(\frac{D_2 - x - \mu_1 q_{21}^*(0)}{\sigma_1 q_{21}^*}\right) - w_2 \leq p\mu_2 E\Phi\left(\frac{D_2 - x - \mu_1\hat{q}_1(x)}{\sigma_1\hat{q}_1(x)}\right) - w_2 = 0;$$

otherwise, $\Phi((D_2 - x - \mu_1)/\sigma_1 q_{21}^*) \approx 0$ when $D_2 < x$ as we assume $\mu_1 > 3\sigma_1$. Due to the joint concavity of $\Pi_2(q_{21}, q_{22})$ on q_{21} and q_{22}, the optimal ordering quantity from supplier 2 is 0. In other words, the retailer will choose single sourcing from supplier 1. Through similar procedure, we can show that, if (2.7) is true, the retailer will only source from supplier 2.

Finally, if

$$\mu_1 w_2 - \mu_2 w_1 < p\sigma_1\mu_2 E\phi\left(\frac{D_2 - x - \mu_1\hat{q}_1}{\sigma_1\hat{q}_1}\right),$$

similarly it can be found that $(\Pi_2(q_{21}(0), 0))'_{q_{22}} > 0$, and hence the optimal ordering quantity q_{22}^* should be positive. Furthermore, through similar analysis it can be found that $(\Pi_2(0, q_{22}(0)))'_{q_{21}} > 0$ under the condition

$$\mu_2 w_1 - \mu_1 w_2 < p\sigma_2\mu_1 E\phi\left(\frac{D_2 - x - \mu_2\hat{q}_2(x)}{\sigma_2\hat{q}_2(x)}\right).$$

Thus the optimal ordering quantity q_{21}^* should be positive. Therefore, the retailer will choose dual sourcing from both suppliers. □

From the previous theorem, the optimal ordering quantity is $q_{21}^* = q_{21}(0)$ while $q_{22}^* = 0$, if (2.6) is true; the optimal ordering quantity $q_{21}^* = 0$ while $q_{22}^* = q_{22}(0)$, if (2.7) is true; otherwise, $q_{21}^* = q_{21}(q_{22}^*)$ while $q_{22}^* = q_{22}(q_{21}^*)$.

Notice that $\hat{q}_i$ given in (2.10) and (2.11) is function of inventory level x, we find the following property.

Proposition 2.3.3. *$\hat{q}_i(x)$ is decreasing in x, furthermore, the retailer is more likely to choose single sourcing than dual sourcing as the ending inventory of first period increases.*

Proof. Recall that $\hat{q}_1(x)$ is the solution of (2.10). Taking derivative of (2.10) with respect to x and applying Implicit Function Theorem yields,

$$p\mu_2 \int_{x+\mu_1\hat{q}_1(x)}^{\infty} \phi\left(\frac{\xi - x - \mu_1\hat{q}_1(x)}{\sigma_1\hat{q}_1(x)}\right) \frac{-\hat{q}_1(x) - (\xi - x)\hat{q}_1'(x)}{\sigma_1\hat{q}_1^2(x)} f(\xi)\mathrm{d}\xi = 0, \quad (2.12)$$

as $\Phi\left(\frac{D_2 - x - \mu_1\hat{q}_1(x)}{\sigma_1\hat{q}_1(x)}\right) \approx 0$ when $D_2 - x < 0$. Thus

$$\hat{q}_1'(x) < 0.$$

Since $\hat{q}_1'(x) < 0$, note that

$$\begin{aligned}
&E\left[\phi\left(\frac{D_2 - x - \mu_1\hat{q}_1(x)}{\sigma_1\hat{q}_1(x)}\right)\right]' \\
&= \int_{x+\mu_1\hat{q}_1(x)}^{\infty} -\frac{\xi - x - \mu_1\hat{q}_1(x)}{\sigma_1\hat{q}_1(x)} \phi\left(\frac{\xi - x - \mu_1\hat{q}_1(x)}{\sigma_1\hat{q}_1(x)}\right) \frac{-\hat{q}_1(x) - (\xi - x)\hat{q}_1'(x)}{\sigma_1\hat{q}_1^2(x)} f(\xi)\mathrm{d}\xi \\
&= \int_{x+\mu_1\hat{q}_1(x)}^{\infty} \xi\phi\left(\frac{\xi - x - \mu_1\hat{q}_1(x)}{\sigma_1\hat{q}_1(x)}\right) \frac{\hat{q}_1(x) + (\xi - x)\hat{q}_1'(x)}{\sigma_1^2\hat{q}_1^3(x)} f(\xi)\mathrm{d}\xi \\
&= \int_{x+\mu_1\hat{q}_1(x)}^{x-\hat{q}_1(x)/\hat{q}_1'(x)} \xi\phi\left(\frac{\xi - x - \mu_1\hat{q}_1(x)}{\sigma_1\hat{q}_1(x)}\right) \frac{\hat{q}_1(x) + (\xi - x)\hat{q}_1'(x)}{\sigma_1^2\hat{q}_1^3(x)} f(\xi)\mathrm{d}\xi \\
&\quad + \int_{x-\hat{q}_1(x)/\hat{q}_1'(x)}^{+\infty} \xi\phi\left(\frac{\xi - x - \mu_1\hat{q}_1(x)}{\sigma_1\hat{q}_1(x)}\right) \frac{\hat{q}_1(x) + (\xi - x)\hat{q}_1'(x)}{\sigma_1^2\hat{q}_1^3(x)} f(\xi)\mathrm{d}\xi \\
&\leq [x - \hat{q}_1(x)/\hat{q}_1'(x)] \int_{x+\mu_1\hat{q}_1(x)}^{x-\hat{q}_1(x)/\hat{q}_1'(x)} \phi\left(\frac{\xi - x - \mu_1\hat{q}_1(x)}{\sigma_1\hat{q}_1(x)}\right) \frac{\hat{q}_1(x) + (\xi - x)\hat{q}_1'(x)}{\sigma_1^2\hat{q}_1^3(x)} f(\xi)\mathrm{d}\xi \\
&\quad \times [x - \hat{q}_1(x)/\hat{q}_1'(x)] \int_{x-\hat{q}_1(x)/\hat{q}_1'(x)}^{+\infty} \phi\left(\frac{\xi - x - \mu_1\hat{q}_1(x)}{\sigma_1\hat{q}_1(x)}\right) \frac{\hat{q}_1(x) + (\xi - x)\hat{q}_1'(x)}{\sigma_1^2\hat{q}_1^3(x)} f(\xi)\mathrm{d}\xi \\
&= [x - \hat{q}_1(x)/\hat{q}_1'(x)] \int_{x+\mu_1\hat{q}_1(x)}^{\infty} \phi\left(\frac{\xi - x - \mu_1\hat{q}_1(x)}{\sigma_1\hat{q}_1(x)}\right) \frac{\hat{q}_1(x) + (\xi - x)\hat{q}_1'(x)}{\sigma_1^2\hat{q}_1^3(x)} f(\xi)\mathrm{d}\xi = 0.
\end{aligned}$$

Where the second and last equalities follow from (2.12), and the inequality follows by noting that $\frac{\hat{q}_1(x) + (\xi - x)\hat{q}_1'(x)}{\sigma_1^2\hat{q}_1^3(x)} \geq 0$ when $\xi \leq x - \hat{q}_1(x)/\hat{q}_1'(x)$ and $\frac{\hat{q}_1(x) + (\xi - x)\hat{q}_1'(x)}{\sigma_1^2\hat{q}_1^3(x)} \leq 0$ when $\xi \geq x - \hat{q}_1(x)/\hat{q}_1'(x)$. Thus $E\left[\phi\left(\frac{D_2 - x - \mu_1\hat{q}_1(x)}{\sigma_1\hat{q}_1(x)}\right)\right]$ is increasing in x. Finally observing the conditions for identifying single or dual sourcing, it is concluded that the retailer will be more likely to choose single sourcing than dual sourcing as inventory increases. □

Further noting that the optimal ordering quantity depends on w_i, there are the following comparative statistics results.

Proposition 2.3.4. *$q^*_{2i}(w_i, w_{3-i})$ is decreasing in w_i and increasing in w_{3-i}, where $i = 1, 2$.*

Proof. According to the first order conditions and Implicit Function Theorem, it can be found that

$$q^{*\prime}_{21}(w_1) = \frac{(\Pi_2(q_{21}, q_{22}))''_{q_{22}}}{(\Pi_2(q_{21}, q_{22}))''_{q_{21}}(\Pi_2(q_{21}, q_{22}))''_{q_{22}} - (\Pi_2(q_{21}, q_{22}))''_{q_{21}q_{22}}} \leq 0$$

and

$$q^{*\prime}_{21}(w_2) = \frac{(\Pi_2(q_{21}, q_{22})''_{q_{21}q_{22}}}{(\Pi_2(q_{21}, q_{22}))''_{q_{21}q_{22}})^2 - (\Pi_2(q_{21}, q_{22}))''_{q_{21}}\Pi_2(q_{21}, q_{22})_{q_{22}}} \geq 0,$$

where the first inequality follows from the joint concavity of $\Pi_2(q_{21}, q_{22})$, and the second inequality follows further from the submodularity of $\Pi_2(q_{21}, q_{22})$. □

2.4 Analysis of the Two-Period Problem

This section will discuss the retailer's optimal sourcing strategies and ordering quantity in the first period.

Following from Lemma 2.3.1, there is the following lemma.

Lemma 2.4.1. *$\Pi_1(q_{11}, q_{12})$ is jointly concave in q_{11} and q_{12}.*

If

$$x \geq \max\left\{F^{-1}\left(1 - \frac{w_1}{p\mu_1}\right), F^{-1}\left(1 - \frac{w_2}{p\mu_2}\right)\right\}, \tag{2.13}$$

then the retailer orders nothing, and the optimal profit of the second period is

$$\Pi_2(0, 0) = pE\min\{D_2, x\} = pd_2 - pE[(D_2 - x)^+].$$

From Theorem 2.3.2, if

$$F^{-1}\left(1 - \frac{w_2}{p\mu_2}\right) \leq x < F^{-1}\left(1 - \frac{w_1}{p\mu_1}\right)$$
$$\text{or } x < \min\left\{F^{-1}\left(1 - \frac{w_1}{p\mu_1}\right), F^{-1}\left(1 - \frac{w_2}{p\mu_2}\right)\right\} \text{ and (2.6)}, \tag{2.14}$$

then the retailer chooses single sourcing from supplier 1. The optimal ordering quantity $q^*_{21}(x)$ is given by

$$p\mu_1 E\Phi\left(\frac{D_2 - x - \mu_1 q^*_{21}(x)}{\sigma_1 q^*_{21}(x)}\right) - p\sigma_1 E\phi\left(\frac{D_2 - x - \mu_1 q^*_{21}(x)}{\sigma_1 q^*_{21}(x)}\right) - w_1 = 0.$$

The optimal profit of the second period is

$$\begin{aligned}\Pi_2(q_{21}^*(x),0) &= pE\{\min[D_2, x + Y_1 q_{21}^*(x)]\} - w_1 q_{21}^*(x)\\ &= pd_2 - pE\left[(D_2 - x)\Phi\left(\frac{D_2 - x - \mu_1 q_{21}^*(x)}{\sigma_1 q_{21}^*(x)}\right)\right].\end{aligned}$$

Similarly if

$$\begin{aligned}&F^{-1}\left(1 - \frac{w_1}{p\mu_1}\right) \le x < F^{-1}\left(1 - \frac{w_2}{p\mu_2}\right)\\ &\text{or } \; x < \min\left\{F^{-1}\left(1 - \frac{w_1}{p\mu_1}\right), F^{-1}\left(1 - \frac{w_2}{p\mu_2}\right)\right\} \text{ and (2.7)},\end{aligned} \tag{2.15}$$

then the retailer chooses single sourcing from supplier 2. The optimal ordering quantity $q_{22}^*(x)$ is given by

$$p\mu_2 E\Phi\left(\frac{D_2 - x - \mu_2 q_{22}^*(x)}{\sigma_2 q_{22}^*(x)}\right) - p\sigma_2 E\phi\left(\frac{D_2 - x - \mu_2 q_{22}^*(x)}{\sigma_2 q_{22}^*(x)}\right) - w_2 = 0.$$

The optimal profit of the second period is

$$\Pi_2(0, q_{22}^*(x)) = pd_2 - pE\left[(D_2 - x)\Phi\left(\frac{D_2 - x - \mu_2 q_{22}^*(x)}{\sigma_2 q_{22}^*(x)}\right)\right].$$

Further if

$$x < \min\left\{F^{-1}\left(1 - \frac{w_1}{p\mu_1}\right), F^{-1}\left(1 - \frac{w_2}{p\mu_2}\right)\right\} \text{ and (2.8) and (2.9)}, \tag{2.16}$$

then the retailer chooses dual sourcing in the second period. Then

$$p\mu_1 EG(D_2 - x, q_{21}^*(x), q_{22}^*(x)) - p\sigma_1^2 q_{21}^*(x) E[g(D_2 - x, q_{21}^*(x), q_{22}^*(x))] - w_1 = 0,$$

$$p\mu_2 EG(D_2 - x, q_{21}^*(x), q_{22}^*(x)) - p\sigma_2^2 q_{22}^*(x) E[g(D_2 - x, q_{21}^*(x), q_{22}^*(x))] - w_2 O = 0.$$

The optimal profit of the second period is then

$$\Pi_2(q_{21}^*(x), q_{22}^*(x)) = pd_2 - pE\left[(D_2 - x)\Phi\left(\frac{D_2 - x - \mu_1 q_{21}^*(x) - \mu_2 q_{22}^*(x)}{\sqrt{\sigma_1^2 q_{21}^*(x)^2 + \sigma_2^2 q_{22}^*(x)^2}}\right)\right].$$

Therefore, the optimal profit-to-go is

$$V_2(x) = \begin{cases} pd_2 - pE[(D_2 - x)^+], & (2.13) \text{ true}, \\ pd_2 - pE\left[(D_2 - x)\Phi\left(\frac{D_2 - x - \mu_1 q_{21}^s(x)}{\sigma_1 q_{21}^s(x)}\right)\right], & (2.14) \text{ true}, \\ pd_2 - pE\left[(D_2 - x)\Phi\left(\frac{D_2 - x - \mu_1 q_{21}^d(x) - \mu_2 q_{22}^*(x)}{\sqrt{\sigma_1^2 (q_{21}^d(x))^2 + \sigma_2^2 q_{22}^*(x)^2}}\right)\right], & (2.16) \text{ true}, \\ pd_2 - pE\left[(D_2 - x)\Phi\left(\frac{D_2 - x - \mu_2 q_{22}^s(x)}{\sigma_2 q_{22}^s(x)}\right)\right], & (2.15) \text{ true}. \end{cases} \tag{2.17}$$

Before we turn to the optimization problem of the first period, we find that the following lemma is useful.

Lemma 2.4.2.

$$\frac{\mathrm{d}V_2(x)}{\mathrm{d}x} = \begin{cases} p(1 - F(x)), & (2.13) \textit{ true}, \\ pE\Phi\left(\dfrac{D_2 - x - \mu_1 q_{21}^s(x)}{\sigma_1 q_{21}^s(x)}\right), & (2.14) \textit{ true}, \\ pE\Phi\left(\dfrac{D_2 - x - \mu_1 q_{21}^d(x) - \mu_2 q_{22}^*(x)}{\sqrt{\sigma_1^2 q_{21}^d(x)^2 + \sigma_2^2 q_{22}^*(x)^2}}\right), & (2.16) \textit{ true}, \\ pE\Phi\left(\dfrac{D_2 - x - \mu_2 q_{22}^s(x)}{\sigma_2 q_{22}^s(x)}\right), & (2.15) \textit{ true}. \end{cases}$$

Proof. The result is straightforward when Lemma (2.13) is true. If Lemma (2.14) is true, then $q_{21}^*(x)$ satisfies the first order condition

$$p\mu_1 E\Phi\left(\frac{D_2 - x - \mu_1 q_{21}^*(x)}{\sigma_1 q_{21}^*(x)}\right) - p\sigma_1 E\phi\left(\frac{D_2 - x - \mu_1 q_{21}^*(x)}{\sigma_1 q_{21}^*(x)}\right) - w_1 = 0.$$

Take derivative with respect to x yields

$$E\left[\frac{D_2 - x}{\sigma_1 q_{21}^*(x)}\phi\left(\frac{D_2 - x - \mu_1 q_{21}^*(x)}{\sigma_1 q_{21}^*(x)}\right)\frac{\partial \frac{D_2 - x}{q_{21}^*(x)}}{\partial x}\right] = 0.$$

Thus, as $q_{21}^*(x) > 0$,

$$\begin{aligned}\frac{\mathrm{d}V_2(x)}{\mathrm{d}x} = pE\left[\Phi\left(\frac{D_2 - x - \mu_1 q_{21}^*(x)}{\sigma_1 q_{21}^*(x)}\right)\right] \\ -pE\left[(D_2 - x)\phi\left(\frac{D_2 - x - \mu_1 q_{21}^*(x)}{\sigma_1 q_{21}^*(x)}\right)\frac{1}{\sigma_2}\frac{\partial \frac{D_2 - x}{q_{21}^*(x)}}{\partial x}\right] \\ = pE\left[\Phi\left(\frac{D_2 - x - \mu_1 q_{21}^* 0(x)}{\sigma_1 q_{21}^*(x)}\right)\right].\end{aligned}$$

The proof is similar when (2.15) is true.

When (2.16) is true, the first order conditions which $q_{21}^*(x)$ and $q_{22}^*(x)$ satisfy can be written as

$$pE\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{\frac{D_2 - x - q_{22}^*(x) y_2}{q_{21}^*(x)}} y_1 g_1(y_1) g_2(y_2) \mathrm{d}y_1 \mathrm{d}y_2\right] - w_1 = 0,$$

$$pE\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{\frac{D_2 - x - q_{22}^*(x) y_2}{q_{21}^*(x)}} y_2 g_1(y_1) g_2(y_2) \mathrm{d}y_1 \mathrm{d}y_2\right] - w_2 = 0.$$

Taking derivative with respect to x yields

$$\begin{aligned}pE\Bigg[\int_{-\infty}^{+\infty}\left(\frac{D_2 - x - y_2 q_{22}^*(x)}{q_{21}^*(x)}\right)'_x \frac{D_2 - x - y_2 q_{22}^*(x)}{q_{21}^*(x)} g_1 \\ \times\left(\frac{D_2 - x - y_2 q_{22}^*(x)}{q_{21}^*(x)}\right) g_2(y_2)\mathrm{d}y_2\Bigg] = 0,\end{aligned} \tag{2.18}$$

$$\begin{aligned}pE\Bigg[\int_{-\infty}^{+\infty}\left(\frac{D_2 - x - y_2 q_{22}^*(x)}{q_{21}^*(x)})\right)'_x y_2 g_1 \\ \times\left(\frac{D_2 - x - y_2 q_{22}^*(D_2 - x)}{q_{21}^*(x)}\right) g_2(y_2)\mathrm{d}y_2\Bigg] = 0.\end{aligned} \tag{2.19}$$

Therefore, we find that

$$
\begin{aligned}
\frac{\mathrm{d}V_2(x)}{\mathrm{d}x} &= pE\left[\Phi\left(\frac{D_2 - x - \mu_1 q_{21}^*(x) - \mu_2 q_{22}^*(x)}{\sqrt{(\sigma_1^2)(q_{21}^*(x))^2 + (\sigma_2^2)(q_{22}^*(x))^2}}\right)\right] \\
&\quad - pE\left[(D_2 - x)\left(\int_{-\infty}^{\infty}\int_{-\infty}^{\frac{D_2 - x - q_{22}^*(x)y_2}{q_{21}^*(x)}} g_1(y_1)g_2(y_2)\mathrm{d}y_1\mathrm{d}y_2\right)_x'\right] \\
&= pE\left[\Phi\left(\frac{D_2 - x - \mu_1 q_{21}^*(x) - \mu_2 q_{22}^*(x)}{\sqrt{\sigma_1^2(q_{21}^*(x))^2 + \sigma_2^2(q_{22}^*(x))^2}}\right)\right] \\
&\quad - pE\left[(D_2 - x)\int_{-\infty}^{+\infty}\left(\frac{D_2 - x - y_2 q_{22}^*(x)}{q_{21}^*(x)}\right)_x' \right. \\
&\qquad \left. g_1\left(\frac{D_2 - x - y_2 q_{22}^*(x)}{q_{21}^*(x)}\right) g_2(y_2)\mathrm{d}y_2\right] \\
&= pE\left[\Phi\left(\frac{D_2 - x - \mu_1 q_{21}^*(x) - \mu_2 q_{22}^*(x)}{\sqrt{\sigma_1^2(q_{21}^*(x))^2 + \sigma_2^2(q_{22}^*(x))^2}}\right)\right].
\end{aligned}
$$

where the penultimate equality follows from that

$$
\begin{aligned}
& pE\left[(D_2 - x)\int_{-\infty}^{+\infty}\left(\frac{D_2 - x - y_2 q_{22}^*(x)}{q_{21}^*(x)}\right)_x' g_1\left(\frac{D_2 - x - y_2 q_{22}^*(x)}{q_{21}^*(x)}\right) g_2(y_2)\mathrm{d}y_2\right] \\
&= q_{21}^*(x)pE\left[\int_{-\infty}^{+\infty}\left(\frac{D_2 - x - y_2 q_{22}^*(x)}{q_{21}^*(x)}\right)_x' \frac{D_2 - x - y_2 q_{22}^*(x)}{q_{21}^*(x)} \right. \\
&\qquad \left. g_1\left(\frac{D_2 - x - y_2 q_{22}^*(x)}{q_{21}^*(x)}\right) g_2(y_2)\mathrm{d}y_2\right] \\
&\quad + q_{22}^*(x)pE\left[\int_{-\infty}^{+\infty}\left(\frac{D_2 - x - y_2 q_{22}^*(x)}{q_{21}^*(x)}\right)_x' y_2 \right. \\
&\qquad \left. g_1\left(\frac{D_2 - x - y_2 q_{22}^*(D_2 - x)}{q_{21}^*(x)}\right) g_2(y_2)\mathrm{d}y_2\right] = 0.
\end{aligned}
$$

Since $V_2(x)$ is continuous in x across different ranges, the lemma is proved. □

Next we proceed to the optimization problem of the first period. For notational convenience, define, for $i = 1, 2$, the different conditions for no sourcing (NS), single sourcing (SS), and dual sourcing (DS)

$$(\text{NS}) : Y_i q_{1,i} + Y_{3-i} q_{1,3-i} - D_1 \geq \max\left\{ F^{-1}\left(1 - \frac{w_i}{p\mu_i}\right), F^{-1}\left(1 - \frac{w_{3-i}}{p\mu_{3-i}}\right)\right\},$$

$$(\text{SS}_i) : F^{-1}\left(1 - \frac{w_{3-i}}{p\mu_{3-i}}\right) \leq Y_i q_{1,i} + Y_{3-i} q_{1,3-i} - D_1 < F^{-1}\left(1 - \frac{w_i}{p\mu_i}\right),$$

$$\text{or } Y_i q_{1,i} + Y_{3-i} q_{1,3-i} - D_1 < \min\left\{ F^{-1}\left(1 - \frac{w_i}{p\mu_i}\right), F^{-1}\left(1 - \frac{w_{3-i}}{p\mu_{3-i}}\right)\right\}$$

$$\text{and } \mu_i w_{3-i} - \mu_{3-i} w_i$$

$$\geq p\sigma_i \mu_{3-i} E\phi\left(\frac{D_1 + D_2 - Y_i q_{1i} - Y_{3-i} q_{13-i} - \mu_i \hat{q}_i (Y_i q_{1,i} + Y_{3-i} q_{1,3-i} - D_1)}{\sigma_i \hat{q}_i (Y_i q_{1,i} + Y_{3-i} q_{1,3-i} - D_1)}\right),$$

$$(\text{DS}_i) : Y_i q_{1,i} + Y_{3-i} q_{1,3-i} - D_1 < \min\left\{ F^{-1}\left(1 - \frac{w_i}{p\mu_i}\right), F^{-1}\left(1 - \frac{w_{3-i}}{p\mu_{3-i}}\right)\right\},$$

$$\text{and } \mu_i w_{3-i} - \mu_{3-i w_i}$$

$$< p\sigma_i \mu_{3-i} E\phi\left(\frac{D_1 + D_2 - Y_i q_{1,i} - Y_{3-i} q_{1,3-i} - \mu_i \hat{q}_i (Y_i q_{1,i} + Y_{3-i} q_{1,3-i} - D_1)}{\sigma_i \hat{q}_i (Y_i q_{1,i} + Y_{3-i} q_{1,3-i} - D_1)}\right).$$

Then the total profit function for the retailer can be shown as

$$\begin{aligned}
&\Pi_1(q_{11}, q_{12}) \\
&\quad = E\{pD_1 - w_1 q_{11} - w_2 q_{12} + V_2(Y_1 q_{11} + Y_2 q_{12} - D_1)\} \\
&\quad = p(d_1 + d_2) - w_1 q_{11} - w_2 q_{12} \\
&\qquad - pE\Bigg\{(D_1 + D_2 - Y_1 q_{11} - Y_2 q_{12})\Bigg[\mathbf{1}_{\{D_2 \geq Y_1 q_{11} + Y_2 q_{12} - D_1\}} \mathbf{1}_{\{(\text{NS})\}} \\
&\qquad + \Phi\left(\frac{D_1 + D_2 - Y_1 q_{11} - Y_2 q_{12} - \mu_1 q_{21}^*(Y_1 q_{11} + Y_2 q_{12} - D_1)}{\sigma_1 q_{21}^*(Y_1 q_{11} + Y_2 q_{12} - D_1)}\right) \mathbf{1}_{\{(\text{SS}_1)\}} \\
&\qquad + \Phi\left(\frac{D_1 + D_2 - Y_1 q_{11} - Y_2 q_{12} - \mu_1 q_{21}^*(Y_1 q_{11} + Y_2 q_{12} - D_1) - \mu_2 q_{22}^*(Y_1 q_{11} + Y_2 q_{12} - D_1)}{\sqrt{\sigma_1^2 (q_{21}^*(Y_1 q_{11} + Y_2 q_{12} - D_1))^2 + \sigma_2^2 (q_{22}^*(Y_1 q_{11} + Y_2 q_{12} - D_1))^2}}\right) \\
&\qquad \times \mathbf{1}_{\{(\text{DS}_1)(\text{DS}_2)\}} \\
&\qquad + \Phi\left(\frac{D_1 + D_2 - Y_1 q_{11} - Y_2 q_{12} - \mu_2 q_{22}^*(Y_1 q_{11} + Y_2 q_{12} - D_1)}{\sigma_2 q_{22}^*(Y_1 q_{11} + Y_2 q_{12} - D_1)}\right) \mathbf{1}_{\{(\text{SS}_2)\}}\Bigg]\Bigg\}.
\end{aligned} \tag{2.20}$$

Let $\Gamma(Y_1 q_{11} + Y_2 q_{12} - D_1)$ be the function in square brackets after taking expectation over D_2. There is

$$\Pi_1(q_{11}, q_{12}) = p(d_1 + d_2) - w_1 q_{11} - w_2 q_{12} - pE[(D_1 + D_2 - Y_1 q_{11} - Y_2 q_{12})\Gamma(Y_1 q_{11} + Y_2 q_{12} - D_1)].$$

It can be found that $p\Gamma(x) = (V_2(x))'$. With different sourcing strategies in the second period, $\Gamma(Y_1 q_{11} + Y_2 q_{12} - D_1)$ reflects different probabilities that the demand of the second period is not satisfied given inventory $Y_1 q_{11} + Y_2 q_{12} - D_1$. Notice that $0 \leq \Gamma(Y_1 q_{11} + Y_2 q_{12} - D_1) \leq 1$.

The first order conditions for the optimization problem of the first period are given by

$$\begin{aligned}(\Pi_1(q_{11},q_{12}))'_{q_{11}} &= pE[Y_1V_2(Y_1q_{11}+Y_2q_{12}-D_1)]'-w_1\\ &= pE[Y_1\Gamma(Y_1q_{11}+Y_2q_{12}-D_1)]-w_1\\ &= 0,\end{aligned}$$

and

$$(\Pi_1(q_{11},q_{12}))'_{q_{12}} = pE[Y_2\Gamma(Y_1q_{11}+Y_2q_{12}-D_1)]-w_2=0.$$

Then straightforwardly, the retailer's optimal sourcing strategy in the first period is provided in the following theorem.

Theorem 2.4.3. *In the first period, the retailer's optimal sourcing strategy is*

(i) Single sourcing from supplier 1 *if*

$$pE[Y_1\Gamma(Y_1\hat{q}_1^1-D_1)]-w_1\geq 0,$$

(ii) Single sourcing from supplier 2 *if*

$$pE[Y_2\Gamma(Y_2\hat{q}_2^1-D_1)]-w_2\geq 0,$$

(iii) Dual sourcing from the two suppliers if

$$\begin{aligned}pE[Y_1\Gamma(Y_1\hat{q}_1^1-D_1)]-w_1<0,\\ pE[Y_2\Gamma(Y_2\hat{q}_2^1-D_1)]-w_2<0,\end{aligned}$$

where $\hat{q}_1^1$ *and* $\hat{q}_2^1$ *are given by*

$$\begin{aligned}p\mu_2E[\Gamma(Y_1\hat{q}_1^1-D_1)]-w_2=0,\\ p\mu_1E[\Gamma(Y_2\hat{q}_2^1-D_1)]-w_1=0.\end{aligned}$$

Proof. The proof is similar to that for Theorem 2.3.2. Firstly we discuss the condition for the retailer to determine whether to choose single sourcing from supplier 1. Let $q_{12}=0$, the optimal solution q_{11}^* satisfies the first order condition

$$\frac{\partial\Pi_1(q_{11}^*,0)}{\partial q_{11}} = pE[Y_1\Gamma(D_2,Y_1q_{11}^*-D_1)]-w_1=0.$$

Given $\hat{q}_1^1$ so that $p\mu_2E[\Gamma(D_2,Y_1\hat{q}_1^1-D_1)]-w_2=0$, if $pE[Y_1\Gamma(D_2,Y_1\hat{q}_1^1-D_1)]-w_1\geq 0$, then $\frac{\partial\Pi_1(\hat{q}_1^1,0)}{\partial q_{11}}\geq 0$. Since $\Pi_1(q_{11},q_{12})$ is concave in q_{11}, we find that $q_{11}^*\geq\hat{q}_1^1$. Thus given $q_{11}=q_{11}^*$ and $q_{12}=0$, the partial derivative on q_{12} is

$$\frac{\partial\Pi_1(q_{11}^*,0)}{\partial q_{12}} = p\mu_2E[\Gamma(D_2,Y_1q_{11}^*-D_1)]-w_2\leq p\mu_2E[\Gamma(D_2,Y_1\hat{q}_1^1-D_1)]-w_2=0.$$

Where the inequality follows from Lemma 2.3.1 by noting that $V_2(x)$ is concave in x, and hence $\frac{\mathrm{d}V_2(x)}{\mathrm{d}x} = pE\Gamma(D_2, x)$ is decreasing in x. Next due to the joint concavity of $\Pi_1(q_{11}, q_{12})$ on q_{11} and q_{12}, the optimal ordering quantity from supplier 2 is 0. In other words, the retailer will choose single sourcing from supplier 1. The other proof follows similar procedure to the proof for Theorem 2.3.2. □

The following proposition indicates that there is no incentive for the retailer to switch suppliers.

Proposition 2.4.4. *The retailer will not choose single sourcing from different suppliers in different periods.*

Proof. Without loss of generality, assume the retailer choose single sourcing from supplier 2 in the second period, i.e., inequality (2.7) is satisfied. It is needed to prove the retailer will not choose single sourcing from supplier 1 in the first period. First notice that (2.7) indicates $\mu_2 w_1 - \mu_1 w_2 > 0$. Then we find that

$$
\begin{aligned}
& pE[Y_1\Gamma(Y_1\hat{q}_1^1 - D_1)] - w_1 \\
&= pE\left[\int_{-\infty}^{\infty} y_1\Gamma(y_1\hat{q}_1^1 - D_1)g_1(y_1)\mathrm{d}y_1\right] - w_1 \\
&= p\mu_1 E\left[\int_{-\infty}^{+\infty} \Gamma(y_1\hat{q}_1^1 - D_1)g_1(y_1)\mathrm{d}y_1\right] \\
&\quad - p\sigma_1^2 E\left[\int_{-\infty}^{+\infty} \Gamma(y_1\hat{q}_1^1 - D_1)g_1'(y_1)\mathrm{d}y_1\right] - w_1 \\
&= p\mu_1 \frac{w_2}{p\mu_2} - p\sigma_1^2 E\Bigg[\Gamma(y_1\hat{q}_1^1 - D_1)g_1(y_1)|_{-\infty}^{+\infty} \\
&\quad - \int_{-\infty}^{+\infty} (\partial\Gamma(y_1\hat{q}_1^1 - D_1))(y_1)g_1(y_1)\mathrm{d}y_1\Bigg] - w_1 \\
&= p\mu_1 \frac{w_2}{p\mu_2} + p\sigma_1^2 E\left[\int_{-\infty}^{+\infty} (\partial\Gamma(y_1\hat{q}_1^1 - D_1))(y_1)g_1(y_1)\mathrm{d}y_1\right] - w_1 \\
&\le \frac{\mu_1 w_2}{\mu_2} - w_1 \\
&< 0,
\end{aligned}
$$

where the third equality follows from the definition of $\hat{q}_1^1$, and the penultimate inequality is due to the fact that $V_2(x)$ is concave and hence $E[\Gamma(y_1\hat{q}_1^1 - D_1)] = (V_2(y_1\hat{q}_1^1 - D_1))'$ is decreasing in y_1. Therefore, the condition for the retailer to choose single sourcing from supplier 1 in Theorem 2.4.3 can not be satisfied, i.e., the retailer will not choose single sourcing from supplier 1. □

When demand is deterministic, more structural results and sights can be found. In Sect. 2.5, we report our findings.

2.5 A Special Case: Deterministic Demand

In this section, to gain more insight and structrual results, we consider the case that the demand is deterministic, $D = d$. Note that the first order conditions (2.4) and (2.5) change to

$$p\mu_1 G(d_2 - x, q_1, q_2) - p\sigma_1^2 q_1 g(d_2 - x, q_1, q_2) - w_1 = 0,$$
$$p\mu_2 G(d_2 - x, q_1, q_2) - p\sigma_2^2 q_2 g(d_2 - x, q_1, q_2) - w_2 = 0.$$

Notice that $\hat{q}_1$ and $\hat{q}_2$ given in (2.10) and (2.11) can be derived in close form. Then the retailer's optimal sourcing strategy could be simplified as follows.

Theorem 2.5.1. *In the second period, the retailer's optimal sourcing strategy is independent of demand as well as ending inventory of the previous period. Specifically, it is optimal for the retailer to choose*

(i) Single sourcing from supplier 1 *if*

$$\mu_1 w_2 - \mu_2 w_1 \geq p\sigma_1\mu_2\phi\left(\Phi^{-1}\left(\frac{w_2}{p\mu_2}\right)\right), \tag{2.21}$$

(ii) Single sourcing from supplier 2 *if*

$$\mu_2 w_1 - \mu_1 w_2 \geq p\sigma_2\mu_1\phi\left(\Phi^{-1}\left(\frac{w_1}{p\mu_1}\right)\right), \tag{2.22}$$

(iii) Dual sourcing from the two suppliers if

$$\mu_1 w_2 - \mu_2 w_1 < p\sigma_1\mu_2\phi\left(\Phi^{-1}\left(\frac{w_2}{p\mu_2}\right)\right), \tag{2.23}$$

$$\mu_2 w_1 - \mu_1 w_2 < p\sigma_2\mu_1\phi\left(\Phi^{-1}\left(\frac{w_1}{p\mu_1}\right)\right). \tag{2.24}$$

We find more properties characterizing the optimal sourcing strategy. Define

$$T_i^w(w_i) = \left[\mu_i w_{3-i} - p\sigma_{3-i}\mu_i\phi\left(\Phi^{-1}\left(\frac{w_i}{p\mu_i}\right)\right)\right] \Big/ \mu_i$$

the functions of w_i.

Proposition 2.5.2. $T_i^w(w_i), i = 1, 2$ *are increasing and convex in* w_i. *By drawing the two functions onto the coordinate planes with* w_1 *and* w_2 *being the coordinate axis, the two curves intersect at* $(w_1, w_2) = (p\mu_1, p\mu_2)$ *and a point near* $(w_1, w_2) =$ $(0, 0)$. *Finally the area between the two curves displays where the retailer chooses dual sourcing.*

Proof. Taking derivatives of $T_1^w(w_1)$ on w_1 yields

$$\left(T_1^w(w_1)\right)' = \left[\mu_2 + \sigma_2\Phi^{-1}\left(\frac{w_1}{p\mu_1}\right)\right] \Big/ \mu_1 > 0,$$

$$\left(T_1^w(w_1)\right)'' = \sigma_2 \Big/ \left[p\mu_1^2\phi\left(\Phi^{-1}\left(\frac{w_1}{p\mu_1}\right)\right)\right] \geq 0,$$

where the first inequality follows from $\frac{w_1}{p\mu_1} \geq G_2(0) = \Phi\left(-\frac{\mu_2}{\sigma_2}\right)$ due to the assumption. Therefore, $T_1^w(w_1)$ is increasing and convex in w_1, and similarly $T_2^w(w_2)$ is increasing and convex in w_2.

Furthermore, noting that $T_i^w(p\mu_i) = p\mu_{3-i}$ and $T_i^w(0) \approx 0$, it is found that the two curves intersect at $(w_1, w_2) = (p\mu_1, p\mu_2)$ and the point near the origin. Finally from Theorem 2.5.1 it is concluded that wholesale price pairs of (w_1, w_2) under which the retailer chooses dual sourcing are between the two curves $w_2 = T_1^w(w_1)$ and $w_1 = T_2^w(w_2)$. □

Similar to Proposition 2.5.2, by defining

$$T_i^\mu(\mu_i) = \left[\mu_i w_{3-i} + p\sigma_{3-i}\mu_i\phi\left(\Phi^{-1}\left(\frac{w_i}{p\mu_i}\right)\right)\right] \Big/ \mu_i,$$

the following proposition characterizes the retailer's optimal sourcing strategy in terms of the mean value of the yield rate.

Proposition 2.5.3. $T_i^\mu(\mu_i), i = 1, 2$ *are increasing and concave in* μ_i*. By drawing the two functions onto the coordinate planes with* μ_1 *and* μ_2 *being the coordinate axis, the two curves intersect at* $(\mu_1, \mu_2) = (0, 0)$ *and* $(\mu_1, \mu_2) = (w_1/p, w_2/p)$*. Finally the area between the two curves and lines* $\mu_1 = 1 - 3\sigma_1$ *and* $\mu_2 = 1 - 3\sigma_2$ *displays where the retailer chooses dual sourcing.*

Proof. Taking derivative of $T_1^\mu(\mu_1)$ on μ_1 yields

$$\left(T_1^\mu(\mu_1)\right)' = \left[w_2 + p\sigma_2\phi\left(\Phi^{-1}\left(\frac{w_1}{p\mu_1}\right)\right) - p\sigma_2\mu_1\Phi^{-1}\left(\frac{w_1}{p\mu_1}\right)\right.$$

$$\left. \times\, \phi\left(\Phi^{-1}\left(\frac{w_1}{p\mu_1}\right)\right) \frac{\frac{-w_1}{p\mu_1^2}}{\phi\left(\Phi^{-1}\left(\frac{w_1}{p\mu_1}\right)\right)}\right] \Big/ w_1$$

$$= \left[w_2 + p\sigma_2\phi\left(\Phi^{-1}\left(\frac{w_1}{p\mu_1}\right)\right) + p\sigma_2\Phi^{-1}\left(\frac{w_1}{p\mu_1}\right)\frac{w_1}{p\mu_1}\right] \Big/ w_1$$

$$= [w_2 + p\sigma_2\phi(z) + p\sigma_2 z\Phi(z)]/w_1.$$

Where the last equality is due to the transformation $z = \Phi^{-1}\left(\frac{w_1}{p\mu_1}\right)$. Taking derivative of $\phi(z) + z\Phi(z)$ on z yields

$$-z\phi(z) + \Phi(z) + z\phi(z) = \Phi(z) \geq 0.$$

Thus $\left(T_1^{\mu}(\mu_1)\right)'$ is decreasing in μ_1, and $T_1^{\mu}(\mu_1)$ is concave in μ_1. Notice that $\phi(z) + z\Phi(z)$ approximates to zero when z approximates to $-\infty$. Therefore, it is found that $\phi(z) + z\Phi(z) \geq 0$ and hence $\left(T_1^{\mu}(\mu_1)\right)' > 0$, and further $T_1^{\mu}(\mu_1)$ is increasing in μ_1. Similarly it is found that $T_2^{\mu}(\mu_2)$ is increasing and concave in μ_2.

Furthermore, since $T_i^{\mu}(0) = 0$ and $T_i^{\mu}(w_i/p) = w_{3-i}/p$, the two curves intersect at the origin and $(\mu_1, \mu_2) = (w_1/p, w_2/p)$. Finally by noting that $w_i/p \leq \mu_i \leq 1 - 3\sigma_i$ we conclude that the pairs of (μ_1, μ_2) with which the retailer chooses dual sourcing are between the two curves $\mu_2 = T_1^{\mu}(\mu_1)$ and $\mu_1 = T_2^{\mu}(\mu_2)$, and the two lines $\mu_1 = 1 - 3\sigma_1$ and $\mu_2 = 1 - 3\sigma_2$. □

Finally we study the impact of the variance of yield rate on the retailer's optimal sourcing strategy.

Proposition 2.5.4. *(a) The retailer is more likely to choose dual sourcing if either variance of yield rates increases.*
(b) The retailer is more likely to source from supplier i if μ_i increases and/or w_i decreases.

The proof follows from Theorem 2.5.1 by noting that $p\sigma_{3-i}\mu_i\phi\left(\Phi^{-1}\left(\frac{w_i}{p\mu_i(\bar{Y}_i, \bar{q}_i)}\right)\right)$ is increasing in σ_{3-i}.

Furthermore, by comparing the single or dual sourcing conditions under deterministic demand and normal distributed demand, we find following property.

Proposition 2.5.5. *If demand is random and normally distributed, then the retailer is more likely to choose dual sourcing under deterministic demand than under random demand.*

Proof. If demand D_2 follows normal distribution with mean μ_{D_2} and variance $\sigma_{D_2}^2$, then from (2.10) we find that

$$\begin{aligned}
p\mu_2 E\Phi\left(\frac{D_2 - x - \mu_1\hat{q}_1(x)}{\sigma_1\hat{q}_1(x)}\right) - w_2 \\
&= p\mu_2 \int_{-\infty}^{\infty} \Phi\left(\frac{z - x - \mu_1\hat{q}_1(x)}{\sigma_1\hat{q}_1(x)}\right) f(z)\mathrm{d}z - w_2 \\
&= p\mu_2 \Phi\left(\frac{-x - \mu_1\hat{q}_1(x) + \mu_{D_2}}{\sqrt{\sigma_1^2\hat{q}_1^2(x) + \sigma_{D_2}^2}}\right) - w_2.
\end{aligned}$$

Thus $\hat{q}_1(x|\bar{Y}_1, \bar{q}_1, \bar{Y}_2, \bar{q}_2)$ can be obtained as

$$\frac{-x - \mu_1\hat{q}_1(x) + \mu_{D_2}}{\sqrt{\sigma_1^2\hat{q}_1^2(x) + \sigma_D^2}} = \Phi^{-1}\left(\frac{w_2}{p\mu_2}\right).$$

Furthermore,

$$p\sigma_1\mu_2 E\phi\left(\frac{D_2 - x - \mu_1\hat{q}_1(x)}{\sigma_1\hat{q}_1(x)}\right) = p\sigma_1\mu_2\int_{-\infty}^{\infty}\phi\left(\frac{z - x - \mu_1\hat{q}_1(x)}{\sigma_1\hat{q}_1(x)}\right)f(z)\mathrm{d}z$$
$$= p\sigma_1\mu_2\frac{\sigma_1\hat{q}_1(x)}{\sqrt{\sigma_1^2\hat{q}_1^2(x) + \sigma_D^2}}\phi\left(\frac{-x - \mu_1\hat{q}_1(x) + \mu_{D_2}}{\sqrt{\sigma_1^2\hat{q}_1^2(x) + \sigma_D^2}}\right)$$
$$= p\sigma_1\mu_2\frac{\sigma_1^2\hat{q}_1(x)}{\sqrt{(\sigma_1^2\hat{q}_1^2(x) + \sigma_D^2}}\phi\left(\Phi^{-1}\left(\frac{w_2}{p\mu_2}\right)\right) < p\sigma_1\mu_2\phi\left(\Phi^{-1}\left(\frac{w_2}{p\mu_2}\right)\right).$$

Similarly there is

$$p\sigma_2\mu_1 E\phi\left(\frac{D_2 - x - \mu_2\hat{q}_2(x)}{\sqrt{\sigma_2^2\hat{q}_2(x)}}\right) < p\sigma_2\mu_1\phi\left(\Phi^{-1}\left(\frac{w_1}{p\mu_1}\right)\right).$$

Therefore, by comparing Theorems 2.3.2 and 2.5.1, we conclude that the retailer will be more likely to choose dual sourcing under the deterministic demand. □

In what follows we propose the optimality analysis of the two-period problem.

For the following analysis, we find it convenient to define $\varphi(z|\alpha, \beta^2) = \alpha\Phi\left(\frac{z-\alpha}{\beta}\right) - \beta\phi\left(\frac{z-\alpha}{\beta}\right)$ given α and β. Since $\frac{\mathrm{d}\varphi(z|\alpha,\beta^2)}{\mathrm{d}z} = \frac{z}{\beta}\phi\left(\frac{z-\alpha}{\beta}\right)$, then $\varphi(z|\alpha, \beta^2)$ is an increasing function of z when $z > 0$.

Notice that if $D_2 \leq x$, then the retailer will order nothing. If inequality $\mu_i w_{3-i} - \mu_{3-i} w_i \geq p\sigma_i\mu_{3-i}\phi\left(\Phi^{-1}\left(\frac{w_{3-i}}{p\mu_{3-i}}\right)\right)$ is satisfied, the retailer chooses single sourcing from supplier i and the optimal ordering quantity q_{is}^* is given by

$$p\int_{-\infty}^{\frac{d_2-x}{q_{is}^*}} y_i g_i(y_i)\mathrm{d}y - w_i = p\mu_i G_i\left(\frac{d_2 - x}{q_{is}^*}\right) - (\sigma_i^2)g_i\left(\frac{d_2 - x}{q_{is}^*}\right)$$
$$= p\varphi\left(\frac{d_2 - x}{q_{is}^*}|\mu_i, \sigma_i^2\right) - w_i = 0,$$

with $d_2 > x$. And the corresponding optimal profit is given by

$$pd_2 - p(d_2 - x)^+ G_i\left(\frac{d_2 - x}{q_{is}^*}\right) = pd_2 - p(d_2 - x)^+ \Phi\left(\frac{\varphi^{-1}\left(\frac{w_i}{p}|\mu_i, \sigma_i^2\right) - \mu_i}{\sigma_i}\right).$$

Furthermore, if inequalities (2.23) and (2.24) are satisfied, the retailer chooses dual sourcing and the optimal ordering quantity $q_{1d}^*(d_2 - x)$ and $q_{2d}^*(d_2 - x)$ are given by the first order conditions

$$p\mu_1 G(d_2 - x, q_1, q_2) - p\sigma_1^2 q_1 g(d_2 - x, q_1, q_2) - w_1 = 0,$$
$$p\mu_2 G(d_2 - x, q_1, q_2) - p\sigma_2^2 q_2 g(d_2 - x, q_1, q_2) - w_2 = 0.$$

And the optimal profit is given by

$$\begin{aligned} pd_2 - p(d_2 - x)^+ &\int_{-\infty}^{+\infty} \int_{-\infty}^{\frac{d_2 - x - y_2 q_{2d}^*(d_2 - x)}{q_{1d}^*(d_2 - x)}} g_1(y_1)\mathrm{d}y_1 g_2(y_2)\mathrm{d}y_2 \\ = pd_2 - p(d_2 - x)^+ &\Phi\left(\frac{d_2 - x - \mu_1 q_{1d}^*(d_2 - x) - \mu_2 q_{2d}^*(d_2 - x)}{\sqrt{\sigma_1^2(q_{1d}^*(d_2 - x))^2 + \sigma_2^2 q_{2d}^*(d_2 - x))^2}}\right). \end{aligned}$$

In fact, according to the following lemma it can be found that $\Phi\left(\frac{d_2 - x - \mu_1 q_{1d}^*(d_2 - x) - \mu_2 q_{2d}^*(d_2 - x)}{\sqrt{\sigma_1^2(q_{1d}^*(d_2 - x))^2 + \sigma_2^2(q_{2d}^*(d_2 - x))^2}}\right)$ is independent of $d_2 - x$ when $d_2 - x > 0$.

Lemma 2.5.6. *When the retailer chooses dual sourcing and $d_2 - x > 0$, then*

$$\partial\left[\int_{-\infty}^{+\infty} \int_{-\infty}^{\frac{D_2 - x - y_2 q_{2d}^*(D_2 - x)}{q_{1d}^*(D_2 - x)}} g_1(y_1)\mathrm{d}y_1 g_2(y_2)\mathrm{d}y_2\right] \Big/ \partial[D_2 - x] = 0.$$

Proof. The proof is similar to that for Lemma 2.4.2. If the retailer chooses dual sourcing, the first order conditions can be rewritten as

$$p\int_{-\infty}^{+\infty} \int_{-\infty}^{\frac{D_2 - x - y_2 q_{2d}^*(D_2 - x)}{q_{1d}^*(D_2 - x)}} y_1 g_1(y_1)\mathrm{d}y_1 g_2(y_2)\mathrm{d}y_2 - w_1 = 0,$$

$$p\int_{-\infty}^{+\infty} \int_{-\infty}^{\frac{D_2 - x - y_2 q_{2d}^*(D_2 - x)}{q_{1d}^*(D_2 - x)}} y_2 g_1(y_1)\mathrm{d}y_1 g_2(y_2)\mathrm{d}y_2 - w_2 = 0.$$

Taking the derivative of the first order conditions on $D_2 - x$ yields

$$p\int_{-\infty}^{+\infty} \frac{\partial\left(\frac{D_2-x-y_2q_{2d}^*(D_2-x)}{q_{1d}^*(D_2-x)}\right)}{\partial(D_2-x)} \frac{D_2-x-y_2q_{2d}^*(D_2-x)}{q_{1d}^*(D_2-x)} g_1$$

$$\times\left(\frac{D_2-x-y_2q_{2d}^*(D_2-x)}{q_{1d}^*(D_2-x)}\right) g_2(y_2)\mathrm{d}y_2=0, \tag{2.25}$$

$$p\int_{-\infty}^{+\infty} \frac{\partial\left(\frac{D_2-x-y_2q_{2d}^*(D_2-x)}{q_{1d}^*(D_2-x)}\right)}{\partial(D_2-x)} y_2 g_1$$

$$\times\left(\frac{D_2-x-y_2q_{2d}^*(D_2-x)}{q_{1d}^*(D_2-x)}\right) g_2(y_2)\mathrm{d}y_2=0. \tag{2.26}$$

Therefore, it is found that

$$\partial\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{\frac{D_2-x-y_2q_{2d}^*(D_2-x)}{q_{1d}^*(D_2-x)}} g_1(y_1)\mathrm{d}y_1 g_2(y_2)\mathrm{d}y_2\right] \Big/ \partial[D_2-x]$$

$$= p\int_{-\infty}^{+\infty} \frac{\partial\left(\frac{D_2-x-y_2q_{2d}^*(D_2-x)}{q_{1d}^*(D_2-x)}\right)}{\partial(D_2-x)} g_1\left(\frac{D_2-x-y_2q_{2d}^*(D_2-x)}{q_{1d}^*(D_2-x)}\right) g_2(y_2)\mathrm{d}y_2$$

$$= [q_{1d}^*(D_2-x)(*^1) + q_{2d}^*(D_2-x)(*^2)]/(D_2-x) = 0,$$

where $*^1$ and $*^2$ denote the left side of equalities for (2.25) and (2.26) respectively. □

In what follows denote $\Phi\left(\frac{D_2-x-\mu_1q_{1d}^*(D_2-x)-\mu_2q_{2d}^*(D_2-x)}{\sqrt{\sigma_1^2(q_{1d}^*(D_2-x))^2+\sigma_2^2(q_{2d}^*(D_2-x))^2}}\right)$ by $\Psi(\mu_1,\sigma_1^2,\mu_2,\sigma_2^2)$ which is independent of the difference of demand and inventory but depends on the other model parameters. $\Psi(\mu_1,\sigma_1^2,\mu_2,\sigma_2^2)$ indicates the probability that the demand of the second period is not satisfied when the retailer chooses dual sourcing with the optimal ordering quantity. There is $0 \le \Psi(\mu_1,\sigma_1^2+\tau_1^2,\mu_2,\sigma_2^2+\tau_2^2) \le 1$.

Therefore, the profit-to-go can be shown as

$$V_2(x) = \max_{q_1 \geq 0, q_2 \geq 0} pE\{\min[d_2, x + Y_1 q_1 + Y_2 q_2]\} - w_1 q_1 - w_2 q_2$$
$$= \begin{cases} pd_2 - p(d_2 - x)^+ \Phi\left(\frac{\varphi^{-1}\left(\frac{w_1}{p}|\mu_1, \sigma_1^2 + \tau_1^2\right) - \mu_1}{\sigma_1}\right), & (2.21) \text{ true}, \\ pd_2 - p(d_2 - x)^+ \Psi(\mu_1, \sigma_1^2, \mu_2, \sigma_2^2), & (2.23) \text{ and } (2.24) \text{ true}, \\ pd_2 - p(d_2 - x)^+ \Phi\left(\frac{\varphi^{-1}\left(\frac{w_2}{p}|\mu_2, \sigma_2^2\right) - \mu_2}{\sigma_2}\right), & (2.22) \text{ true}. \end{cases} \tag{2.27}$$

Next we return to the optimization problem of the first period. The discussion depends on which sourcing strategy the retailer chooses. First the following lemma is useful, with the proof following from similar procedure to that for Proposition 2.5.3.

Lemma 2.5.7. *$z\phi\left(\Phi^{-1}\left(\frac{w_2}{p\mu_2 z}\right)\right)$ is increasing in z.*

Next we discuss the cases under different sourcing strategy.

Case 1: If inequality of (2.21) is satisfied, then the profit function can be shown as

$$\Pi_1(q_1, q_2) = E\{pd_1 - w_1 q_1 - w_2 q_2 + V_2(Y_1 q_1 + Y_2 q_2 - d_1)\}$$
$$= p(d_1 + d_2) - w_1 q_1 - w_2 q_2 - p\Phi\left(\frac{\varphi^{-1}\left(\frac{w_1}{p}|\mu_1, \sigma_1^2\right) - \mu_1}{\sigma_1}\right)$$
$$\times E(d_1 + d_2 - Y_1 q_1 - Y_2 q_2)^+.$$

The first order conditions are

$$\Phi\left(\frac{\varphi^{-1}\left(\frac{w_1}{p}|\mu_1, \sigma_1^2\right) - \mu_1}{\sigma_1}\right) E[Y_1 \mathbf{1}_{\{Y_1 q_1 + Y_2 q_2 \leq d_1 + d_2\}}] - w_1 = 0,$$

$$\Phi\left(\frac{\varphi^{-1}\left(\frac{w_1}{p}|\mu_1, \sigma_1^2\right) - \mu_1}{\sigma_1}\right) E[Y_2 \mathbf{1}_{\{Y_1 q_1 + Y_2 q_2 \leq d_1 + d_2\}}] - w_2 = 0.$$

Similar to Theorem 2.5.1, it is found that the retailer determines whether to choose single sourcing from supplier 1 or dual sourcing by comparing $\mu_1 w_2 - \mu_2 w_1$ and

$$p\sigma_1\mu_2\Phi\left(\frac{\varphi^{-1}\left(\frac{w_1}{p}|\mu_1,\sigma_1^2\right)-\mu_1}{\sigma_1}\right)$$
$$\phi\left(\Phi^{-1}\left(\frac{w_2}{p\mu_2\Phi\left(\frac{\varphi^{-1}\left(\frac{w_1}{p}|\mu_1,\sigma_1^2\right)-\mu_1}{\sigma_1}\right)}\right)\right). \tag{2.28}$$

However, inequality of (2.21) and Lemma 2.5.7 allow us to show that

$$\begin{aligned}\mu_1 w_2 - \mu_2 w_1 &\geq p\sigma_1\mu_2\phi\left(\Phi^{-1}\left(\frac{w_2}{p\mu_2}\right)\right)\\ &\geq p\sigma_1\mu_2\Phi\left(\frac{\varphi^{-1}\left(\frac{w_1}{p}|\mu_1,\sigma_1^2\right)-\mu_1}{\sigma_1}\right)\\ &\quad\times\phi\left(\Phi^{-1}\left(\frac{w_2}{p\mu_2\Phi\left(\frac{\varphi^{-1}\left(\frac{w_1}{p}|\mu_1,\sigma_1^2\right)-\mu_1}{\sigma_1}\right)}\right)\right).\end{aligned}$$

Therefore,the retailer will choose single sourcing from supplier 1 in the first period if (2.21) is satisfied. Similarly, it is found that the retailer will choose single sourcing from supplier 2 in the first period if (2.22) is satisfied.

Case 2: If (2.23) and (2.24) are satisfied, since $\Psi(\mu_1,\sigma_1^2,\mu_2,\sigma_2^2) \leq 1$, it is found that

$$\begin{aligned}p\sigma_1\mu_2\phi\left(\Phi^{-1}\left(\frac{w_2}{p\mu_2}\right)\right) &\geq p\sigma_1\mu_2\Psi(\mu_1,\sigma_1^2,\mu_2,\sigma_2^2)\\ &\times\phi\left(\Phi^{-1}\left(\frac{w_2}{p\mu_2\Psi(\mu_1,\sigma_1^2,\mu_2,\sigma_2^2)}\right)\right).\end{aligned}$$

Similarly, it is concluded that the retailer will choose dual sourcing in the first period if $\mu_i w_{3-i} - \mu_{3-i} w_i$ is less than $p\sigma_i\mu_{3-i}\Psi(\mu_1,\sigma_1^2,\mu_2,\sigma_2^2)\phi\left(\Phi^{-1}\left(\frac{w_{3-i}}{p\mu_{3-i}\Psi(\mu_1,\sigma_1^2,\mu_2,\sigma_2^2)}\right)\right)$. If $\mu_i w_{3-i} - \mu_{3-i} w_i$ is between $p\sigma_i\mu_{3-i}\Psi(\mu_1,\sigma_1^2,\mu_2,\sigma_2^2)\phi\left(\Phi^{-1}\left(\frac{w_{3-i}}{p\mu_{3-i}\Psi(\mu_1,\sigma_1^2,\mu_2,\sigma_2^2)}\right)\right)$ and $p\sigma_i\mu_{3-i}\phi\left(\Phi^{-1}\left(\frac{w_{3-i}}{p\mu_{3-i}}\right)\right)$, the retailer will choose single sourcing from supplier i.

Finally we summarize the retailer's optimal sourcing strategy for the optimization problem of the first period with deterministic demand.

Theorem 2.5.8. *When the demand is deterministic, the retailer's optimal sourcing strategy is to choose*

(i) Single sourcing from supplier 1 if

$$\mu_1 w_2 - \mu_2 w_1 \geq p\sigma_1\mu_2\Psi(\mu_1, \sigma_1^2, \mu_2, \sigma_2^2)\phi\left(\Phi^{-1}\left(\frac{w_2}{p\mu_2\Psi(\mu_1, \sigma_1^2, \mu_2, \sigma_2^2)}\right)\right),$$

(ii) Single sourcing from supplier 2 if

$$\mu_2 w_1 - \mu_1 w_2 \geq p\sigma_2\mu_1\Psi(\mu_1, \sigma_1^2, \mu_2, \sigma_2^2)\phi\left(\Phi^{-1}\left(\frac{w_1}{p\mu_1\Psi(\mu_1, \sigma_1^2, \mu_2, \sigma_2^2)}\right)\right),$$

(iii) Dual sourcing from the two suppliers if

$$\mu_1 w_2 - \mu_2 w_1 \leq p\sigma_1\mu_2\Psi(\mu_1, \sigma_1^2, \mu_2, \sigma_2^2)\phi\left(\Phi^{-1}\left(\frac{w_2}{p\mu_2\Psi(\mu_1, \sigma_1^2, \mu_2, \sigma_2^2)}\right)\right),$$

and

$$\mu_2 w_1 - \mu_1 w_2 \leq p\sigma_2\mu_1\Psi(\mu_1, \sigma_1^2, \mu_2, \sigma_2^2)\phi\left(\Phi^{-1}\left(\frac{w_1}{p\mu_1\Psi(\mu_1, \sigma_1^2, \mu_2, \sigma_2^2)}\right)\right).$$

From Theorems 2.5.1 and 2.5.8, it is optimal for the retailer to choose single sourcing from supplier i in the first period and dual sourcing in the second period if

$$\begin{aligned} p\sigma_i\mu_{3-i}\Psi(\mu_1, \sigma_1^2, \mu_2, \sigma_2^2)\phi\left(\Phi^{-1}\left(\frac{w_{3-i}}{p\mu_{3-i}\Psi(\mu_1, \sigma_1^2, \mu_2, \sigma_2^2)}\right)\right) \\ \leq \mu_i w_{3-i} - \mu_{3-i} w_i < p\sigma_i\mu_{3-i}\phi\left(\Phi^{-1}\left(\frac{w_{3-i}}{p\mu_{3-i}}\right)\right). \end{aligned}$$

or in other words, the retailer will be more likely to choose single sourcing in the first period, and dual sourcing in the second period. The reason can be attributed to the fact that in the first period the unsatisfied demand could be backlogged, and hence the retailer can take more risk. But in the second period, the unsatisfied demand will be lost. Thus the retailer needs to make the ordering more reliable, i.e., choose dual sourcing.

2.6 Numerical Examples

In this section, numerical examples are presented to demonstrate the optimal sourcing strategy and ordering quantity, and their dependences on wholesale price w. The model parameters are given as follows: $p = 3$, $\mu_1 = 0.5$, $\sigma_1 = 0.14$, $\mu_2 = 0.4$, $\sigma_2 = 0.12$, and deterministic demand $d = 6$.

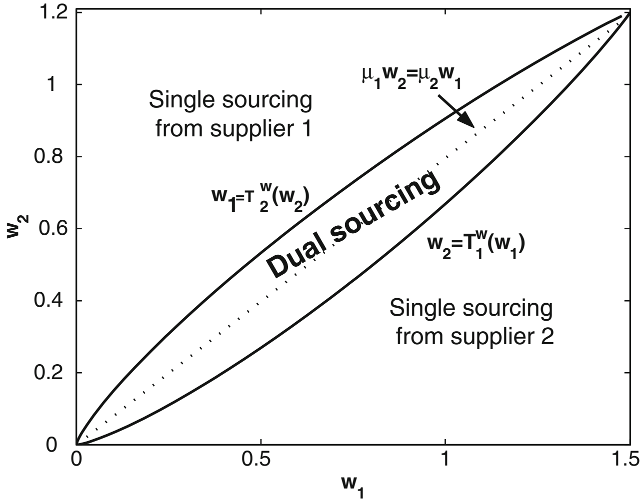

Fig. 2.1 Region of sourcing strategy with respect to wholesale price

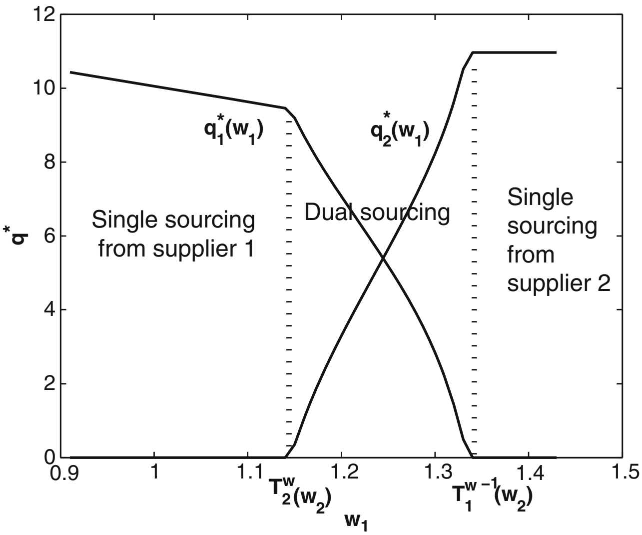

Fig. 2.2 Optimal ordering quantity in terms of wholesale price

Figure 2.1 identifies regions of wholesale price w_1 and w_2 in which the retailer will choose different sourcing strategies. As Proposition 2.5.2 presented, the retailer chooses dual sourcing in the region between the two curves $T_1^w(w_1)$ and $T_2^w(w_2)$.

Figure 2.2 shows the dependence of the optimal ordering quantity q_i^* on wholesale price w_1. It shows that q_1^* is decreasing in w_1, while q_2^* is increasing in w_1. The figure also identifies the regions of w_1 in which the retailers choose single sourcing or dual sourcing.

2.7 Concluding Remarks

This chapter addresses a two-periods inventory control problems faced by a retailer who is served by two unreliable suppliers. The retailer facing stochastic demand needs to determine the sourcing strategy, i.e., which supplier to select and further how much to order. For each period, we identify the conditions under which the retailer will choose single sourcing or dual sourcing, and find that the supplier selection process is the trade-off between the ordering cost and the randomicity of the yield rate. It is further pointed out that more structural results can be found under the setting of deterministic demand. Specifically, the sourcing threshold, the trade-off between the ordering cost and parameters of the random yield rate can be in explicit form. We also complement these results with useful comparative statistics.

Many interesting issues remain to be investigated. For example, the issue of yield information update can be incorporated into the supplier selection model. Specifically, the retailer can collect yield information after the first period based on how many units the retailer orders and actually delivered by the supplier. The more the ordered units, then the more information about the yield rate of the supplier. Following the basic normal sampling process structure as in Pratt et al. (1995), the information collected in the first period can be used to generate a posterior distribution for estimate of yield rate in the second period. The optimal sourcing strategy and the ordering quantity need to be computed in the new setting.

In addition, it is interesting to consider the Stackelberg game in which the suppliers determine the wholesale price and then the retailer choose the sourcing strategy as well as the ordering quantity. The problem of a single supplier controlling wholesale price while selling to a newsvendor has been addressed by Lariviere and Porteus (2001). Babich et al. (2007a, 2007b) added a possibility of supplier's default to the problem in Lariviere and Porteus (2001) and focussed on the effect of the supply risk on the performance of the supply chain. In the following chapter we further generalize the problem by considering a game with more than one supplier.

Chapter 3
Sourcing Strategy of Retailer and Pricing Strategies of Suppliers

3.1 Introduction

Supply disruption management has received increasing attention from both industry and academia[1]. Firms are starting to realize that supply disruption severely affects their ability to successfully manage their supply chains. The academia has devoted much research effort to studying this issue. Many papers have been published that advise firms on how to manage their supply chains in the presence of supply disruption.

While the literature on supply disruption management is growing, the vast majority of these studies only investigated strategies of retailers or strategies of suppliers. Different from the existing literature on supply disruption management, this chapter investigates not only the sourcing strategies of the retailer but also the pricing game played between suppliers in a single-retailer and two-supplier supply chain in the presence of supply disruption. We examine the pricing game under two scenarios, namely, one between non-cooperative suppliers and the other between cooperative suppliers. As such, the literature on supply disruption management, the wholesale price setting problem, and non-cooperative and cooperative games in supply chains is all relevant to our study.

There is a large body of literature on the broad topic of supply disruption management. Based on the number of supplier, these studies can be classified into two categories: singular supplier models and multi supplier models. With no alternative source available for single-supplier systems, inventory mitigation is the only disruption management strategy under consideration in singular supplier models. The focus of singular supplier models is to identify the optimal inventory policy or the optimal parameters for particular inventory policy when there is supply disruption risk. In multi supplier models, it is assumed that retailer sources is from

[1]The following discussion in this chapter is largely based on the ideas and results presented in Li et al. (2010).

J. Li et al., *Risk Management of Supply and Cash Flows in Supply Chains*, International Series in Operations Research & Management Science 165,
DOI 10.1007/978-1-4614-0511-5_3,

two or more suppliers. The inter-failure time and the repair time are scholastic for all suppliers. The disruption management strategies include sourcing mitigation, contingent rerouting, dual sourcing, emergency sourcing, and demand management. Recent literature dealing with supply disruption management includes, and are not limited to, Parlar and Perry (1995), Moinzadeh and Aggarwal (1997), Arreola-Risa and DeCroix (1998), Abboud (2001), Gupta (1996), Parlar (1997), Özekici and Parlar (1999), Burke et al. (2004), Li et al. (2004), Lewis et al. (2006), Ross et al. (2008), Parlar and Perry (1996), Gürler and Parlar (1997), Tomlin (2005, 2006), Yu et al. (2009), and Sarkar and Mohapatra (2009). For the details, one can refer to Sect. 1.2.4.3 of this book.

The supply disruption papers cited above only investigated the strategies of retailers under the assumption that suppliers are exogenous. However, suppliers' responses, e.g., their pricing strategies, are also crucial factors that impact the supply chain. The wholesale price setting problem has been extensively studied in the literature. Recent literature includes Lariviere and Porteus (2001), Wang and Gerchak (2003), Tomlin (2003), Bernstein and DeCroix (2004), and Cachon and Lariviere (2001), among others. They gave the optimal pricing strategies of the suppliers under different scenarios. However, most of the above work assumed perfectly-reliable supply. The price setting problem with unreliable supply has received much less attention.

We now turn our attention to the literature on game analysis of supply chains. Game theory can be divided broadly into two approaches, namely, the non-cooperative and the cooperative approaches (Deng et al. 2005). In the last several years, it has been recognized that game theory is an effective tool for the analysis of supply chains with multiple agents. In recent years, there is a wide variety of research papers that apply non-cooperative game theory to the field of supply chain management. For the sake of conciseness, we do not provide a comprehensive review of the literature in this area. For an excellent survey, readers can refer to Cachon and Netessine (2004). Research employing cooperative game theory to study supply chain management are much less prevalent, but are becoming more popular. This trend is probably due to the prevalence of bargaining and negotiation in supply chain relationships. One can refer to Nagarajan and Sošić (2008) for a detailed survey of the existing literature on applications of cooperative games to supply chain management.

Motivated by the above observations, this chapter is set out to study a supply chain consisting of one retailer and two suppliers and consider the price setting problem in the presence of supply disruption. In this chapter we investigate both a centralized supply chain, and a decentralized supply chain. Furthermore, we consider two scenarios for the decentralized supply chain, i.e., the two suppliers are competitive and cooperative. We seek to find the optimal order quantities and the optimal wholesale prices in both the scenarios. Babich (2006) and Babich et al. (2007a, 2007b) developed similar models to investigate supplier pricing decisions with supply disruption. Babich (2006) investigated how the supplier default risk and default co-dependence affect the procurement and production decisions of the manufacturer, supplier pricing decisions, and the value of the supplier's option to postpone its pricing decisions. Babich et al. (2007a, 2007b) examined the effects of co-dependence among supplier defaults on the performance of firms and the consequences of the suppliers offering different payment policies. The main difference between their papers and our study is about the treatment of unfilled demand due to supply disruption. In this chapter, the unfilled demand is filled from a spot market rather than it is lost. This is always true because the demand can be filled by emergency sourcing or global sourcing, which is a common business practice with advances in transportation and information technology. The incorporation of the spot market in the model alters supplier competition. Moreover, we consider a situation in which the suppliers are cooperative. The study of the pricing decisions of cooperative suppliers in this setting is not far-fetched. Firstly, examples of real-world supplier alliances in supply chains abound. For example, Greene (2002) presented several instances of alliances between component manufacturers in the semiconductor industry. Secondly, retailers encourage cooperation between suppliers in hopes of converting difficult suppliers into supportive suppliers through cooperation, which provides opportunities for the sharing of good practices and experiences between suppliers. The pricing decisions of cooperative suppliers are of interest to our study.

We intend to contribute to the knowledge in this area by addressing two key questions: How does the supply disruption affect the suppliers' pricing and the retailer's ordering behaviours? How shall we coordinate the behaviours of cooperative suppliers in the presence of supply disruption? This chapter provides valuable managerial guidance for retailers to allocate their orders between different suppliers and for suppliers to price their supplies when facing supply disruption. Specially, we make four main contributions:

1. We show the existence of an equilibrium price in the competitive scenario for two typical customer demand distributions, namely, the uniform distribution and the exponential distribution.
2. Based on the uniform demand distribution, we obtain an explicit form of the unique equilibrium price.
3. We investigate the impacts of supply disruption on the retailer's sourcing strategy and the suppliers' pricing strategy by both theoretical and computational analyses.
4. We devise a coordination mechanism to maximize the profits of cooperative suppliers.

The rest of this chapter is organized as follows. The problem under consideration is introduced and formulated in Sect. 3.2. Section 3.3 analyzes a benchmark scenario in which the whole channel (i.e., the supply chain) is centrally controlled. Section 3.4 investigates a decentralized supply chain under two scenarios, one with non-cooperative suppliers and the other with cooperative suppliers. In Sect. 3.5 numerical results are presented to illustrate the theoretical results. Conclusions and suggestions for future research are given in Sect. 3.6. All the proofs of the theoretical results are given in the Appendix.

3.2 The Problem

In this chapter we study a supply chain consisting of one retailer and two suppliers with unreliable supply. All three firms are assumed to be risk neutral and pursue expected profit maximization. In addition, we assume that there is a spot market as a contingent supplier that is perfectly reliable.

The retailer buys a short-life product from the two suppliers and from the spot market, and sells the product to its customers in a single selling season. The uncertain source of supply is a state of the suppliers, which are subject to random failures. If a supplier is in the success state, the orders placed with it will be delivered on time. However, if a supplier is in the failure state, no orders can be supplied. We assume that there are two types of failure: common-cause and supplier-specific failures. A common-cause failure affects both suppliers. For example, an earthquake may affect all the suppliers in a region. A supplier may still fail for some supplier-specific reason even if there is no common-cause failure. For example, equipment failure might affect one supplier but not the other supplier. We assume that supplier 2 is affected only by the common-cause failure but supplier 1 is affected by both types of failure. First, supplier 1 and supplier 2 decide their individual wholesale prices. Then the retailer allocates its orders between the two suppliers before the states of the suppliers are realized. After the states have been realized, the retailer has a chance to make an emergency order from the spot market. We assume that the replenishment rate is infinite and the lead time is zero.

The following notation is used in the model:

$i = 1, 2, 3$ stands for supplier 1, supplier 2 and the spot market, respectively.
b is the goodwill cost of a unit of unmet demand.
c_i is the delivery cost of a unit of the product of supplier i, $i = 1, 2$.
D is the positive stochastic customer demand.
f is the positive probability density function of D.
F is the differentiable and strictly increasing cumulative distribution function of D.
p is the fixed selling price of a unit of the product.
Q_i is the order quantity placed with supplier i, $i = 1, 2$.
Q_3 is the inventory level after making an emergency order from the spot market.
s is the salvage value of a unit of the residual product.
w_i is the wholesale price of a unit of the product offered by supplier i, $i = 1, 2$.
w_3 is the fixed wholesale price of a unit of the product offered by the spot market.
α is the probability of a common-cause failure not occurring, where $0 < \alpha < 1$.
β is the probability that supplier 1 does not fail conditional on a common-cause failure not occurring, where $0 < \beta < 1$.
γ is the total proportion of the marginal delivery cost in the event of a failure, where $0 < \gamma < 1$.
η is the proportion of the cost incurred by the supplier who fails in the event of a failure, where $0 \leq \eta \leq 1$.

Among the above variables, w_1, w_2, Q_1, Q_2, and Q_3 are decision variables and the others are exogenous variables, which are known to all the members of the supply chain. In this chapter the revenues of supplier 1, supplier 2, and the retailer are our focus. We do not care about the revenue of the spot market and have no regard for its delivery cost. The spot market is not a decision-maker in the supply chain.

It should be noted that a marginal cost γc_i is incurred in the event of a failure. We expect that the failing supplier and the retailer assume this cost jointly. The marginal cost assumed by the failing supplier is $\eta\gamma c_i$ and the marginal cost assumed by the retailer is $(1-\eta)\gamma c_i$. This cost structure is different from that used in most of the literature in which only the retailer assumes the cost in the event of a failure. But this is not always true. In fact, before supply failures are realized, both the retailer and the suppliers usually have incurred some costs, which may include fixed set-up costs and variable costs. For simplicity of analysis, we assume that all the setup costs are zero and all the variable costs in the event of a supply failure are proportional to the delivery cost and to the order quantity.

Based on the reliability of the suppliers, it is reasonable to assume that $c_1 < c_2 < w_3$. In addition, we assume that $0 \leq s < c_1 < c_2 < w_3 < p$. These inequalities ensure that each firm makes a positive profit and the chain will not produce infinite quantities of the product.

In the following section we consider a centralized system in which all the decisions are centralized to maximize the performance of the entire supply chain (including the retailer, supplier 1, and supplier 2). We give the conditions for both suppliers being placed with positive orders and the corresponding optimal order quantities. The centralized system solution serves as a benchmark for the decentralized setting. Then we consider a decentralized supply chain under two different scenarios in which the suppliers are competitive or cooperative. For the two decentralized problems, information on each player's demand function, cost structure, and decision rules is common knowledge to all the parties concerned. The decentralized supply chain with competitive suppliers, in which the players act independently and make decisions that maximize their individual profits, can be viewed as two static nested games. The first is a static non-cooperative game between supplier 1 and supplier 2. They choose their wholesale prices simultaneously and do not collude. The second is a Stackelberg game, which is nested within the static non-cooperative game. In the Stackelberg game, the leaders (supplier 1 and supplier 2) select the wholesale prices, and the followers (the retailer facing random yields) respond by selecting their order quantities. For the decentralized supply chain with competitive suppliers, we give the equilibrium wholesale price of the two suppliers and the optimal order quantities of the retailer. Finally we investigate the decentralized supply chain with cooperative suppliers in which supplier 1 and supplier 2 choose their individual wholesale prices to maximize their total profits. To ensure stability and robustness of the cooperation, the Nash bargaining game in cooperative game theory is used to divide the profit pie created through cooperation.

3.3 The Centralized Supply Chain

It is obvious that the supply chain will perform best if the channel is centrally controlled. Since the wholesale price is only used to divide the profit between the retailer and the suppliers, w_1 and w_2 are no longer decision variables in the centralized supply chain. The decision variables are only Q_1, Q_2 and Q_3. We seek to determine the channel's optimal order allocation decisions when there is supply uncertainty for a seasonal product. The sequence of events in the centralized supply chain is as follows:

1. Orders are placed with supplier 1 and supplier 2, respectively, in anticipation of supply disruption and demand (Stage 1).
2. An emergency order is placed with the spot market after a supply disruption has occurred but before demand occurs (Stage 2).
3. When the selling season arrives, the product is sold at a fixed price in the market. Any unmet demand incurs a goodwill cost to the whole channel. After the selling season, the residual product will be salvaged (Stage 3).

Denote z and Q_{3c} as the inventory level of the supply chain before and after the emergency order is placed, respectively. Let $\ddot{\pi}_c(Q_{3c}|z)$ be the channel's random profit in stage 2, i.e., the random profit after the emergency order $Q_{3c} - z$ is placed (hereafter the subscript 'c' stands for the centralized supply chain and the superscript '¨' stands for stage 2). We have

$$\ddot{\pi}_c(Q_{3c}|z) = p(Q_{3c} \wedge D) - w_3(Q_{3c} - z)^+ + s(Q_{3c} - D)^+ - b(D - Q_{3c})^+. \quad (3.1)$$

Then we can deduce the channel's expected profit in stage 2, enoted as $\ddot{\Pi}_c(Q_{3c}|z)$, which is given by

$$\ddot{\Pi}_c(Q_{3c}|z) = \begin{cases} (p+b-w_3)Q_{3c} - (p+b-s)\int_0^{Q_{3c}} F(x)\mathrm{d}x + w_3 z - bE[X], & Q_{3c} \geq z, \\ (p+b)z - (p+b-s)\int_0^{z} F(x)\mathrm{d}x - bE[X], & Q_{3c} = z, \end{cases} \quad (3.2)$$

where $E[X]$ is the mean of the random demand D.

The channel's order problem in stage 2 is to choose the emergency order quantity $Q_{3c} - z$ to maximize its expected profit for any given initial inventory level z. This is the classical newsvendor problem. By using the first- and second-order optimality conditions, we can obtain that the order-up-to-level (OUL) policy is optimal for the channel and the threshold value of the inventory level is $\hat{Q}_{3c} = F^{-1}\left(\frac{p+b-w_3}{p+b-s}\right)$.

Hence, the optimal inventory level after the retailer placing an emergency order is as follows:

$$Q^*_{3c} = \hat{Q}_{3c} \vee z. \tag{3.3}$$

Then the maximum expected revenue of the channel in stage 2 for any given initial inventory level z is deduced as follows:

$$\ddot{\Pi}^*_c(z) = \begin{cases} (p+b-w_3)\hat{Q}_{3c} - (p+b-s)\int_0^{\hat{Q}_{3c}} F(x)\mathrm{d}x + w_3 z - bE[X], & z \le \hat{Q}_{3c}, \\ (p+b)z - (p+b-s)\int_0^{z} F(x)\mathrm{d}x - bE[X], & z \ge \hat{Q}_{3c}. \end{cases} \tag{3.4}$$

It is obvious that the overall probability of supplier 1 not failing is $\alpha\beta$ and the overall probability of supplier 2 not failing is α. Hence the channel's expected profit in stage 1 after the retailer placing orders with supplier 1 and supplier 2, denoted as $\dot{\Pi}_c(Q_{1c}, Q_{2c})$, is given by (hereafter the superscript '$\cdot$' stands for stage 1)

$$\begin{aligned} \dot{\Pi}_c(Q_{1c}, Q_{2c}) = {} & \alpha\beta \left[\ddot{\Pi}^*_c(Q_{1c} + Q_{2c}) - c_1 Q_{1c} - c_2 Q_{2c}\right] \\ & + \alpha(1-\beta)\left[\ddot{\Pi}^*_c(Q_{2c}) - \gamma c_1 Q_{1c} - c_2 Q_{2c}\right] \\ & + (1-\alpha)\left[\ddot{\Pi}^*_c(0) - \gamma c_1 Q_{1c} - \gamma c_2 Q_{2c}\right]. \end{aligned} \tag{3.5}$$

The channel's order problem in stage 1 is to choose order quantities Q_{1c} and Q_{2c} to maximize its expected profit. We have the following conclusions about the optimal sourcing strategy of the centralized supply chain.

Theorem 3.3.1. *After a supply disruption has occurred, the optimal ordering strategy of the centralized supply chain from the spot market is the OUL policy and the threshold value of the inventory level is* $\hat{Q}_{3c} = F^{-1}\left(\frac{p+b-w_3}{p+b-s}\right)$. *The optimal sourcing strategies from supplier 1 and supplier 2 are as follows:*

1. *If* $\alpha\beta(w_3 - c_1 + \gamma c_1) - \gamma c_1 < 0$ *and* $\alpha(w_3 - c_2 + \gamma c_2) - \gamma c_2 < 0$, *both supplier 1 and supplier 2 are placed with zero order quantity and the centralized supply chain only sources from the spot market. The emergency order quantity is* $\hat{Q}_{3c}$.
2. *If* $\alpha\beta(w_3 - c_1 + \gamma c_1) - \gamma c_1 < 0$ *and* $\alpha(w_3 - c_2 + \gamma c_2) - \gamma c_2 \ge 0$, *the optimal quantity ordered from supplier 1 is zero and the optimal quantity ordered from supplier 2 is* $F^{-1}\left(\frac{\alpha(p+b-c_2+\gamma c_2)-\gamma c_2}{\alpha(p+b-s)}\right)$.
3. *If* $0 \le \alpha\beta(w_3 - c_1 + \gamma c_1) - \gamma c_1 \le \beta\left[\alpha(w_3 - c_2 + \gamma c_2) - \gamma c_2\right]$, *the optimal quantity ordered from supplier 1 is zero and the optimal quantity ordered from supplier 2 is* $F^{-1}\left(\frac{\alpha(p+b-c_2+\gamma c_2)-\gamma c_2}{\alpha(p+b-s)}\right)$.
4. *If* $0 \le \beta\left[\alpha(w_3 - c_2 + \gamma c_2) - \gamma c_2\right] \le \alpha\beta(w_3 - c_1 + \gamma c_1) - \gamma c_1 \le \alpha(w_3 - c_2 + \gamma c_2) - \gamma c_2$, *both supplier 1 and supplier 2 are selected to be placed with positive orders, which are given by*

$$Q_{1c}^* = F^{-1}\left(\frac{\alpha\beta(p+b-c_1+\gamma c_1)-\gamma c_1}{\alpha\beta(p+b-s)}\right)$$
$$-F^{-1}\left(\frac{\alpha(p+b-c_2+\gamma c_2)-\gamma c_2-(\alpha\beta(p+b-c_1+\gamma c_1)-\gamma c_1)}{\alpha(1-\beta)(p+b-s)}\right), \tag{3.6}$$

$$Q_{2c}^* = F^{-1}\left(\frac{\alpha(p+b-c_2+\gamma c_2)-\gamma c_2-(\alpha\beta(p+b-c_1+\gamma c_1)-\gamma c_1)}{\alpha(1-\beta)(p+b-s)}\right). \tag{3.7}$$

5. *If* $\alpha\beta(w_3-c_1+\gamma c_1)-\gamma c_1 \geq 0$ *and* $\alpha\beta(w_3-c_1+\gamma c_1)-\gamma c_1 \geq \alpha(w_3-c_2+\gamma c_2)-\gamma c_2$*, the optimal quantity ordered from supplier 2 is zero and the optimal quantity ordered from supplier 1 is* $F^{-1}\left(\frac{\alpha\beta(p+b-c_1+\gamma c_1)-\gamma c_1}{\alpha\beta(p+b-s)}\right)$.

We obtain the conditions for both suppliers being placed with positive order quantities as the following corollary.

Corollary 3.3.2. *In the centralized supply chain, both suppliers are placed with positive order quantities if and only if the following conditions hold:*

C1: $\alpha\beta(w_3-c_1+\gamma c_1)-\gamma c_1 \geq 0$
C2: $\alpha(w_3-c_2+\gamma c_2)-\gamma c_2 \geq \alpha\beta(w_3-c_1+\gamma c_1)-\gamma c_1$
C3: $\alpha\beta(w_3-c_1+\gamma c_1)-\gamma c_1 \geq \beta\,(\alpha(w_3-c_2+\gamma c_2)-\gamma c_2)$

For the centralized supply chain, it is possible that both supplier 1 and supplier 2 are not selected, i.e., the channel only sources from the spot market if $\alpha\beta(w_3-c_1+\gamma c_1)-\gamma c_1 < 0$ and $\alpha(w_3-c_2+\gamma c_2)-\gamma c_2 < 0$. However, if $Q_{1c}^*+Q_{2c}^* > 0$, then $Q_{1c}^*+Q_{2c}^* > \hat{Q}_{3c}^*$, i.e., the total order quantity always exceeds the threshold value $\hat{Q}_{3c}^*$ if any supplier is placed with a positive order quantity. This indicates that once the channel selects one supplier or two suppliers, it prefers the supplier(s) to the spot market.

From Theorem 3.3.1, we see that the sourcing strategy of the retailer in the centralized supply chain is affected mainly by two key factors, i.e., $\alpha\beta(w_3-c_1+\gamma c_1)-\gamma c_1$ and $\alpha(w_3-c_2+\gamma c_2)-\gamma c_2$. These two factors can be regarded as the competitiveness of the two suppliers in the centralized supply chain. The larger the value of a factor is, the more powerful is the corresponding supplier, i.e., a higher probability that the supplier will be placed with a positive order quantity. Furthermore, the other factors that affect supplier competitiveness include the fixed wholesale price of the spot market, the delivery cost of a unit of the product of the supplier, the probability of delivering orders on time, and the total proportion of the marginal delivery cost in the event of a failure. The supplier can improve his competitiveness by decreasing the delivery cost or improving the probability of delivering orders on time. However, stable delivery usually increases the marginal delivery cost. Thus, a trade-off exists between the probability of on-time delivery and the marginal cost of delivery.

3.4 The Decentralized Supply Chain

Consider the decentralized supply chain in which the firms make their decisions independently. The sequence of events in the decentralized supply chain is as follows:

1. The suppliers decide their individual wholesale prices either without cooperation or with cooperation (Stage 0).
2. The retailer places orders of Q_1 units and Q_2 units with supplier 1 and supplier 2, respectively, in anticipation of supply disruption and demand (Stage 1).
3. The retailer makes an emergency order from the spot market after a supply disruption has occurred but before demand occurs (Stage 2).
4. When the selling season arrives, the retailer sells the product at a fixed price in the market. Any unmet demand incurs a goodwill cost to the retailer. After the selling season, the residual product will be salvaged (Stage 3).

As mentioned in Sect. 3.2, the situation with competitive suppliers can be viewed as two static nested games. In the following we investigate the response function of the retailer for any given wholesale price. Then based on the optimal response function, we derive the optimal wholesale price decisions for the suppliers without cooperation. A sufficient condition for the existence of an equilibrium is provided. Finally we devise a coordination mechanism to maximize the profits of both suppliers when they are cooperative.

3.4.1 The Optimal Strategy of the Retailer

This section aims to determine the retailer's optimal order allocation decisions to maximize its expected profit in stage 1 for any given wholesale price when the supply chain is decentralized.

It is straightforward to deduce that the retailer adopts the same optimal strategies as those in the centralized channel in stage 2 after a supply disruption has occurred. Both of them apply the OUL policy with the same threshold $\hat{Q}_{3\mathrm{d}} = \hat{Q}_{3\mathrm{c}} = F^{-1}\left(\frac{p+b-w_3}{p+b-s}\right)$ (hereafter the subscript 'd' stands for the decentralized supply chain). Hence the maximum expected profit of the retailer in stage 2, denoted by $\ddot{\Pi}_r^*(z)$, is also the same as the maximum expected profit of the centralized supply chain in stage 2 with the same initial inventory level z defined by (3.4).

Denote $\dot{\Pi}_r(Q_1, Q_2)$ as the retailer's expected profit in stage 1 for given wholesale prices w_1 and w_2, which is given by

$$\begin{aligned}\dot{\Pi}_r(Q_1, Q_2) = \alpha\beta&\left[\ddot{\Pi}_r^*(Q_1 + Q_2) - w_1 Q_1 - w_2 Q_2\right]\\&+\alpha(1-\beta)\left[\ddot{\Pi}_r^*(Q_2) - (1-\eta)\gamma c_1 Q_1 - w_2 Q_2\right]\\&+(1-\alpha)\left[\ddot{\Pi}_r^*(0) - (1-\eta)\gamma c_1 Q_1 - (1-\eta)\gamma c_2 Q_2\right].\qquad(3.8)\end{aligned}$$

Derivation of the retailer's optimal strategy is similar to that of the centralized supply chain. We only state the main conclusions in the following theorem.

Theorem 3.4.1. *After a supply disruption has occurred, the optimal order strategy of the retailer from the spot market is the OUL policy and the threshold value of the inventory level is* $\hat{Q}_{3d} = F^{-1}\left(\frac{p+b-w_3}{p+b-s}\right)$. *The optimal sourcing strategies from supplier 1 and supplier 2 of the retailer are given as follows:*

1. *If* $\alpha\beta(w_3 - w_1 + (1-\eta)\gamma c_1) - (1-\eta)\gamma c_1 < 0$ *and* $\alpha(w_3 - w_2 + (1-\eta)\gamma c_2) - (1-\eta)\gamma c_2 < 0$, *then both supplier 1 and supplier 2 are placed with zero order quantity and the retailer only sources from the spot market. The emergency order quantity is* $\hat{Q}_{3d}$.
2. *If* $\alpha\beta(w_3 - w_1 + (1-\eta)\gamma c_1) - (1-\eta)\gamma c_1 < 0$ *and* $\alpha(w_3 - w_2 + (1-\eta)\gamma c_2) - (1-\eta)\gamma c_2 \geq 0$, *then the optimal quantity ordered from supplier 1 is zero and the optimal quantity ordered from supplier 2 is* $F^{-1}\left(\frac{\alpha(p+b-w_2+(1-\eta)\gamma c_2)-(1-\eta)\gamma c_2}{\alpha(p+b-s)}\right)$.
3. *If* $0 \leq \alpha\beta(w_3 - w_1 + (1-\eta)\gamma c_1) - (1-\eta)\gamma c_1 \leq \beta[\alpha(w_3 - w_2 + (1-\eta)\gamma c_2) - (1-\eta)\gamma c_2]$, *then the optimal quantity ordered from supplier 1 is zero and the optimal quantity ordered from supplier 2 is* $F^{-1}\left(\frac{\alpha(p+b-w_2+(1-\eta)\gamma c_2)-(1-\eta)\gamma c_2}{\alpha(p+b-s)}\right)$.
4. *If* $0 \leq \beta\,[\alpha(w_3 - w_2 + (1-\eta)\gamma c_2) - (1-\eta)\gamma c_2] \leq \alpha\beta(w_3 - w_1 + (1-\eta)\gamma c_1) - (1-\eta)\gamma c_1 \leq \alpha(w_3 - w_2 + (1-\eta)\gamma c_2) - (1-\eta)\gamma c_2$, *both supplier 1 and supplier 2 are selected to be placed with positive order quantities, which are given by*

$$Q_{1d}^* = F^{-1}\left(\frac{\alpha\beta(p+b-w_1+(1-\eta)\gamma c_1)-(1-\eta)\gamma c_1}{\alpha\beta(p+b-s)}\right)$$
$$-F^{-1}\left(\frac{\alpha(1-\beta)(p+b) - \alpha w_2 + \alpha\beta w_1 - (1-\alpha)(1-\eta)\gamma c_2 + (1-\alpha\beta)(1-\eta)\gamma c_1}{\alpha(1-\beta)(p+b-s)}\right), \tag{3.9}$$

and

$$Q_{2d}^* = F^{-1}\left(\frac{\alpha(1-\beta)(p+b) - \alpha w_2 + \alpha\beta w_1 - (1-\alpha)(1-\eta)\gamma c_2 + (1-\alpha\beta)(1-\eta)\gamma c_1}{\alpha(1-\beta)(p+b-s)}\right). \tag{3.10}$$

5. *If* $\alpha\beta(w_3 - w_1 + (1-\eta)\gamma c_1) - (1-\eta)\gamma c_1 \geq 0$ *and* $\alpha\beta(w_3 - w_1 + (1-\eta)\gamma c_1) - (1-\eta)\gamma c_1 > \alpha(w_3 - w_2 + (1-\eta)\gamma c_2) - (1-\eta)\gamma c_2$, *then the optimal quantity ordered from supplier 2 is zero and the optimal quantity ordered from supplier 1 is* $F^{-1}\left(\frac{\alpha\beta(p+b-w_1+(1-\eta)\gamma c_1)-(1-\eta)\gamma c_1}{\alpha\beta(p+b-s)}\right)$.

Similar to the conclusions reached for the centralized supply chain, we have the following conditions for both suppliers being placed with positive order quantities.

Corollary 3.4.2. *In the decentralized supply chain, both suppliers are placed with positive order quantities if and only if the following conditions hold:*

C4: $\alpha\beta(w_3 - w_1 + (1-\eta)\gamma c_1) - (1-\eta)\gamma c_1 \geq 0$
C5: $\alpha(w_3 - w_2 + (1-\eta)\gamma c_2) - (1-\eta)\gamma c_2 \geq \alpha\beta(w_3 - w_1 + (1-\eta)\gamma c_1) - (1-\eta)\gamma c_1$
C6: $\alpha\beta(w_3 - w_1 + (1-\eta)\gamma c_1) - (1-\eta)\gamma c_1 \geq \beta[\alpha(w_3 - w_2 + (1-\eta)\gamma c_2) - (1-\eta)\gamma c_2]$

For the decentralized supply chain, we also find that the sourcing strategy of the retailer is affected mainly by two key factors, i.e., $\alpha\beta(w_3 - w_1 + (1-\eta)\gamma c_1) - (1-\eta)\gamma c_1$ and $\alpha(w_3 - w_2 + (1-\eta)\gamma c_2) - (1-\eta)\gamma c_2$. These two factors can be regarded as the competitiveness of the two suppliers in the decentralized supply chain. The larger the value of a factor is, the more powerful is the corresponding supplier. Furthermore, the other factors that affect supplier competitiveness include the wholesale price offered by the supplier and the proportion of the cost incurred by the supplier in addition to all the factors in the centralized supply chain. The supplier in the centralized supply chain can improve the competitiveness by decreasing his wholesale price in addition to decreasing the delivery cost or improving the probability of delivering orders on time. However, there is a trade-off between the order quantity and the wholesale price.

3.4.2 The Optimal Strategies of Competitive Suppliers

In this section supplier 1 and supplier 2 are assumed to be competitive, i.e., supplier 1 and supplier 2 set their individual wholesale prices simultaneously to maximize their respective expected profits before the retailer places its orders and the suppliers do not collude. As mentioned in Sect. 3.2, this is a static non-cooperative game between supplier 1 and supplier 2. We first derive the feasible strategy space of both suppliers. Then we derive a sufficient condition for the existence of an equilibrium price strategy in this game. Based on the assumption of a uniform demand distribution, we further obtain an explicit form of the equilibrium strategy.

We have obtained the conditions in Corollary 3.4.2 in which both supplier are placed with positive order quantities. Based on these conditions in Corollary 3.4.2, we obtain the feasible strategy spaces of both suppliers to make a positive expected profit and the conditions for the existence of the spaces as follows:

Theorem 3.4.3. *If* $\beta(\alpha - \alpha\gamma + \gamma)c_2 > (\alpha\beta - \alpha\beta\gamma + \gamma)c_1$, *then the feasible strategy space of supplier 1 is* $[\frac{\alpha\beta - \alpha\beta\eta\gamma + \eta\gamma}{\alpha\beta}c_1, \frac{\alpha - \alpha\gamma + \gamma}{\alpha}c_2 - \frac{(1-\alpha\beta)(1-\eta)\gamma}{\alpha\beta}c_1]$. *When supplier 1 sets its wholesale price in the interval, it will obtain a positive profit. Similarly, if* $\alpha w_3 - (\alpha - \alpha\gamma + \gamma)c_2 > \alpha\beta w_3 - (\alpha\beta - \alpha\beta\gamma + \gamma)c_1$, *then the feasible strategy space of supplier 2 is* $[\frac{\alpha - \alpha\gamma\eta + \gamma\eta}{\alpha}c_2, (1-\beta)w_3 - \frac{(1-\alpha)(1-\eta)\gamma}{\alpha}c_2 + \frac{\alpha\beta - \alpha\beta\gamma + \gamma}{\alpha}c_1]$. *When supplier 2 sets its wholesale price in the interval, it will obtain a positive profit.*

Note the two conditions in Theorem 3.4.3 are $C2$ and $C3$ in Corollary 3.3.2. This means that if both suppliers are placed with positive order quantities in the centralized supply chain, then there exist feasible strategy spaces for the two competitive suppliers to obtain a positive profit in the decentralized supply chain. Hence, we have the following results.

Proposition 3.4.4. *The two competitive suppliers can obtain a positive expected profit in the decentralized supply chain if both of them are placed with positive order quantities in the centralized supply chain.*

From Theorem 3.4.3, we obtain the feasible strategy spaces for both suppliers. Consequently, we discuss the existence of an equilibrium solution for the game. For any wholesale price, supplier 1 and supplier 2 can correctly anticipate the retailer's demand curves, i.e., $Q_1(w_1, w_2)$ and $Q_2(w_1, w_2)$, which are given by (3.9) and (3.10). Hence, the suppliers face the inverse demand curves

$$w_1(Q_1, Q_2) = p + b - (p + b - s)F(Q_1 + Q_2) - \frac{(1-\alpha\beta)(1-\eta)\gamma}{\alpha\beta}c_1, \quad (3.11)$$

and

$$\begin{aligned} w_2(Q_2, Q_1) = {} & p + b - \beta(p + b - s)F(Q_1 + Q_2) - (1-\beta)(p + b - s)F(Q_2) \\ & - \frac{(1-\alpha)(1-\eta)\gamma}{\alpha}c_2. \end{aligned} \quad (3.12)$$

Because $F(x)$ is continuous and strictly increasing, it is easy to verify that the corresponding feasible spaces for Q_1 and Q_2 are also closed intervals if the feasible spaces for w_1 and w_2 are closed intervals. Moreover, the revenue functions are equivalent to

$$\begin{aligned} \Pi_{s_1}(Q_1, Q_2) &= [\alpha\beta w_1(Q_1, Q_2) - (\alpha\beta - \alpha\beta\eta\gamma + \eta\gamma)c_1]Q_1 \\ &= [\alpha\beta(p + b) - \alpha\beta(p + b - s)F(Q_1 + Q_2) \\ &\quad -(\alpha\beta - \alpha\beta\gamma + \gamma)c_1]Q_1, \end{aligned} \quad (3.13)$$

and

$$\begin{aligned} \Pi_{s_2}(Q_2, Q_1) &= [\alpha w_2(Q_2, Q_1) - (\alpha - \alpha\gamma\eta + \gamma\eta)c_2]Q_2 \\ &= [\alpha(p + b) - \alpha\beta(p + b - s)F(Q_1 + Q_2) \\ &\quad -\alpha(1-\beta)(p + b - s)F(Q_2) - (\alpha - \alpha\gamma + \gamma)c_2]Q_2. \end{aligned} \quad (3.14)$$

The problems of supplier 1 and supplier 2 are equivalent for setting quantities Q_1 and Q_2 to maximize $\Pi_{s_1}(Q_1, Q_2)$ and $\Pi_{s_2}(Q_2, Q_1)$ simultaneously. Maximizing $\Pi_{s_1}(Q_1, Q_2)$ and $\Pi_{s_2}(Q_2, Q_1)$ are straightforward if they are unimodal. However the objective functions $\Pi_{s_1}(Q_1, Q_2)$ and $\Pi_{s_2}(Q_2, Q_1)$ are dependent on the demand distribution. It should be noted that not all demand distributions result in a unimodal objective function. In the following, we derive a sufficient and less restrictive condition to ensure the objective functions are unimodal.

Define a new function $g(Q_2|Q_1, \beta)$ as follows:

$$g(Q_2|Q_1, \beta) \triangleq \frac{Q_2[\beta(p + b - s)f(Q_2 + Q_1) + (1-\beta)(p + b - s)f(Q_2)]}{\beta[p + b - (p + b - s)F(Q_2 + Q_1)] + (1-\beta)[p + b - (p + b - s)F(Q_2)]}. \quad (3.15)$$

We have the following conclusions about the objective functions.

Lemma 3.4.5. *Suppose $F(x)$ has a support $[a, b)$. If $g(Q_2|Q_1, \beta)$ is weakly increasing for Q_2, then supplier 2's revenue function is unimodal for $Q_2 \in [0, +\infty)$. Moreover, supplier 1's revenue function $\Pi_{s_1}(Q_1, Q_2)$ is also unimodal for $Q_1 \in [0, +\infty)$.*

As pointed out in the above analysis, the strategy space for each supplier's decision is a closed interval; hence, it is a nonempty compact convex set of the Eculidean space. Along with the results in Lemma 3.4.5, we have the following theorem about the existence of a Nash equilibrium of this game.

Theorem 3.4.6. *If $g(Q_2|Q_1, \beta)$ is weakly increasing for Q_2, then a pure strategy Nash equilibrium exists.*

In the proof of Lemma 3.4.5, a new function $g(Q_2|Q_1, \beta)$ was defined. If $s = 0$, $Q_1 = 0$ and $\beta = 1$, $g(Q_2|Q_1, \beta)$ is the so-called generalized failure rate was defined by Lariviere and Porteus (2001). They proved that if the demand follows an increasing generalized failure rate ($IGFR$), the objective is unimodal. They also pointed out that most demand distributions follow an *IGFR*. In this chapter we obtain a similar condition, i.e., $g(Q_2|Q_1, \beta)$ is weakly increasing. It should be noted that it is difficult to verify that all the demand distributions meet this condition. Fortunately, it is straightforward to verify that both the uniform and exponential distributions possess this important property.

Corollary 3.4.7. *If the demand follows a uniform distribution or an exponential distribution, then a pure strategy Nash equilibrium exists.*

Our analysis so far has not imposed any restrictions on the demand distribution. Further analysis (e.g., the uniqueness of the equilibrium and the explicit expression of the equilibrium) for a general demand distribution is difficult. In order to gain further insights, we assume that the demand D is uniformly distributed in some interval, which without loss of generality can be taken as the interval [0, 1]. Here the game between supplier 1 and supplier 2 is a non-cooperative static game. We have the following theorem.

Theorem 3.4.8. *Assuming that the demand D is uniformly distributed in the interval* [0, 1]. *If $\beta(\alpha - \alpha\gamma + \gamma)c_2 > (\alpha\beta - \alpha\beta\gamma + \gamma)c_1$ and $\alpha w_3 - (\alpha - \alpha\gamma + \gamma)c_2 > \alpha\beta w_3 - (\alpha\beta - \alpha\beta\gamma + \gamma)c_1$, then the unique Nash equilibrium strategy of the game between the suppliers is given by:*

$$(w_{1n}^*, w_{2n}^*) = \begin{cases} (w_{11}, w_{21}), & w_{11} \leq \hat{w}_1 \bigwedge \overline{w}_1 \text{ and } w_{21} \leq \hat{w}_2 \bigwedge \overline{w}_2, \\ (w_{12}, w_{23}), & \overline{w}_1 \geq w_{12} > \hat{w}_1 \text{ and } w_{23} \leq \hat{w}_2 \bigwedge \overline{w}_2, \\ (w_{13}, w_{22}), & w_{13} \leq \hat{w}_1 \bigwedge \overline{w}_1 \text{ and } \overline{w}_2 \geq w_{22} > \hat{w}_2, \\ (w_{13}, w_{23}), & \overline{w}_1 \geq w_{13} > \hat{w}_1 \text{ and } \overline{w}_2 \geq w_{23} > \hat{w}_2. \end{cases} \tag{3.16}$$

(All the variables appearing in this theorem are defined in the proof. Hereafter the subscript 'n' stands for the decentralized supply chain with non-cooperation suppliers.)

3.4.3 The Optimal Strategies of Cooperative Suppliers

In this section, supplier 1 and supplier 2 are assumed to be cooperative, i.e., the two suppliers set their individual wholesale prices in order to maximize their total expected profits before the retailer places its orders. Obviously, a necessary precondition for their cooperation is that none of them is intended to set its wholesale price low enough to monopolize the market. It seems that the wholesale prices of both suppliers are set by one decision-maker to ensure both suppliers are placed with non-negative order quantities and to maximize the total profit of the two suppliers. We first derive the optimal wholesale price of the two cooperative suppliers. Then we discuss how to divide the profit and how to pool the cost between the two suppliers to execute the cooperative wholesale price successfully. Finally, we discuss the coordination of the whole channel.

From the analysis in Sect. 3.4.1, if and only if the conditions in Corollary 3.4.2 hold, both suppliers will be placed with positive order quantities, which are given by (3.9) and (3.10). Then the total revenue function of the two suppliers is given by

$$\Pi_{\mathrm{dc}}(w_1, w_2) = [\alpha\beta w_1 - (\alpha\beta - \alpha\beta\eta\gamma + \eta\gamma)c_1]Q_1 + [\alpha w_2 - (\alpha - \alpha\gamma\eta + \gamma\eta)c_2]Q_2. \tag{3.17}$$

(Hereafter the subscript '*dc*' stands for the decentralized supply chain with cooperative suppliers).

The problem of cooperative suppliers is as follows:

$$\begin{aligned}
&\max\ \Pi_{\mathrm{dc}}(w_1, w_2)\\
&\text{s.t.}\ \ 0 \le w_1 \le w_3 - \frac{(1-\alpha\beta)(1-\eta)\gamma c_1}{\alpha\beta};\\
&\qquad w_2 \ge w_1 + \frac{(1-\alpha\beta)(1-\eta)\gamma c_1}{\alpha\beta} - \frac{(1-\alpha)(1-\eta)\gamma c_2}{\alpha};\\
&\qquad w_2 \le w_3 - \frac{(1-\alpha)(1-\eta)\gamma c_2}{\alpha} - \frac{\alpha\beta(w_3 - w_1) - (1-\alpha\beta)(1-\eta)\gamma c_1}{\alpha}.
\end{aligned} \tag{3.18}$$

It is easy to verify that the Hessian matrix of $\Pi_{\mathrm{dc}}(w_1, w_2)$ is negative definite. Hence, $\Pi_{\mathrm{dc}}(w_1, w_2)$ is jointly concave with respect to w_1 and w_2. Moreover, the feasible spaces for w_1 and w_2 are convex. Problem (3.18) is a convex quadratic programming problem, which can be solved by some popular mathematical software such as MatLab or Mathematica.

It is obvious that if supplier 1 and supplier 2 are cooperative, they can obtain more total expected revenue than that under the non-cooperative scenario. How to divide the profit pie created through cooperation is crucial for stability and robustness of the cooperation. The Nash bargaining game in cooperative game theory can be used to ascertain the allocation ratio of the expected profit pie to ensure that both suppliers earn a rational expected revenue. It should be noted that the bargaining solution of the game is not a randomized outcome. However, all the revenues of the suppliers are random. Thus, only to ascertain the allocation ratio of the expected profit pie is not enough to guarantee supplier cooperation. An effective way to allocate the randomized revenue should be developed to the effect that the total expected profit can be allocated according to the bargaining solution. In the following we primarily focus on two issues, i.e., allocation of the expected revenue and allocation of the randomized revenue.

We start by building a basic bargaining model initiated by Nash (1951). Recall that the Nash bargaining game requires us to identify a feasible set of payoffs F and a disagreement point d that are pre-determined and are independent of the negotiations. To do so, let us first suppose that the two suppliers negotiate on individual expected revenues denoted by $(\Pi_{\mathrm{dcs}_1}, \Pi_{\mathrm{dcs}_2})$. Obviously, this negotiation is conducted over the sharing of some fixed profit pie. Denote the optimal value of Problem (3.18), i.e., the pie to be allocated between the two suppliers, as Π^*_{dc}. Thus, the feasible set of the bargaining is $F = (\Pi_{\mathrm{dcs}_1}, \Pi_{\mathrm{dcs}_2} | \Pi_{\mathrm{dcs}_1} + \Pi_{\mathrm{dcs}_2} = \Pi^*_{\mathrm{dc}})$. Furthermore, according to the rule of negotiation, the disagreement point is defined as the two suppliers' equilibrium expected revenues under the non-cooperative scenario, i.e., $d = (\Pi^*_{\mathrm{s1}}, \Pi^*_{\mathrm{s2}})$. Hence, the Nash bargaining solution between the two suppliers is obtained by solving the following optimization problem:

$$\arg \max_{(\Pi_{\mathrm{dcs}_1}, \Pi_{\mathrm{dcs}_2}) \in F, (\Pi_{\mathrm{dcs}_1}, \Pi_{\mathrm{dcs}_2}) \geq d} (\Pi_{\mathrm{dcs}_1} - \Pi^*_{\mathrm{s1}})(\Pi_{\mathrm{dcs}_2} - \Pi^*_{\mathrm{s2}}). \tag{3.19}$$

It is straightforward to obtain the solution of Problem (3.19) as follows:

$$(\Pi^*_{\mathrm{dcs}_1}, \Pi^*_{\mathrm{dcs}_2}) = \left(\frac{\Pi^*_{\mathrm{dc}} + \Pi^*_{\mathrm{s1}} - \Pi^*_{\mathrm{s2}}}{2}, \frac{\Pi^*_{\mathrm{dc}} - \Pi^*_{\mathrm{s1}} + \Pi^*_{\mathrm{s2}}}{2} \right). \tag{3.20}$$

Since all revenue uncertainty is due to random demand falls on the retailer, the uncertainty of the suppliers' revenues is caused only by their reliability. Supplier 1 and 2 can allocate the randomized profit according to ratios θ_1, θ_2, and θ_3 as follows:

1. If both suppliers are in the success state, the total profit of them is $(w_1 - c_1)Q_1 + (w_2 - c_2)Q_2$. The profits of supplier 1 and 2 are $\theta_1[(w_1 - c_1)Q_1 + (w_2 - c_2)Q_2]$ and $(1 - \theta_1)[(w_1 - c_1)Q_1 + (w_2 - c_2)Q_2]$, respectively. The probability of this case occurring is $\alpha\beta$.
2. If supplier 1 is in the failure state and supplier 2 is in the success state, the total profit of them is $(w_2 - c_2)Q_2 - \eta\gamma c_1 Q_1$. The profits of supplier 1 and 2 are $\theta_2[(w_2 - c_2)Q_2 - \eta\gamma c_1 Q_1]$ and $(1 - \theta_2)[(w_2 - c_2)Q_2 - \eta\gamma c_1 Q_1]$, respectively. The probability of this case occurring is $\alpha(1 - \beta)$.
3. If both suppliers are in the failure state, the total profit of them is $-\eta\gamma(c_1 Q_1 + c_2 Q_2)$. The profits of supplier 1 and 2 are $\theta_3[-\eta\gamma(c_1 Q_1 + c_2 Q_2)]$ and $(1 - \theta_3)[-\eta\gamma(c_1 Q_1 + c_2 Q_2)]$, respectively. The probability of this case occurring is $1 - \alpha$.

Obviously, the expected profit of supplier 1 can be written equivalently as

$$\Pi_{s_1}(\theta_1, \theta_2, \theta_3) = \alpha\beta\theta_1[(w_1 - c_1)Q_1 + (w_2 - c_2)Q_2] - (1-\alpha)\theta_3\eta\gamma(c_1 Q_1 + c_2 Q_2)$$
$$+\alpha(1-\beta)\theta_2[(w_2 - c_2)Q_2 - \eta\gamma c_1 Q_1].$$

Similarly, the expected profit of supplier 2 can be written as

$$\begin{aligned}\Pi_{s_2}(\theta_1, \theta_2, \theta_3) &= \alpha\beta(1-\theta_1)[(w_1 - c_1)Q_1 + (w_2 - c_2)Q_2] \\ &\quad - (1-\alpha)(1-\theta_3)\eta\gamma(c_1 Q_1 + c_2 Q_2) \\ &\quad + \alpha(1-\beta)(1-\theta_2)[(w_2 - c_2)Q_2 - \eta\gamma c_1 Q_1].\end{aligned}$$

To obtain the bargaining solution, supplier 1 and 2 can negotiate parameters θ_1, θ_2 and θ_3, which are subject to the following equations:

$$\begin{cases} \Pi_{s_1}(\theta_1, \theta_2, \theta_3) = \Pi^*_{dcs_1}, \\ \Pi_{s_2}(\theta_1, \theta_2, \theta_3) = \Pi^*_{dcs_2}. \end{cases} \tag{3.21}$$

Equation (3.21) is a system of 2 linear equations in 3 unknowns. This system of linear equations has infinitely many solutions. Supplier 1 and 2 can select a combination of $(\theta_1, \theta_2, \theta_3)$ subject to (3.21) to allocate their randomized profits.

In sum, the cooperative suppliers can obtain more profits by the following mechanism:

1. Supplier 1 and 2 decide their individual wholesale prices with cooperation to maximize their total expected profits.
2. Adopt the Nash bargaining framework to examine the expected total profit allocations.
3. All the parameters $(\theta_1, \theta_2, \theta_3)$ subject to (3.21) are negotiated to allocate their randomized profits.

We now turn our attention to coordination of the whole channel. Note that the retailer in the decentralized supply chain adopts the same OUL policy as the centralized channel after a supply disruption has occurred. Thus, the whole channel is coordinated if and only if the retailer in the decentralized system chooses the same inventory vector as in the centralized system. Letting $Q^*_{1n} = Q^*_{1c}$ and $Q^*_{2n} = Q^*_{2c}$, we have

$$w^*_{1c} = \frac{(\alpha\beta - \alpha\beta\gamma + \gamma) - (1-\alpha\beta)(1-\eta)\gamma}{\alpha\beta} c_1, \tag{3.22}$$

$$w^*_{2c} = \frac{(\alpha - \alpha\gamma + \gamma) - (1-\alpha)(1-\eta)\gamma}{\alpha} c_2. \tag{3.23}$$

Hence, the maximum expected revenue of the whole supply chain can be achieved if the corresponding wholesale prices are identical to (3.22) and (3.23). However, even if the corresponding wholesale prices are identical to (3.22) and (3.23), it is difficult to achieve full coordination only by the wholesale prices. The reason is that, it is difficult to allocate the total revenue only by the wholesale prices between the retailer and the suppliers since the revenue of the retailer is stochastic due to the random demand.

3.5 Numerical Examples

In this section we present numerical examples to illustrate the theoretical results and explore the differences between the centralized supply chain and the decentralized supply chain. We studied all 36 problems created by all possible combinations of the following parameters: $w_3 = \{16\}$; $c_1 = \{10.5\}$; $c_2 = \{12\}$; $p = \{18\}$; $b = \{5\}$; $s = \{3\}$; $\gamma = \{0.2, 0.3, 0.4\}$; $\eta = \{0.2, 0.5, 0.8\}$; $\alpha = \{0.7, 0.9\}$; and $\beta = \{0.7, 0.9\}$. $\gamma = \{0.2, 0.3, 0.4\}$ denotes that the marginal delivering cost in the event of a failure is low, moderate, and high, respectively. The meanings of $\eta = \{0.2, 0.5, 0.8\}$ are similar. $\alpha = \{0.7, 0.9\}$ denotes a low and high probability of a common-cause failure not occurring, respectively. The meanings of $\beta = \{0.7, 0.9\}$ are similar. In these problems, demand was uniformly distributed over [300,700] and thus the corresponding mean demand was 500.

All the different problems and the computational results of the centralized supply chain are listed in Table 3.1. The computational results of the decentralized supply chain with non-cooperative are listed in Table 3.2. The computational results of the decentralized supply chain with cooperative are listed in Table 3.3.

From the computational results in Table 3.1, the following observations can be made:

- The total profit of the supply chain decreases as γ increases.
- The order quantity from supplier 2 increases as γ increases. However, the order quantity from supplier 1 decreases. This indicates that the larger the total proportion of the marginal delivery cost in the event of a failure, the more important is supply stability.
- If the supply stability of supplier 1 is high, the advantage of the low cost is obvious. But if supply stability is too low, the advantage does not exist any more.

From the computational results in Tables 3.2 and 3.3, the following observations can be made:

- If the two suppliers are cooperative, supplier 1 will set its wholesale price high enough to compel the retailer to source only from supplier 2, who sets its wholesale price substantially higher than the equilibrium wholesale price. Hence, the sum of the suppliers' profits increases and the profit of the retailer decreases at the same time.

Table 3.1 The centralized supply chain

	γ	η	α	β	w_1^* [1]	w_2^*	Q_1^*	Q_2^*	Π_t^*
1	0.2	0.2	0.7	0.7	11.1557	12.3086	22.8571	483.4286	1068.9
2	0.2	0.2	0.7	0.9	I [2]	I	I	I	I
3	0.2	0.2	0.9	0.7	10.8700	12.0800	35.5556	489.7778	1699.8
4	0.2	0.2	0.9	0.9	I	I	I	I	I
5	0.2	0.5	0.7	0.7	11.5929	12.5143	22.8571	483.4286	1068.9
6	0.2	0.5	0.7	0.9	I	I	I	I	I
7	0.2	0.5	0.9	0.7	11.1167	12.1333	35.5556	489.7778	1699.8
8	0.2	0.5	0.9	0.9	I	I	I	I	I
9	0.2	0.8	0.7	0.7	12.2486	12.8229	22.8571	483.4286	1068.9
10	0.2	0.8	0.7	0.9	I	I	I	I	I
11	0.2	0.8	0.9	0.7	11.4867	12.2133	35.5556	489.7778	1699.8
12	0.2	0.8	0.9	0.9	I	I	I	I	I
13	0.3	0.2	0.7	0.7	II [3]	II	II	II	II
14	0.3	0.2	0.7	0.9	I	I	I	I	I
15	0.3	0.2	0.9	0.7	10.8700	12.0800	3.3333	509.6667	1632.3
16	0.3	0.2	0.9	0.9	I	I	I	I	I
17	0.3	0.5	0.7	0.7	II	II	II	II	II
18	0.3	0.5	0.7	0.9	I	I	I	I	I
19	0.3	0.5	0.9	0.7	11.4250	12.2000	3.3333	509.6667	1632.3
20	0.3	0.5	0.9	0.9	I	I	I	I	I
21	0.3	0.8	0.7	0.7	II	II	II	II	II
22	0.3	0.8	0.7	0.9	I	I	I	I	I
23	0.3	0.8	0.9	0.7	11.9800	12.3200	3.3333	509.6667	1632.3
24	0.3	0.8	0.9	0.9	I	I	I	I	I
25	0.4	0.2	0.7	0.7	II	II	II	II	II
26	0.4	0.2	0.7	0.9	I	I	I	I	I
27	0.4	0.2	0.9	0.7	II	II	II	II	II
28	0.4	0.2	0.9	0.9	I	I	I	I	I
29	0.4	0.5	0.7	0.7	II	II	II	II	II
30	0.4	0.5	0.7	0.9	I	I	I	I	I
31	0.4	0.5	0.9	0.7	II	II	II	II	II
32	0.4	0.5	0.9	0.9	I	I	I	I	I
33	0.4	0.8	0.7	0.7	II	II	II	II	II
34	0.4	0.8	0.7	0.9	I	I	I	I	I
35	0.4	0.8	0.9	0.7	II	II	II	II	II
36	0.4	0.8	0.9	0.9	I	I	I	I	I

[1] For the centralized supply chain, w_1^* and w_2^* denote the wholesale prices at which the best performance of the whole supply chain can be achieved.
[2] Hereafter 'I' indicates that the retailer only orders from supplier 1.
[3] Hereafter 'II' indicates that the retailer only orders from supplier 2.

- When the two suppliers are cooperative, the total profit of the whole supply chain is lower than the profit when they are competitive. This indicates that cooperation of the suppliers does not necessarily lead to supply chain efficiency.

Table 3.2 The decentralized supply chain with competitive suppliers

	w_1^*	w_2^*	Q_1^*	Q_2^*	Π_r^*	Π_{s1}^*	Π_{s2}^*	Π_t^*
1	11.4986	12.9600	43.4286	456.0000	849.1680	7.2960	207.9360	1064.4
2	∣	∣	∣	∣	∣	∣	∣	∣
3	11.4033	12.8267	49.7778	464.8889	1364.7	16.7253	312.4053	1693.8
4	∣	∣	∣	∣	∣	∣	∣	∣
5	11.9357	13.1657	43.4286	456	849.1680	7.2960	207.9360	1064.4
6	∣	∣	∣	∣	∣	∣	∣	∣
7	11.6500	12.8800	49.7778	464.8889	1364.7	16.7253	312.4053	1693.8
8	∣	∣	∣	∣	∣	∣	∣	∣
9	12.5914	13.4743	43.4286	456.0000	849.1680	7.2960	207.9360	1064.4
10	∣	∣	∣	∣	∣	∣	∣	∣
11	12.0200	12.9600	49.7778	464.8889	1364.7	16.7253	312.4053	1693.8
12	∣	∣	∣	∣	∣	∣	∣	∣
13	∥	∥	∥	∥	∥	∥	∥	∥
14	∣	∣	∣	∣	∣	∣	∣	∣
15	10.9200	13.1250	69.6667	442.3333	1183.5	2.1945	416.0145	1601.7
16	∣	∣	∣	∣	∣	∣	∣	∣
17	∥	∥	∥	∥	∥	∥	∥	∥
18	∣	∣	∣	∣	∣	∣	∣	∣
19	11.4750	13.2450	69.6667	442.3333	1183.5	2.1945	416.0145	1601.7
20	∣	∣	∣	∣	∣	∣	∣	∣
21	∥	∥	∥	∥	∥	∥	∥	∥
22	∣	∣	∣	∣	∣	∣	∣	∣
23	12.0300	13.3650	69.6667	442.3333	1183.5	2.1945	416.0145	1601.7
24	∣	∣	∣	∣	∣	∣	∣	∣
25	∥	∥	∥	∥	∥	∥	∥	∥
26	∣	∣	∣	∣	∣	∣	∣	∣
27	∥	∥	∥	∥	∥	∥	∥	∥
28	∣	∣	∣	∣	∣	∣	∣	∣
29	∥	∥	∥	∥	∥	∥	∥	∥
30	∣	∣	∣	∣	∣	∣	∣	∣
31	∥	∥	∥	∥	∥	∥	∥	∥
32	∣	∣	∣	∣	∣	∣	∣	∣
33	∥	∥	∥	∥	∥	∥	∥	∥
34	∣	∣	∣	∣	∣	∣	∣	∣
35	∥	∥	∥	∥	∥	∥	∥	∥
36	∣	∣	∣	∣	∣	∣	∣	∣

Comparing the computational results in Tables 3.1, 3.2 and 3.3, the following observations can be made:

- If both suppliers are selected in the centralized supply chain, they will also be placed with positive order quantities when they are competitive in the decentralized supply chain.

Table 3.3 The decentralized supply chain with cooperative suppliers

	w_1^*	w_2^*	Q_1^*	Q_2^*	Π_r^*	Π_{s1}^*	Π_{s2}^*	Π_t^*
1	14.4700	15.2800	0	440.0000	90.0000	0	915.2000	1005.2
2	I	I	I	I	I	I	I	I
3	15.1367	15.8133	0	440.0000	90.0000	0	1478.4	1568.4
4	I	I	I	I	I	I	I	I
5	14.9071	15.4857	0	440.0000	90.0000	0	915.2000	1005.2
6	I	I	I	I	I	I	I	I
7	15.3833	15.8667	0	440.0000	90.0000	0	1478.4	1568.4
8	I	I	I	I	I	I	I	I
9	15.5629	15.7943	0	440.0000	90.0000	0	915.2000	1005.2
10	I	I	I	I	I	I	I	I
11	15.7533	15.9467	0	440.0000	90.0000	0	1478.4	1568.4
12	I	I	I	I	I	I	I	I
13	II	II	II	II	II	II	II	II
14	I	I	I	I	I	I	I	I
15	14.5200	15.6800	0	440.0000	90.0000	0	1425.6	1515.6
16	I	I	I	I	I	I	I	I
17	II	II	II	II	II	II	II	II
18	I	I	I	I	I	I	I	I
19	15.0750	15.8000	0	440.0000	90.0000	0	1425.6	1515.6
20	I	I	I	I	I	I	I	I
21	II	II	II	II	II	II	II	II
22	I	I	I	I	I	I	I	I
23	15.6300	15.9200	0	440.0000	90.0000	0	1425.6	1515.6
24	I	I	I	I	I	I	I	I
25	II	II	II	II	II	II	II	II
26	I	I	I	I	I	I	I	I
27	II	II	II	II	II	II	II	II
28	I	I	I	I	I	I	I	I
29	II	II	II	II	II	II	II	II
30	I	I	I	I	I	I	I	I
31	II	II	II	II	II	II	II	II
32	I	I	I	I	I	I	I	I
33	II	II	II	II	II	II	II	II
34	I	I	I	I	I	I	I	I
35	II	II	II	II	II	II	II	II
36	I	I	I	I	I	I	I	I

- In the decentralized supply chain with non-cooperative or cooperative suppliers, the total profit of the supply chain is less than that of the centralized supply chain. Moreover, the wholesale prices are higher than the corresponding wholesale prices at which the best performance of the whole supply chain can be achieved.
- Under the three scenarios, the impact of η on the supply chain is not as significant as the impact of γ. Only the impact on the wholesale prices is obvious.

3.6 Concluding Remarks

In this chapter we considered both upstream and downstream uncertainty in determining appropriate sourcing strategies for retailers and pricing strategies for suppliers. By assuming that demand is uniformly distributed, we derived the optimal order quantities from different suppliers. This allows the retailer to examine the critical trade-off between the low-cost supplier's reliability versus its cost advantage relative to the other suppliers. We derived a sufficient condition for the existence of an equilibrium price in a decentralized system when the suppliers are competitive. Based on the assumption of a uniform demand distribution, we obtained an explicit form of the solutions when the suppliers are competitive. We also constructed a coordination mechanism to maximize the profits of the suppliers. These findings can guide suppliers to find a trade-off between order quantity and wholesale price and a trade-off between the probability of on-time delivery and the marginal cost of delivery. Comparing with the benchmark scenario, i.e., a centralized supply chain, we found that it is difficult to achieve full coordination by wholesale-price-only contracts. How to devise a mechanism to coordinate the whole channel is a potential topic for future research.

Appendix

Proof of Theorem 3.3.1

Proof. We discuss the optimal strategies Q_{1c} and Q_{2c} of the channel based on the following different cases.

Case 1. $Q_{2c} \leq Q_{1c} + Q_{2c} \leq \hat{Q}_{3c}$.
Since the first-order derivative of the function in (3.4) with respect to z is given by

$$\frac{\mathrm{d}\ddot{\Pi}_c^*(z)}{\mathrm{d}z} = \begin{cases} w_3, & z \leq \hat{Q}_{3c}, \\ (p+b) - (p+b-s)F(z), & z \geq \hat{Q}_{3c}, \end{cases} \tag{3.24}$$

we obtain the first-order partial derivatives of $\dot{\Pi}_c(Q_{1c}, Q_{2c})$ with respect to Q_{1c} and Q_{2c} in this case as follows:

$$\frac{\partial \dot{\Pi}_c(Q_{1c}, Q_{2c})}{\partial Q_{1c}} = \alpha\beta w_3 - (\alpha\beta - \alpha\beta\gamma + \gamma)c_1, \tag{3.25}$$

$$\frac{\partial \dot{\Pi}_c(Q_{1c}, Q_{2c})}{\partial Q_{2c}} = \alpha w_3 - (\alpha - \alpha\gamma + \gamma)c_2. \tag{3.26}$$

From (3.25) and (3.26), we reach the following conclusions:

1. If $\alpha\beta w_3 - (\alpha\beta - \alpha\beta\gamma + \gamma)c_1 \geq 0$ and $\alpha w_3 - (\alpha - \alpha\gamma + \gamma)c_2 \geq 0$, then the expected revenue will increase as the order quantities Q_1 and Q_2 increase. Hence, $Q_{1c}^* + Q_{2c}^* \geq \hat{Q}_{3c}$.
2. If $\alpha\beta w_3 - (\alpha\beta - \alpha\beta\gamma + \gamma)c_1 < 0$ and $\alpha w_3 - (\alpha - \alpha\gamma + \gamma)c_2 < 0$, then the expected revenue will increase as the order quantities Q_1 and Q_2 decrease. Hence, $Q_{1c}^* = Q_{2c}^* = 0$.
3. If $\alpha\beta w_3 - (\alpha\beta - \alpha\beta\gamma + \gamma)c_1 < 0$ and $\alpha w_3 - (\alpha - \alpha\gamma + \gamma)c_2 \geq 0$, then the expected revenue will increase as Q_1 decreases or as Q_2 increases. Hence, $Q_{1c}^* = 0$ and $Q_{2c}^* \geq \hat{Q}_{3c}$.
4. If $\alpha\beta w_3 - (\alpha\beta - \alpha\beta\gamma + \gamma)c_1 \geq 0$ and $\alpha w_3 - (\alpha - \alpha\gamma + \gamma)c_2 < 0$, then the expected revenue will increase as Q_1 increases or as Q_2 decreases. Hence, $Q_{1c}^* \geq \hat{Q}_{3c}$ and $Q_{2c}^* = 0$.

From conclusions 2, 3 and 4, it can be observed that there is at least one supplier that is not selected to receive orders when $\alpha\beta w_3 - (\alpha\beta - \alpha\beta\gamma + \gamma)c_1 < 0$ and/or $\alpha w_3 - (\alpha - \alpha\gamma + \gamma)c_2 < 0$. The reason is that the supplier's supply reliability is too low or its delivery cost is too high.

Case 2. $Q_{2c} \leq \hat{Q}_{3c} \leq Q_{1c} + Q_{2c}$.
The first-order partial derivatives of $\dot{\Pi}_c(Q_{1c}, Q_{2c})$ with respect to Q_{1c} and Q_{2c} in this case are given by

$$\frac{\partial \dot{\Pi}_c(Q_{1c}, Q_{2c})}{\partial Q_{1c}} = \alpha\beta\left[(p+b) - (p+b-s)F(Q_{1c}+Q_{2c})\right] - (\alpha\beta - \alpha\beta\gamma + \gamma)c_1, \tag{3.27}$$

and

$$\begin{aligned}\frac{\partial \dot{\Pi}_c(Q_{1c}, Q_{2c})}{\partial Q_{2c}} &= \alpha\beta\left[(p+b) - (p+b-s)F(Q_{1c}+Q_{2c})\right] \\ &\quad + \alpha(1-\beta)w_3 - (\alpha - \alpha\gamma + \gamma)c_2. \end{aligned} \tag{3.28}$$

It is straightforward to verify that the Hessian matrix of $\dot{\Pi}_c(Q_{1c}, Q_{2c})$ is negative definite. Hence, $\dot{\Pi}_c(Q_{1c}, Q_{2c})$ is jointly concave with respect to Q_{1c} and Q_{2c}. The optimal order quantity can be easily deduced via the first-order optimality condition.

From the assumption $Q_{2c} \leq \hat{Q}_{3c} \leq Q_{1c} + Q_{2c}$ and the analysis in Case 1, it is straightforward to deduce that $\alpha\beta w_3 - (\alpha\beta - \alpha\beta\gamma + \gamma)c_1 \geq 0$. So we have

$$\left.\frac{\partial \dot{\Pi}_c(Q_{1c}, Q_{2c})}{\partial Q_{1c}}\right|_{Q_{1c}+Q_{2c}=\hat{Q}_{3c}} = \alpha\beta w_3 - (\alpha\beta - \alpha\beta\gamma + \gamma)c_1 \geq 0.$$

Moreover, we have

$$\left.\frac{\partial \dot{\Pi}_c(Q_{1c}, Q_{2c})}{\partial Q_{1c}}\right|_{Q_{1c}+Q_{2c}=+\infty} = \alpha\beta s - (\alpha\beta - \alpha\beta\gamma + \gamma)c_1 < 0,$$

and $F(x)$ is strictly increasing. Hence, $\frac{\partial \dot{\Pi}_c(Q_{1c}, Q_{2c})}{\partial Q_{1c}}$ has an unique zero point as follows:

$$(Q_{1c} + Q_{2c})^* = F^{-1}\left(\frac{\alpha\beta(p+b) - (\alpha\beta - \alpha\beta\gamma + \gamma)c_1}{\alpha\beta(p+b-s)}\right). \tag{3.29}$$

Substituting (3.29) into (3.28), we deduce that

$$\frac{\partial \dot{\Pi}_c(Q_{1c}, Q_{2c})}{\partial Q_{2c}} = \alpha w_3 - (\alpha - \alpha\gamma + \gamma)c_2 - \alpha\beta w_3 + (\alpha\beta - \alpha\beta\gamma + \gamma)c_1. \tag{3.30}$$

So we have the following conclusions.

1. If $\alpha\beta w_3 - (\alpha\beta - \alpha\beta\gamma + \gamma)c_1 > \alpha w_3 - (\alpha - \alpha\gamma + \gamma)c_2$, i.e., $\frac{\partial \dot{\Pi}_c(Q_{1c}, Q_{2c})}{\partial Q_{2c}} < 0$, then the expected revenue will increase as Q_2 decreases. Hence, $Q_{1c}^* = (Q_{1c} + Q_{2c})^*$ and $Q_{2c}^* = 0$.
2. If $\alpha\beta w_3 - (\alpha\beta - \alpha\beta\gamma + \gamma)c_1 \leq \alpha w_3 - (\alpha - \alpha\gamma + \gamma)c_2$, i.e., $\frac{\partial \dot{\Pi}_c(Q_{1c}, Q_{2c})}{\partial Q_{2c}} \geq 0$, then the expected revenue will increase as Q_2 increases. Hence, $Q_{1c}^* + Q_{2c}^* = (Q_{1c} + Q_{2c})^*$ and $Q_{2c}^* \geq \hat{Q}_{3c}$.

Case 3. $\hat{Q}_{3c} \leq Q_{2c} \leq Q_{1c} + Q_{2c}$.
The first-order partial derivatives of $\dot{\Pi}_c(Q_{1c}, Q_{2c})$ with respect to Q_{1c} and Q_{2c} in this case are given by

$$\frac{\partial \dot{\Pi}_c(Q_{1c}, Q_{2c})}{\partial Q_{1c}} = \alpha\beta[(p+b) - (p+b-s)F(Q_{1c} + Q_{2c})] - (\alpha\beta - \alpha\beta\gamma + \gamma)c_1, \tag{3.31}$$

$$\frac{\partial \dot{\Pi}_c(Q_{1c}, Q_{2c})}{\partial Q_{2c}} = \alpha(p+b) - (p+b-s)[\alpha\beta F(Q_{1c} + Q_{2c}) + \alpha(1-\beta)F(Q_{2c})] - (\alpha - \alpha\gamma + \gamma)c_2. \tag{3.32}$$

It can be deduced that the Hessian matrix of $\dot{\Pi}_c(Q_{1c}, Q_{2c})$ is negative definite. The optimal order quantity can be easily deduced via the first-order derivatives.

The unique optimal total order quantity is deduced as follows:

$$(Q_{1c} + Q_{2c})^* = F^{-1}\left(\frac{\alpha\beta(p+b) - (\alpha\beta - \alpha\beta\gamma + \gamma)c_1}{\alpha\beta(p+b-s)}\right). \tag{3.33}$$

Substituting (3.33) into (3.32), it is straightforward to deduce the unique optimal order quantity from supplier 2 as follows:

$$Q_{2c}^* = F^{-1}\left(\frac{\alpha(1-\beta)(p+b) + (\alpha\beta - \alpha\beta\gamma + \gamma)c_1 - (\alpha - \alpha\gamma + \gamma)c_2}{\alpha(1-\beta)(p+b-s)}\right). \tag{3.34}$$

Furthermore, we have

$$Q_{1c}^* \geq 0 \Leftrightarrow Q_{2c}^* \leq (Q_{1c} + Q_{2c})^* \Leftrightarrow (\alpha\beta - \alpha\beta\gamma + \gamma)c_1 \leq \beta(\alpha - \alpha\gamma + \gamma)c_2. \tag{3.35}$$

From the above analysis of the three different cases, we reach the conclusions about the optimal sourcing strategy of the centralized supply chain. □

Proof of Theorem 3.4.3

Proof. Obviously, only if both suppliers are placed with positive order quantities is it possible for them to obtain a positive expected profit? If both suppliers are placed with positive order quantities, the expected revenue function of supplier 1 in stage 0, denoted by $\Pi_{s_1}(w_1, w_2)$, is given by

$$\Pi_{s_1}(w_1, w_2) = [\alpha\beta w_1 - (\alpha\beta - \alpha\beta\eta\gamma + \eta\gamma)c_1]Q_{1d}^*. \tag{3.36}$$

Similarly, the expected revenue function of supplier 2 in stage 0, denoted by $\Pi_{s_2}(w_2, w_1)$, is given by

$$\Pi_{s_2}(w_2, w_1) = [\alpha w_2 - (\alpha - \alpha\gamma\eta + \gamma\eta)c_2]Q_{2d}^*. \tag{3.37}$$

From (3.36) and (3.37), we observe that w_1 and w_2 should be subject to $w_1 > \frac{\alpha\beta - \alpha\beta\eta\gamma + \eta\gamma}{\alpha\beta}c_1$ and $w_2 > \frac{\alpha - \alpha\gamma\eta + \gamma\eta}{\alpha}c_2$ if both suppliers seek to obtain a positive expected profit.

Along with the results in Theorem 3.4.1, we obtain the following conclusions when supplier 1 adopts a different wholesale price for any given $w_2 \in [\frac{\alpha - \alpha\gamma\eta + \gamma\eta}{\alpha}c_2, w_3 - \frac{(1-\alpha)(1-\eta)\gamma c_2}{\alpha}]$.

1. If $\frac{\alpha\beta - \alpha\beta\eta\gamma + \eta\gamma}{\alpha\beta}c_1 < w_1 < w_3 - \frac{(1-\alpha\beta)(1-\eta)\gamma c_1}{\alpha\beta} - \frac{\alpha(w_3 - w_2) - (1-\alpha)(1-\eta)\gamma c_2}{\alpha\beta}$, then $Q_{1d}^* > 0$ and $Q_{2d}^* = 0$.
2. If $w_3 - \frac{(1-\alpha\beta)(1-\eta)\gamma c_1}{\alpha\beta} - \frac{\alpha(w_3 - w_2) - (1-\alpha)(1-\eta)\gamma c_2}{\alpha\beta} < w_1 < w_2 + \frac{(1-\alpha)(1-\eta)\gamma c_2}{\alpha} - \frac{(1-\alpha\beta)(1-\eta)\gamma c_1}{\alpha\beta}$, then $Q_{1d}^* > 0$ and $Q_{2d}^* > 0$.
3. If $w_2 + \frac{(1-\alpha)(1-\eta)\gamma c_2}{\alpha} - \frac{(1-\alpha\beta)(1-\eta)\gamma c_1}{\alpha\beta} < w_1 \leq w_3 - \frac{(1-\alpha\beta)(1-\eta)\gamma}{\alpha\beta}c_1$, then $Q_{1d}^* = 0$ and $Q_{2d}^* > 0$.

Similarly, these are the following conclusions when supplier 2 adopts a different wholesale price for any given $w_1 \in [\frac{\alpha\beta - \alpha\beta\eta\gamma + \eta\gamma}{\alpha\beta}c_1, w_3 - \frac{(1-\alpha\beta)(1-\eta)\gamma}{\alpha\beta}c_1]$.

4. If $\frac{\alpha - \alpha\gamma\eta + \gamma\eta}{\alpha}c_2 < w_2 < w_1 + \frac{(1-\alpha\beta)(1-\eta)\gamma c_1}{\alpha\beta} - \frac{(1-\alpha)(1-\eta)\gamma c_2}{\alpha}$, then $Q_{1d}^* = 0$ and $Q_{2d}^* > 0$.
5. If $w_1 + \frac{(1-\alpha\beta)(1-\eta)\gamma c_1}{\alpha\beta} - \frac{(1-\alpha)(1-\eta)\gamma c_2}{\alpha} < w_2 < w_3 - \frac{\alpha\beta(w_3 - w_1) - (1-\alpha\beta)(1-\eta)\gamma c_1}{\alpha} - \frac{(1-\alpha)(1-\eta)\gamma c_2}{\alpha}$, then $Q_{1d}^* > 0$ and $Q_{2d}^* > 0$.

6. If $w_3 - \frac{(1-\alpha)(1-\eta)\gamma c_2}{\alpha} - \frac{\alpha\beta(w_3-w_1)-(1-\alpha\beta)(1-\eta)\gamma c_1}{\alpha} < w_2 < w_3 - \frac{(1-\alpha)(1-\eta)\gamma c_2}{\alpha}$, then $Q_{1d}^* > 0$ and $Q_{2d}^* = 0$.

We observe that if $\frac{\alpha\beta-\alpha\beta\eta\gamma+\eta\gamma}{\alpha\beta}c_1 < w_3 - \frac{(1-\alpha\beta)(1-\eta)\gamma c_1}{\alpha\beta} - \frac{\alpha(w_3-w_2)-(1-\alpha)(1-\eta)\gamma c_2}{\alpha\beta}$, then it is possible that supplier 1 sets its wholesale price w_1 in $[\frac{\alpha\beta-\alpha\beta\eta\gamma+\eta\gamma}{\alpha\beta}c_1, w_3 - \frac{(1-\alpha\beta)(1-\eta)\gamma c_1}{\alpha\beta} - \frac{\alpha(w_3-w_2)-(1-\alpha)(1-\eta)\gamma c_2}{\alpha\beta}]$ in order to monopolize the market. To avoid this scenario, the countermeasure of supplier 2 is to set its wholesale price w_2 such that $\frac{\alpha\beta-\alpha\beta\eta\gamma+\eta\gamma}{\alpha\beta}c_1 > w_3 - \frac{(1-\alpha\beta)(1-\eta)\gamma c_1}{\alpha\beta} - \frac{\alpha(w_3-w_2)-(1-\alpha)(1-\eta)\gamma c_2}{\alpha\beta}$, i.e., $w_2 < (1-\beta)w_3 - \frac{(1-\alpha)(1-\eta)\gamma}{\alpha}c_2 + \frac{\alpha\beta-\alpha\beta\gamma+\gamma}{\alpha}c_1$. Moreover, it is straightforward to verify that $(1-\beta)w_3 - \frac{(1-\alpha)(1-\eta)\gamma}{\alpha}c_2 + \frac{\alpha\beta-\alpha\beta\gamma+\gamma}{\alpha}c_1 < w_3 - \frac{(1-\alpha)(1-\eta)\gamma c_2}{\alpha} - \frac{\alpha\beta(w_3-w_1)-(1-\alpha\beta)(1-\eta)\gamma c_1}{\alpha}$ when $w_1 > \frac{\alpha\beta-\alpha\beta\eta\gamma+\eta\gamma}{\alpha\beta}c_1$. Hence, the strategy space for supplier 2 is $[\frac{\alpha-\alpha\gamma\eta+\gamma\eta}{\alpha}c_2, (1-\beta)w_3 - \frac{(1-\alpha)(1-\eta)\gamma}{\alpha}c_2 + \frac{\alpha\beta-\alpha\beta\gamma+\gamma}{\alpha}c_1]$. Obviously, to ensure the existence of this strategy space, $\frac{\alpha-\alpha\gamma\eta+\gamma\eta}{\alpha}c_2 < (1-\beta)w_3 - \frac{(1-\alpha)(1-\eta)\gamma}{\alpha}c_2 + \frac{\alpha\beta-\alpha\beta\gamma+\gamma}{\alpha}c_1$, i.e., $\alpha w_3 - (\alpha - \alpha\gamma + \gamma)c_2 > \alpha\beta w_3 - (\alpha\beta - \alpha\beta\gamma + \gamma)c_1$.

Similarly, the feasible strategy space for supplier 1 is $[\frac{\alpha\beta-\alpha\beta\eta\gamma+\eta\gamma}{\alpha\beta}c_1, \frac{\alpha-\alpha\gamma+\gamma}{\alpha}c_2 - \frac{(1-\alpha\beta)(1-\eta)\gamma}{\alpha\beta}c_1]$ if $\beta(\alpha - \alpha\gamma + \gamma)c_2 > (\alpha\beta - \alpha\beta\gamma + \gamma)c_1$. □

Proof of Lemma 3.4.5

Proof. Denote $(p+b) - \beta(p+b-s)F(Q_1+Q_2) - (1-\beta)(p+b-s)F(Q_2)$ as $U_2(Q_2, Q_1)$ and $U_2(Q_2, Q_1)Q_2$ as $R_2(Q_2, Q_1)$.

The first-order derivative of $R_2(Q_2, Q_1)$ is given by

$$\frac{\mathrm{d}R_2(Q_2, Q_1)}{\mathrm{d}Q_2} = U_2(Q_2, Q_1) + Q_2\frac{\mathrm{d}U_2(Q_2, Q_1)}{\mathrm{d}Q_2} = U_2(Q_2, Q_1)(1 - g(Q_2|Q_1, \beta)). \tag{3.38}$$

The second-order derivative of $R_2(Q_2, Q_1)$ is given by

$$\frac{\mathrm{d}^2R_2(Q_2, Q_1)}{\mathrm{d}Q_2^2} = \frac{\mathrm{d}U_2(Q_2, Q_1)}{\mathrm{d}Q_2}(1 - g(Q_2|Q_1, \beta)) - U_2(Q_2, Q_1)\frac{\mathrm{d}g(Q_2|Q_1, \beta)}{\mathrm{d}Q_2}. \tag{3.39}$$

Assume that the support of $F(x)$ is the interval $[a, b)$, i.e., $0 < F(x) < 1$ for $x \in [a, b)$ and $F(x) = 0$ for $x \notin [a, b)$. Then $g(Q_2|Q_1, \beta) = 0$ and $R_2(Q_2, Q_1) = \alpha(p+b)Q_2$ for $Q_2 \in [0, a)$. Define $\overline{Q}_2$ as the least upper bound on the set of points such that $g(Q_2|Q_1, \beta) \le 1$. Since $g(Q_2|Q_1, \beta) = 0$ for $Q_2 \in [0, a)$, $\overline{Q}_2 \ge a$.

If $\frac{\mathrm{d}g(Q_2|Q_1,\beta)}{\mathrm{d}Q_2} \ge 0$, i.e., $g(Q_2|Q_1, \beta)$ is weakly increasing, then $g(Q_2|Q_1, \beta) \ge 1$ for $Q_2 \in [\overline{Q}_2, \infty)$ and $\frac{\mathrm{d}R_2(Q_2,Q_1)}{\mathrm{d}Q_2} \le 0$ for $Q_2 \in [\overline{Q}_2, \infty)$. It can be deduced that $R_2(Q_2, Q_1)$ is decreasing for $Q_2 \in [\overline{Q}_2, +\infty)$. Hence, $\Pi_{s_2}(Q_2, Q_1) = \alpha R_2(Q_2, Q_1) - (\alpha - \alpha\gamma + \gamma)c_2Q_2$ is decreasing for $Q_2 \in [\overline{Q}_2, +\infty)$, too.

Note that $g(Q_2|Q_1, \beta) \leq 1$ for $Q_2 \in [a, \overline{Q}_2)$ and $w_2(Q_2, Q_1)$ is decreasing in Q_2. Hence $\frac{\mathrm{d}^2 R_2(Q_2,Q_1)}{\mathrm{d}Q_2^2} \leq 0$ for $Q_2 \in [a, \overline{Q}_2)$. Then $R_2(Q_2, Q_1)$ is concave for $a \leq Q_2 \leq \overline{Q}_2$. Hence, it is straightforward to deduce that $\Pi_{s_2}(Q_2, Q_1) = \alpha R_2(Q_2, Q_1) - (\alpha - \alpha\gamma + \gamma)c_2 Q_2$ is concave for $Q_2 \in [a, \overline{Q}_2)$, too.

In addition, it is obvious that $\Pi_{s_2}(Q_2, Q_1)$ is linear and strictly increasing on $[0, a)$.

From the above analysis, it is straightforward to deduce that $\Pi_{s_2}(Q_2, Q_1)$ is unimodal for $Q_2 \in [0, +\infty)$.

The unimodality of $\Pi_{s_1}(Q_1, Q_2)$ can be proved by letting $Q_2 = Q_1$, $Q_1 = 0$ and $\beta = 1$ in (3.15). □

Proof of Theorem 3.4.8

Proof. We first assume that $\alpha(w_3 - w_2) - (1-\alpha)(1-\eta)\gamma c_2 > \alpha\beta(w_3 - w_1) - (1-\alpha\beta)(1-\eta)\gamma c_1 > 0$ and $\beta\,[\alpha w_2 + (1-\alpha)(1-\eta)\gamma c_2] > \alpha\beta w_1 + (1-\alpha\beta)(1-\eta)\gamma c_1$. Under these conditions, the optimal order quantities from supplier 1 and supplier 2 are given by

$$Q_1 = \frac{\beta[\alpha w_2 + (1-\alpha)(1-\eta)\gamma c_2] - [\alpha\beta w_1 + (1-\alpha\beta)(1-\eta)\gamma c_1]}{\alpha\beta(1-\beta)(p+b-s)}, \tag{3.40}$$

and

$$Q_2 = \frac{\alpha(p+b-w_2) - (1-\alpha)(1-\eta)\gamma c_2 - [\alpha\beta(p+b-w_1) - (1-\alpha\beta)(1-\eta)\gamma c_1]}{\alpha(1-\beta)(p+b-s)}. \tag{3.41}$$

Substituting (3.40) and (3.41) into the revenue functions (3.36) and (3.37), we have

$$\Pi_{s_1}(w_1, w_2) = \frac{\beta[\alpha w_2 + (1-\alpha)(1-\eta)\gamma c_2] - [\alpha\beta w_1 + (1-\alpha\beta)(1-\eta)\gamma c_1]}{\alpha\beta(1-\beta)(p+b-s)} \times[\alpha\beta w_1 - (\alpha\beta - \alpha\beta\eta\gamma + \eta\gamma)c_1], \tag{3.42}$$

and

$$\Pi_{s_2}(w_2, w_1) = \frac{\alpha(p+b-w_2) - (1-\alpha)(1-\eta)\gamma c_2 - [\alpha\beta(p+b-w_1) - (1-\alpha\beta)(1-\eta)\gamma c_1]}{\alpha(1-\beta)(p+b-s)} \times[\alpha w_2 - (\alpha - \alpha\gamma\eta + \gamma\eta)c_2]. \tag{3.43}$$

It is straightforward to verify that (3.42) and (3.43) are concave with respect to their own decision variables. By setting the first partial derivative of each player's revenue function with respect to its own decision variable equal to zero, we obtain the unconstrained best response function as follows:

$$w_1(w_2) = \frac{\alpha\beta w_2 + (\alpha\beta - 2\alpha\beta\eta\gamma + \alpha\beta\gamma + 2\eta\gamma - \gamma)c_1 + (1-\alpha)(1-\eta)\gamma\beta c_2}{2\alpha\beta}, \tag{3.44}$$

and

$$w_2(w_1) = \frac{\alpha(1-\beta)(p+b) + \alpha\beta w_1 + (1-\alpha\beta)(1-\eta)\gamma c_1 + (\alpha - 2\alpha\gamma\eta + \alpha\gamma + 2\gamma\eta - \gamma)c_2}{2\alpha}. \tag{3.45}$$

Denote the feasible strategy interval of supplier 1 as $[\underline{w}_1, \overline{w}_1]$, i.e., $\underline{w}_1 = \frac{\alpha\beta - \alpha\beta\eta\gamma + \eta\gamma}{\alpha\beta}c_1$ and $\overline{w}_1 = \frac{\alpha - \alpha\gamma + \gamma}{\alpha}c_2 - \frac{(1-\alpha\beta)(1-\eta)\gamma}{\alpha\beta}c_1$. Similarly, denote the feasible strategy interval of supplier 2 as $[\underline{w}_2, \overline{w}_2]$. It is straightforward to verify that $w_1(w_2) \geq \underline{w}_1$ for $w_2 \in [\underline{w}_2, \overline{w}_2]$. Hence, $\Pi_{s_1}(w_1, w_2)$ is an increasing function of w_1 in the interval $[\underline{w}_1, w_1(w_2)]$ for any given $w_2 \in [\underline{w}_2, \overline{w}_2]$. Similarly, $w_2(w_1) \geq \underline{w}_2$ and $\Pi_{s_2}(w_2, w_1)$ is an increasing function of w_2 in the interval $[\underline{w}_2, w_2(w_1)]$ for any given $w_1 \in [\underline{w}_1, \overline{w}_1]$. Moreover,

$$w_1(w_2) \leq \overline{w}_1 \Leftrightarrow w_2 \leq \hat{w}_2, \tag{3.46}$$

and

$$w_2(w_1) \leq \overline{w}_2 \Leftrightarrow w_1 \leq \hat{w}_1, \tag{3.47}$$

where

$$\hat{w}_2 \triangleq \frac{\gamma - \alpha\gamma + \alpha + \gamma\eta - \alpha\gamma\eta + \alpha}{\alpha}c_2 - \frac{\alpha\beta - \alpha\beta\gamma + \gamma}{\alpha\beta}c_1,$$

and

$$\hat{w}_1 \triangleq \frac{(1-\beta)(2w_3 - p - b)}{\beta} + \frac{(2\alpha\beta - \alpha\beta\gamma + \gamma - \alpha\beta\eta\gamma + \eta\gamma)c_1}{\alpha\beta} - \frac{(\gamma - \alpha\gamma + \alpha)c_2}{\alpha\beta}.$$

Hence, we obtain the best response function for any given feasible w_2 of supplier 1 as follows:

$$w_1^*(w_2) = \begin{cases} \frac{\alpha\beta w_2 + (\alpha\beta - 2\alpha\beta\eta\gamma + \alpha\beta\gamma + 2\eta\gamma - \gamma)c_1 + (1-\alpha)(1-\eta)\gamma\beta c_2}{2\alpha\beta}, & w_2 \leq \hat{w}_2, \\ \frac{\alpha - \alpha\gamma + \gamma}{\alpha}c_2 - \frac{(1-\alpha\beta)(1-\eta)\gamma}{\alpha\beta}c_1, & w_2 > \hat{w}_2. \end{cases} \tag{3.48}$$

Similarly, we obtain the best response function for any given feasible w_1 of supplier 2 as follows:

$$w_2^*(w_1) = \begin{cases} \dfrac{\alpha(1-\beta)(p+b) + \alpha\beta w_1 + (1-\alpha\beta)(1-\eta)\gamma c_1 + (\alpha - 2\alpha\gamma\eta + \alpha\gamma + 2\gamma\eta - \gamma)c_2}{2\alpha}, & w_1 \le \hat{w}_1, \\ (1-\beta)w_3 - \frac{(1-\alpha)(1-\eta)\gamma}{\alpha} c_2 + \dfrac{\alpha\beta - \alpha\beta\gamma + \gamma}{\alpha} c_1, & w_1 > \hat{w}_1. \end{cases} \tag{3.49}$$

Since the absolute value of the first derivative of each best response with respect to its own decision variable is less than 1, the best response mapping is a contraction. Hence, the Nash equilibrium is unique.

By letting $w_1 = w_1(w_2(w_1))$ and $w_2 = w_2(w_1(w_2))$, all the possible wholesale prices of supplier 1 and supplier 2 are derived as follows:

$$w_{11} \triangleq \frac{(\gamma\beta - \gamma\beta\eta + \alpha\beta^2\gamma\eta - \alpha\beta^2\gamma + 2\alpha\beta - 4\alpha\gamma\eta\beta + 2\alpha\gamma\beta + 4\gamma\eta - 2\gamma)c_1 + \beta(\alpha - \alpha\gamma + \gamma)c_2 + \alpha\beta(1-\beta)(p+b)}{\alpha\beta(4-\beta)},$$

$$w_{12} \triangleq \frac{\alpha\beta(1-\beta)w_3 + (\alpha\beta - 2\alpha\beta\eta\gamma + \alpha\beta\gamma + 2\eta\gamma - \gamma + \alpha\beta^2 - \alpha\beta^2\gamma + \beta\gamma)c_1}{2\alpha\beta},$$

$$w_{13} \triangleq \frac{\alpha - \alpha\gamma + \gamma}{\alpha} c_2 - \frac{(1-\alpha\beta)(1-\eta)\gamma}{\alpha\beta} c_1,$$

and

$$w_{21} \triangleq \frac{(\gamma\beta - \gamma\beta\eta + \alpha\beta\eta\gamma - \alpha\beta\gamma + 2\alpha - 4\alpha\gamma\eta + 2\alpha\gamma + 4\gamma\eta - 2\gamma)c_2 + (\alpha\beta - \alpha\beta\gamma + \gamma)c_1 + 2\alpha(1-\beta)(p+b)}{\alpha(4-\beta)},$$

$$w_{22} \triangleq \frac{\alpha(1-\beta)(p+b) + (\alpha - 2\alpha\gamma\eta + \alpha\gamma + 2\gamma\eta - \gamma + \alpha\beta - \alpha\beta\gamma + \beta\gamma)c_2}{2\alpha},$$

$$w_{23} \triangleq (1-\beta)w_3 - \frac{(1-\alpha)(1-\eta)\gamma}{\alpha} c_2 + \frac{\alpha\beta - \alpha\beta\gamma + \gamma}{\alpha} c_1.$$

It is straightforward to verify that all the possible values of supplier 1 are larger than $\underline{w}_1$ and all the possible values of supplier 2 are larger than $\underline{w}_2$. Jointly considering the constraints, we obtain the unique Nash equilibrium strategy of the game between the suppliers as follows:

$$(w_{1n}^*, w_{2n}^*) = \begin{cases} (w_{11}, w_{21}), & w_{11} \le \hat{w}_1 \bigwedge \overline{w}_1 \text{ and } w_{21} \le \hat{w}_2 \bigwedge \overline{w}_2, \\ (w_{12}, w_{23}), & \overline{w}_1 \ge w_{12} > \hat{w}_1 \text{ and } w_{23} \le \hat{w}_2 \bigwedge \overline{w}_2, \\ (w_{13}, w_{22}), & w_{13} \le \hat{w}_1 \bigwedge \overline{w}_1 \text{ and } \overline{w}_2 \ge w_{22} > \hat{w}_2, \\ (w_{13}, w_{23}), & \overline{w}_1 \ge w_{13} > \hat{w}_1 \text{ and } \overline{w}_2 \ge w_{23} > \hat{w}_2. \end{cases}$$

□

Chapter 4
Dynamic Inventory Management with Cash Flow Constraints

4.1 Introduction

Current research efforts on inventory management mainly focus on operational decisions and inventory control, i.e., characterizing replenishment policies based on inventory level over a planning horizon.[1] There is an extensive literature on inventory control in both deterministic and stochastic environments, see e.g., Axsäter (2000), Zipkin (2000), Nahmias (2001), Porteus (2002), and Cheng et al. (2010). Most of them ignore financial status of a firm and assume that the firm is able to obtain infinite capital to implement any operational decisions.

However, capital shortage problem is one of the most common problems faced by start-up and growing companies in real systems. Cash flow is usually one of the key reasons for the bankruptcy of these firms. For example, Bradley (2000) surveyed 531 businesses in the Southwest United States that went bankrupt during the calendar year 1998. After analyzing the primary reasons for these businesses failing, it is indicated that inadequate financial planning is one of the most evident reasons, especially operating capital for the early months of the operation. Similar research has been done in China. A project group in Department Research Center of State Council of P.R. China sent questionnaires to small businesses in the country, with response that 66.9% of them have capital problems, which may lead to the closure of these firms. See Chen and Zhang (2001). Therefore, start-up and growing firms, being short of capital, cannot always adopt optimal production policies as other mature companies.

In this chapter, we consider a classic dynamic inventory control problem of a self-financing retailer who periodically replenishes its stock from a supplier and sells it to the market. Excess demand in each period is lost when insufficient inventory is available. The demands for different periods are independent and identically

[1]The following discussion in this chapter is largely based on the ideas and results presented in Chao et al. (2008).

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distributed random variables. The retailer's operational decisions are constrained by its cash flows, which is updated periodically following the purchasing and the sales in each period. We seek to gain understanding on how operational decisions interact with and are affected by cash flows in a dynamic setting. The objective of the firm is to maximize its expected wealth level at the end of the planning horizon. We obtain the explicit structure on how the optimal inventory control strategy depends on the cash flows. We also study the relationship between the optimal control policy and the system parameters, e.g., purchasing price, interest rate, salvage value, and selling price. Conditions are identified under which the optimal control policies are identical across periods. A simple algorithm is developed to compute the optimal inventory control policy for each period.

There are several papers that deal with budgetary constraints. By assuming the availability of market hedges, Birge (2000) adopted option pricing theory for incorporating risk into planning models by adjusting capacity and resource levels. Rosenblatt and Rothblum (1990) treated capacity as a decision variable in their study of multi-item inventory systems under a single resource capacity constraint. Li et al. (1997) considered a single-product firm that makes production decisions, borrowing decisions, and dividend policies for each period while facing uncertain demand. The firm maximizes the expected present value of the infinite-horizon flow of the dividends subjecting to loan size, production size, and liquidity constraints. The firm can obtain an unbounded single-period loan with a constant interest rate. The authors derive the optimal myopic policies and study their structural properties. Archibald et al. (2002) considered a start-up firm facing discretely distributed demand and the objective is to maximize the long-term survival probability instead of average profit per period. The authors concluded that the start-up firms should be more cautious in their component purchasing strategy than the well-established firms. They also showed that the strategy is not monotone in the amount of capital available. Babich and Sobel (2004) studied the coordination of financial decisions (loan size) and operational decisions (production and sales) to maximize the expected discounted proceeds from an initial public offering (IPO). They modeled the IPO event as a stopping time in an infinite-horizon discounted Markov decision process. Furthermore, they characterized an optimal capacity-expansion policy and obtained sufficient conditions for a monotone threshold rule to yield an optimal IPO decision. Hu and Sobel (2005) studied a dynamic newsvendor model with the criterion of maximizing the expected present value of dividends, and examined the interdependence of a firm's capital structure and its short-term operating decisions concerning inventories, dividends, and liquidity. They obtained interesting results on the interaction between firm's capital structure and operational decisions.

The work that also addresses the interface of inventory management and finance is Buzacott and Zhang (2004) who analyzed a Stackelberg game between the bank and the retailer in a newsvendor inventory model. Buzacott and Zhang (2004) considered a single period inventory management problem where the bank's decisions include the interest rate to charge and the loan limit, and the retailer needs to decide the amount to borrow within the loan limit and the amount of inventory to order from suppliers. Both the bank and retailer maximize their expected returns. Other related work in this includes Xu and Birge (2004a, 2004b). They considered a multi-period dynamic model but do not consider the interactions between the bank and the retailer.

In a sense, financial constraint in inventory management can be considered as a supply capacity constraint on ordering quantity. Production–inventory problems with supply capacity constraints have received a great amount of attention since Federgruen and Zipkin (1986a, 1986b), see e.g., Ciarallo et al. (1994) and Wang and Gerchak (1996). The main result for such systems is that its optimal control strategy is a modified base-stock policy. The main difference between the two classes of models lies in the fact that in inventory control problems with supply capacity constraints, the constraints are given externally, while in inventory models with financial constraints, the financial constraints are the result of the firm's past decisions. Therefore, the financial constraints are themselves decisions. Thus, in making inventory decisions, its impact on future financial constraints has to be taken into consideration.

The rest of this chapter is organized as follows. Section 4.2 presents the model. Then Sect. 4.3 studies the optimal inventory strategy with cash flow constraints. Some numerical studies are included in Sect. 4.4. The chapter concludes in Sect. 4.5 with some remarks and some possible extensions. Throughout the chapter we use "increasing" and "decreasing" in non-strict sense, i.e., they represent "nondecreasing" and "nonincreasing", respectively. Also, for any real number x, $x^+ = \max\{x, 0\}$ and $x^- = \max\{-x, 0\}$.

4.2 Assumptions and Model Formulation

We consider the periodic-review inventory control problem where a self-financing retailer sells a single product to the market. The risk neutral retailer faces random demand and makes replenishment decisions over a finite planning horizon of N periods. The successive periods' demands $D_n (1 \leq n \leq N)$ are independent and identically distributed nonnegative random variables, with $f(\cdot)$ and $F(\cdot)$ being their probability density and cumulative distribution functions respectively. A lost-sales model is considered, that is, unmet demand in each period is lost when insufficient inventory is available. The ordering lead time is zero.

Let p be the unit selling price, and c the unit ordering cost. Assume that any inventory left at the end of the planning horizon has a salvage value γ per unit. To avoid triviality we assume

$$-\infty < \gamma \leq c < p, \tag{4.1}$$

with a negative value of γ representing disposal cost. We further assume $(1 + d)c < p$. If this condition is not satisfied, then the firm would always prefer to have all its capital in the banking account.

The sequence of events in each period is as follows. At the beginning of each period, the retailer places an order with its capital on hand, and deposits the surplus capital in a saving account with fixed interest rate d. During the period demand is realized. At the end of the period the retailer receives its revenue from sales and savings interest.

Let S_n be the capital level at the beginning of period n, let x_n and y_n be the inventory levels, before and after the replenishment decisions respectively, at the beginning of period n, and let S_{N+1} be the terminal wealth at the end of the planning horizon.

Because the firm is self-financed, the ordering decision satisfies the cash flow constraint $c(y_n - x_n) \leq S_n$, and the remaining capital in period n, $S_n - c(y_n - x_n)$, is deposited in the savings account to generate an interest of $d(S_n - c(y_n - x_n))$. The revenue from sales in period n is $p \min\{y_n, D_n\}$. Hence, the total capital level at the end of period n, which is also the capital level at the beginning of period $n + 1$, is

$$S_{n+1} = p\min\{y_n, D_n\} + (1 + d)(S_n - c(y_n - x_n)), \quad n = 1, 2, \ldots, N. \tag{4.2}$$

Since we consider lost-sales model, the inventory level at the beginning of period $n + 1$ is

$$x_{n+1} = (y_n - D_n)^+, \qquad n = 1, 2, \ldots, N. \tag{4.3}$$

Therefore, the decision problem of the retailer is to decide an ordering policy to maximize the expected terminal wealth at the end of the planning horizon, given initial inventory level x_1 and initial capital level S_1, and subject to the cash flow constraint in each period. That is, the decision problem is

$$\max_{y_1,\ldots,y_N} E[S_{N+1}],$$

subject to Eqs. (4.2), (4.3), and

$$0 \leq y_n - x_n \leq S_n/c, \qquad n = 1, 2, \ldots, N.$$

The maximum expected terminal wealth is denoted by, $V_n(x, S)$, given that the inventory level and capital level at the beginning of period n are x and S. The optimality equation is

$$\begin{aligned} V_n(x, S) = \max_{x \leq y \leq x + S/c} E\big[&V_{n+1}((y - D_n)^+, p\min\{y, D_n\} \\ &+ (1 + d)(S - c(y - x)))\big], \end{aligned} \tag{4.4}$$

with boundary condition

$$V_{N+1}(x, S) = S + \gamma x.$$

The trade-off in the dynamic programming equation above is between ordering inventory (and therefore earning profit from sales) and putting cash in savings account (and earning interests). When inventory is ordered, the retailer runs the risk of not selling the inventory and therefore loses the opportunity of earning an interest. Note that the problem in the last period is effectively a newsvendor problem with capital constraint.

4.3 The Optimal Inventory Control Strategy with Cash Flow Constraints

In this section, we investigate the optimal inventory control strategy with cash flow constraints.

To derive the optimal control strategy, several lemmas are needed. The first lemma follows immediately from induction.

Lemma 4.3.1. *For any period n and fixed x, $V_n(x, S)$ is increasing in S.*

Lemma 4.3.1 is intuitively clear: The more initial capital level the better to the final objective. To establish the second order property of the value function V_n, we need the following result.

Lemma 4.3.2. *For any n, $V_n(A - z, B + pz)$ is increasing in z for fixed A and B.*

Proof. Note the relationship

$$V_n(A - z, B + pz) = \max_{A - z \le y \le A + B/c + (p-c)z/c} E[V_{n+1}((y - D_n)^+, p \min\{y, D_n\} + (1 + d)(cA + B + (p - c)z - cy)].$$

It follows from Lemma 4.3.1 that the function being maximized above is increasing in z. Because the feasible region $A - z \le y \le A + B/c + (p - c)z/c$ is also increasing in z, $V_n(A - z, B + pz)$ is increasing in z. □

Lemma 4.3.2 is essential in proving the second order property of the value function. The lemma says that it is better to keep cash than having inventory in stock at the beginning of the period. This can be intuitively explained as follows: Capital at the beginning of a period is more flexible than inventory in stock since the firm can always convert it to inventory by placing an order. However, the reverse is not true. In particular, if the on-hand inventory is higher than necessary, it would have been better to have part of that inventory in the form of cash to earn interest.

Lemma 4.3.3. *For any n, $V_n(x, S)$ is jointly concave in x and S.*

Proof. We prove the lemma by backward induction. Clearly, $V_{N+1}(x, S) = S + \gamma x$ is jointly concave in x and S. Assume that $V_{n+1}(x, S)$ is jointly concave in x and S. We now prove the property for n.

We firstly prove $V_{n+1}((y - D_n)^+,\ p\min\{y, D_n\} + (1 + d)(S - c(y - x)))$ is jointly concave in (y, x, S). For any (y_1, x_1, S_1) and (y_2, x_2, S_2) and $0 \le \lambda \le 1$, we need to prove

$$\begin{aligned}
&V_{n+1}\Big((\lambda y_1 + (1-\lambda)y_2 - D_n)^+, p\min\{\lambda y_1 + (1-\lambda)y_2, D_n\} \\
&\qquad + (1+d)\big(\lambda S_1 + (1-\lambda)S_2 - c(\lambda y_1 + (1-\lambda)y_2 - \lambda x_1 - (1-\lambda)x_2)\big)\Big) \\
&\quad \ge \lambda V_{n+1}\Big((y_1 - D_n)^+, p\min\{y_1, D_n\} + (1+d)\big(S_1 - c(y_1 - x_1)\big)\Big) \\
&\qquad + (1-\lambda)V_{n+1}\Big((y_2 - D_n)^+, p\min\{y_2, D_n\} + (1+d)\big(S_2 - c(y_2 - x_2)\big)\Big).
\end{aligned}$$

Note the relationship $(y - D_n)^+ = y - \min\{y, D_n\}$. For convenience let

$$\begin{aligned}
\bar{y} &= \lambda y_1 + (1-\lambda)y_2, \\
\tilde{y} &= \min\{\lambda y_1 + (1-\lambda)y_2, D_n\}, \\
\hat{y} &= \lambda \min\{y_1, D_n\} + (1-\lambda)\min\{y_2, D_n\}.
\end{aligned}$$

Then by $\tilde{y} \ge \hat{y}$, we have

$$\begin{aligned}
&V_{n+1}\Big((\lambda y_1 + (1-\lambda)y_2 - D_n)^+, p\min\{\lambda y_1 + (1-\lambda)y_2, D_n\} \\
&\qquad +(1+d)\big(\lambda S_1 + (1-\lambda)S_2 - c(\lambda y_1 + (1-\lambda)y_2 - \lambda x_1 - (1-\lambda)x_2)\big)\Big) \\
&\quad = V_{n+1}\Big(\bar{y} - \tilde{y}, p\tilde{y} + (1+d)\big(\lambda S_1 + (1-\lambda)S_2 - c(\lambda y_1 + (1-\lambda)y_2 \\
&\qquad\qquad - \lambda x_1 - (1-\lambda)x_2)\big)\Big) \\
&\quad \ge V_{n+1}\Big(\bar{y} - \hat{y}, p\hat{y} + (1+d)\big(\lambda S_1 + (1-\lambda)S_2 - c(\lambda y_1 + (1-\lambda)y_2 \\
&\qquad\qquad - \lambda x_1 - (1-\lambda)x_2)\big)\Big) \\
&\quad = V_{n+1}\Big(\lambda(y_1 - D_n)^+ + (1-\lambda)(y_2 - D_n)^+, \lambda p\min\{y_1, D_n\} \\
&\qquad\qquad + (1-\lambda)p\min\{y_2, D_n\} + (1+d)\big(\lambda S_1 + (1-\lambda)S_2 \\
&\qquad\qquad - c(\lambda y_1 + (1-\lambda)y_2 - \lambda x_1 - (1-\lambda)x_2)\big)\Big) \\
&\quad \ge \lambda V_{n+1}\Big((y_1 - D_n)^+, p\min\{y_1, D_n\} + (1+d)\big(S_1 - c(y_1 - x_1)\big)\Big) \\
&\qquad +(1-\lambda)V_{n+1}\Big((y_2 - D_n)^+, p\min\{y_2, D_n\} + (1+d)\big(S_2 - c(y_2 - x_2)\big)\Big),
\end{aligned}$$

where the first inequality follows from Lemma 4.3.2 and the second inequality follows from the concavity of $V_{n+1}(x, S)$. Hence, $V_{n+1}((y-D_n)^+,\ p\min\{y, D_n\}+(1+d)(S-c(y-x)))$ is jointly concave in (y, x, S), and so is its expected value. Finally since

$$\mathbb{C} = \{(x, y) : x \geq 0, y \in [x, x + S/c]\}$$

is a convex set, applying Proposition B-4 of Heyman and Sobel (1984) we conclude that $V_n(x, S)$ is jointly concave in x and S. □

We find it convenient to study the value function in terms of x and $R = S + cx$. Define

$$\pi_n(y, R) = E[V_{n+1}((y - D_n)^+, p\min\{y, D_n\} + (1 + d)(R - cy))].$$

Then, the optimality equation (4.4) can be rewritten, after introducing a new function $\tilde{V}_n$, as

$$\tilde{V}_n(x, R) = V_n(x, R - cx) = \max_{x \leq y \leq R/c} \pi_n(y, R).$$

Note that $\pi_n(y, R)$ is jointly concave in (y, R). For given R, let $y_n^*(R)$ be the maximizer of the unconstrained optimization problem $\max_y \pi_n(y, R)$. Then the optimal inventory policy is given in the following result. Its proof follows directly from Lemma 4.3.3. Hence, it is omitted here.

Theorem 4.3.4. *When the state is (x, S) at the beginning of period n, a capital-dependent base stock inventory policy $y_n^*(R)$, where $R = S + cx$, is optimal. More specifically,*

(i) If $x \leq y_n^*(R) - S/c$, it is optimal to order up to R/c.
(ii) If $y_n^*(R) - S/c < x < y_n^*(R)$ then it is optimal to order up to $y_n^*(R)$.
(iii) If $x \geq y_n^*(R)$, then it is optimal not to order.

We refer to $y_n^*(R)$ as the optimal *base-stock level* for period n. Hence, for each state (x, S) with $R = S + cx$ there is an order-up-to level $y_n^*(R)$. Because of the constraint $y \leq R/c$, the base-stock level may not be achieved. The optimal achieved inventory level is $\min\{y_n^*(R), R/c\}$ if $x \leq y_n^*(R)$, and it is x otherwise. This is similar to the inventory control problems with finite supply capacity, for which the optimal strategy is to make the inventory level, within the supply capacity, as close to the order-up-to level as possible. In the following, we let

$$\hat{y}_n^*(R) = \min\{y_n^*(R), R/c\}$$

and refer to $\hat{y}_n^*(R)$ as the *optimal replenishment level* of period n. Therefore, if $x \leq \hat{y}_n^*(R)$ then the inventory level at period n is replenished to $\hat{y}_n^*(R)$, and no order is placed otherwise. Thus, if the state of the system at the beginning of period n is (x, S), then the optimal inventory level for period n after replenishment decision is $\min\{\hat{y}_n^*(S + cx), x\}$.

The optimal strategy can still be messy if $y_n^*(R)$ is a complicated function of R. Fortunately, in the following we are able to present an extremely simple form, as well as a computational algorithm, for $\hat{y}_n^*(R)$. More specifically, we show that $\hat{y}_n^*(R)$ can be completely determined by a single parameter, a_n^*. To that end, we introduce a sequence of concave functions $G_n(y)$ as follows: $G_{N+1}(y) = (\gamma - c)y$ and for $n = 1, \ldots, N$,

$$G_n(y) = (1+d)^{N-n}((p-c)E[\min\{y, D_n\}] - dcy) \\ +E[G_{n+1}(\max\{a_{n+1}^*, (y-D_n)^+\})], \tag{4.5}$$

where $a_{N+1}^* = 0$ and for $n = 1, \ldots, N$, a_n^* is the maximizer of $G_n(y)$, which will be the only number in determining the control function $\hat{y}_n^*(R)$, $0 \le R < \infty$.

Lemma 4.3.5. *The following relationship is satisfied:*

$$F^{-1}\left(\frac{p-(1+d)c}{p-c}\right) \ge a_1^* \ge a_2^* \ge \cdots \ge a_N^* = F^{-1}\left(\frac{p-(1+d)c}{p-\gamma}\right), \tag{4.6}$$

where F^{-1} is the inverse function of F.

Proof. We prove the result by induction. Clearly

$$G_N(y) = (p-c)E\min\{y, D_N\} - dcy + G_{N+1}((y-D_N)^+) \\ = (p-c)E\min\{y, D_N\} - dcy + (\gamma - c)(y-D_N)^+,$$

is concave in y, its maximizer

$$a_N^* = F^{-1}\left(\frac{p-(1+d)c}{p-\gamma}\right) \le F^{-1}\left(\frac{p-(1+d)c}{p-c}\right)$$

is the newsvendor solution. Assume that we have proved the result for $n+1$, i.e., $G_{n+1}, \ldots, G_N$ are concave and

$$F^{-1}\left(\frac{p-(1+d)c}{p-c}\right) \ge a_{n+1}^* \ge a_{n+2}^* \ge \cdots \ge a_N^* = F^{-1}\left(\frac{p-(1+d)c}{p-\gamma}\right),$$

we proceed to prove n. Taking derivatives of $G_n(y)$ with respect to y yields

$$G_n'(y) = (1+d)^{N-n}[(p-c)(1-F(y)) - dc] \\ +E\big[G_{n+1}'(y-D_n)1[D_n \le y - a_{n+1}^*]\big], \tag{4.7}$$

$$G_n''(y) = -(1+d)^{N-n}(p-c)f(y) \\ +E\big[G_{n+1}''(y-D_n)1[D_n \le y - a_{n+1}^*]\big]. \tag{4.8}$$

Hence, it follows from the induction assumption that (4.8) is non-positive and $G_n(y)$ is concave in y.

Substituting $y = a^*_{n+1}$ in (4.7), the second term on the right hand side of (4.7) is 0, and the first term is nonnegative. Thus $G'_n(a^*_{n+1}) \geq 0$ and $a^*_n \geq a^*_{n+1}$. Furthermore, note that the first term on the right hand side of (4.7) vanishes at $y = F^{-1}\left(\frac{p-(1+d)c}{p-c}\right)$, while the second term

$$G'_{n+1}(y - D_n)1[D_n \leq y - a^*_{n+1}] = G'_{n+1}(y - D_n)1[y - D_n \geq a^*_{n+1}]$$

is always non-positive because a^*_{n+1} is the maximizer of G_{n+1}. This shows that $a^*_n \leq F^{-1}\left(\frac{p-(1+d)c}{p-c}\right)$. □

The value a^*_n will serve as the ideal order-up-to level for period n. We note that, the problem in the last period is effectively a newsvendor problem with capital constraint, its optimal order-up-to level is well-known and is given by the last number in (4.6). This gives us the lower bound in Lemma 4.3.5. The most desirable situation for the firm would be to have the option of returning whatever is left to the supplier at the price paid, c, and in this case there would be no risk and the optimal inventory level can be set aggressively, i.e., set the inventory level to the first number in (4.6). This explains the upper bound in Lemma 4.3.5. In general, when there is more period remaining to go, then it is more likely that the on-hand inventory can be successfully used to satisfy future demand, and this explains why the optimal level a^*_n is decreasing in n.

The following decomposition result allows us to completely characterize the optimal replenishment level $\hat{y}^*_n(R)$. Its proof is lengthy and is provided in the Appendix.

Theorem 4.3.6. *(i) For any period n, when $R \geq ca^*_{n+1}$ and $y \leq R/c$, the objective function can be decomposed as*

$$\pi_n(y, R) = (1 + d)^{N+1-n}R + G_n(y).$$

(ii) If $R \geq ca^*_n$, then the optimal order-up-to level is $y^*_n(R) = a^*_n$, and if $R < ca^*_n$, then $y^*_n(R) \geq R/c$.

Theorem 4.3.6 states that, for large R and small inventory level y, the value function $\pi_n(y, R)$ can be decomposed to concave functions of R and y alone. This is not true, however, for small R or large y. Indeed, in general we would expect the value function $\pi_n(y, R)$ to be a complicated function of (y, R), and the separability comes as a surprise. It is this separability result that enables us to significantly simplify the optimal inventory control strategy. Note that $y^*_n(R)$ is the optimal solution for $\max_y \pi_n(y, R)$, which is the desired inventory level for stage n without the capital constraint, while a^*_n is a constant that is the maximizer of concave function $G_n(\cdot)$. Part (ii) of Theorem 4.3.6 states that $y^*_n(R)$ becomes flat and equal to a^*_n on $R \geq ca^*_n$.

The following is the main result of this chapter. Its proof follows directly from (ii) of Theorem 4.3.6 and $\hat{y}_n^*(R) = \min\{y_n^*(R), R/c\}$.

Theorem 4.3.7. *Suppose the state at the beginning of period n is (x, S) and let $R = S + cx$. The optimal replenishment level for period n is*

$$\hat{y}_n^*(R) = \begin{cases} a_n^*, & \text{if } R \geq ca_n^*, \\ R/c, & \text{if } R \leq ca_n^*. \end{cases}$$

That is, for any period n with state (x, R), the optimal inventory control policy is to,

(1) Replenish the inventory level to R/c if $R/c \leq a_n^*$
(2) Replenish the inventory level to a_n^* if $x < a_n^* < R/c$
(3) Do not order anything if $x \geq a_n^*$

Therefore, for each period n, the optimal replenishment level first linearly increases with the wealth level R at rate $1/c$ till ca_n^* and then it becomes flat from $R = ca_n^*$. This gives us an exceedingly simple inventory control policy: The inventory control policy is determined solely by a capital-independent level a_n^*, at the beginning of period n, the firm replenishes its inventory level to a_n^* as long as there is sufficient capital available; if there is no sufficient capital, then it replenishes as much as possible, that is, it uses up all of its capital.

The optimal replenishment level $\hat{y}_n^*(R)$ is determined by a single parameter a_n^*, and the computation of a_n^* is straightforward. A nested algorithm is summarized as follows.

Algorithm:

Step 1. Set $a_{N+1}^* = 0$, and compute G_N by (4.5). Set $n = N$.
Step 2. Computed a_n^* via concave function $G_n(y)$ of (4.5).
Step 3. If $n = 1$ then stop. Otherwise set $n := n - 1$ and repeat Step 2.

Remark 4.3.8. We point out that even though the optimal replenishment level $\hat{y}_n^*(R)$ is very simple, the base-stock level $y_n^*(R)$ can be quite complicated on $R \leq ca_n^*$. As a matter of fact, $y_n^*(R)$ may not be even monotone on $R \in [0, ca_n]$. See the numerical example in Sect. 4.4.

The following theorem presents the comparatively static results for the optimal policy on the selling price p, purchasing price c, salvage value γ, and interest rate d.

Theorem 4.3.9. *(i) The optimal control policy parameters a_n^*, $n = 1, \ldots, N$, are increasing in γ and p, and decreasing in c and d.*
(ii) As γ increases from $-\infty$ to c, a_n^* increases from 0 to $F^{-1}\left(\frac{p-(1-d)c}{p-c}\right)$. In particular, as $\gamma = c$, the optimal inventory policy is the same for each period and is given by

$$a_1^* = a_2^* = \cdots = a_N^* = F^{-1}\left(\frac{p-(1+d)c}{p-c}\right).$$

Proof. (i) We first prove the result on γ, p and c. Since a_n^* is the maximizer of $G_n(y)$, it suffices to prove $G_n'(y)$ is increasing in γ and p, and decreasing in c. By induction. First notice that

$$G_N'(y) = [(p-c)(1-F(y)) - dc] + \int_0^y (\gamma - c)\mathrm{d}F(z),$$

which is clearly increasing in γ and p, and decreasing in c. Suppose G_{n+1}' is increasing in γ and p and decreasing in c, then we have

$$\frac{\partial G_n'(y)}{\partial \gamma} = \int_0^{(y-a_{n+1}^*)^+} \frac{\partial G_{n+1}'(y-z)}{\partial \gamma}\mathrm{d}F(z),$$

$$\frac{\partial G_n'(y)}{\partial p} = (1+d)^{N-n}p(1-F(y)) + \int_0^{(y-a_{n+1}^*)^+} \frac{\partial G_{n+1}'(y-z)}{\partial p}\mathrm{d}F(z),$$

$$\frac{\partial G_n'(y)}{\partial c} = -(1+d)^{N-n}(c(1-F(y))+d) + \int_0^{(y-a_{n+1}^*)^+} \frac{\partial G_{n+1}'(y-z)}{\partial \gamma}\mathrm{d}F(z).$$

Hence, $\partial G_n'(y)/\partial \gamma \geq 0, \partial G_n'(y)/\partial p \geq 0$ and $\partial G_n'(y)/\partial c \leq 0$ follow immediately from the induction hypothesis.

To prove a_n^* is decreasing in d, by Lemma 4.3.5, it suffices to show that $\partial G_n'(y)/\partial d \leq 0$ for $n = 1, \ldots, N$ on the range

$$y \geq F^{-1}\left(\frac{p-(1+d)c}{p-\gamma}\right). \tag{4.9}$$

This is again proved by induction and it is trivially true for N. Suppose it has been established for $n+1$. Then, on range (4.9) we have

$$\begin{aligned}
\frac{\partial G_n'(y)}{\partial d} &= (1+d)^{N-n-1}[(p-c)(1-F(y)) - dc - (1+d)c] \\
&\quad + \int_0^{(y-a_{n+1}^*)^+} \frac{\partial G_{n+1}'(y-z)}{\partial d}\mathrm{d}F(z) \\
&\leq (1+d)^{N-n-1}[(p-c)(1-F(y)) - dc - (1+d)c] \\
&\leq (1+d)^{N-n-1}\left[(p-c)\frac{(1+d)c-\gamma}{p-\gamma} - dc - (1+d)c\right] \\
&= \frac{(1+d)^{N-n-1}}{p-\gamma}[-(c-\gamma)(1+d)c - (p-c)\gamma - (p-\gamma)dc] \\
&\leq 0,
\end{aligned}$$

where the first inequality follows from induction hypothesis, the second inequality follows from (4.9), and the last inequality follows from (4.1).

(ii) If $\gamma = c$, then it follows from Lemma 4 that all a_n^* are equal, completing the proof of Theorem 4.3.9. □

As mentioned earlier, the number a_n^* is basically the ideal order-up-to level for period n. When the salvage value or the selling price is higher, it is more profitable to keep a higher inventory thus a_n^* is increasing in γ and p. The same argument shows that when the purchasing price c is higher, it is better to reduce the inventory level thus a_n^* is decreasing in c. When d increases, savings account becomes a more attractive option, hence the firm will be willing to invest more in the banking account than in the inventory, explaining why a_n^* is decreasing in d. Finally, when the salvage value is the same as the ordering cost, then there is no risk associated with salvaging the inventory at the end. Therefore, the problem in each period is a newsvendor problem with capital constraint. This explains part (ii) of Theorem 4.3.9.

4.4 Numerical Examples

We present numerical examples to demonstrate the optimal inventory policy and its dependency on wealth level R, salvage value γ, and interest rate d. The model parameters in all these numerical examples are $p = 1.3$ and $c = 1$. In Fig. 4.1 the

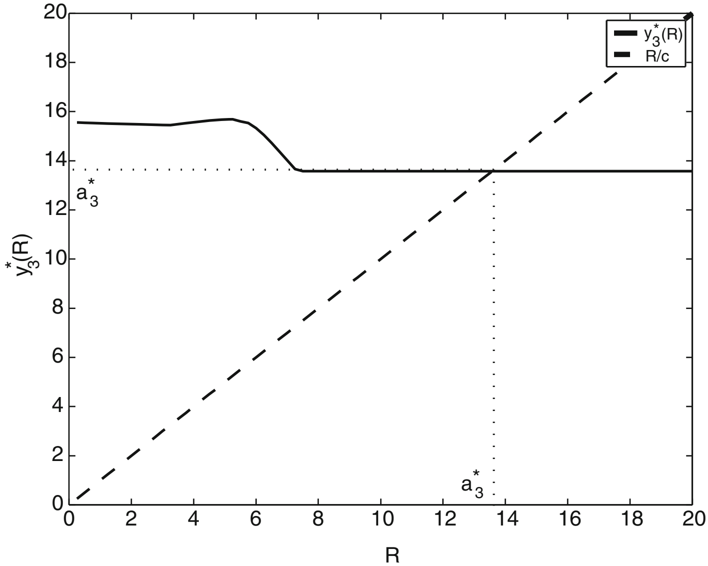

Fig. 4.1 Optimal base stock policy for period 3

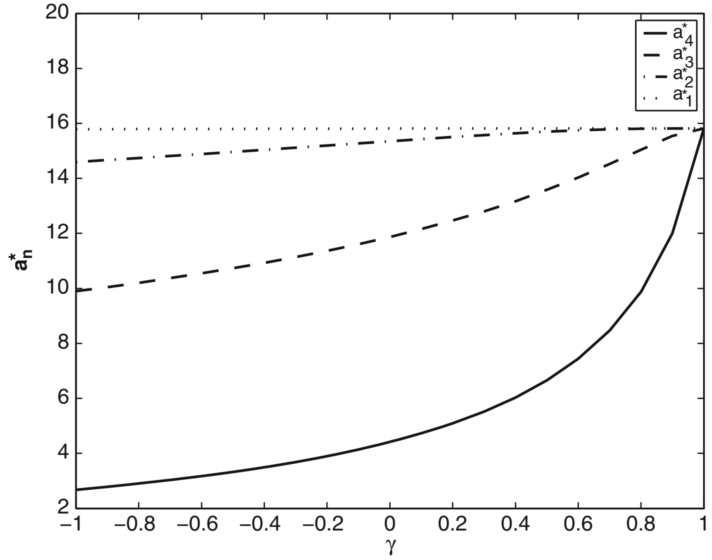

Fig. 4.2 The optimal control strategy a_n^* on γ

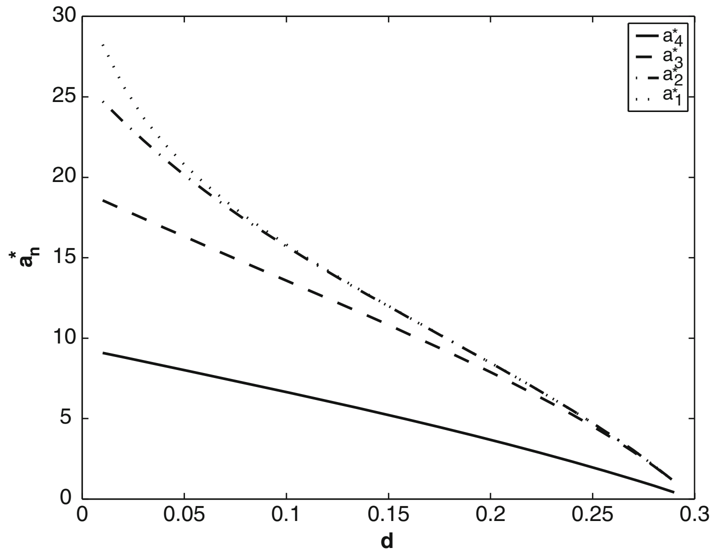

Fig. 4.3 The optimal control strategy a_n^* on d

interest rate is set at $d = 0.1$ and the salvage value is $\gamma = 0.5$; in Fig. 4.2 the interest rate is set at $d = 0.1$; and in Fig. 4.3, the salvage value is $\gamma = 0.5$. The demand has truncated normal distribution with mean 10 and variance 10. Assume there are totally $N = 4$ periods.

First compute the optimal policy parameters a_n^* for $n = 1, 2, 3$ and 4 via optimizing concave functions $G_n(y)$. Notice that the upper and lower bounds for a_n^* are given by $F^{-1}(\frac{p-(1+d)c}{p-c}) = 15.8151$ and $F^{-1}(\frac{p-(1+d)c}{p-\gamma}) = 6.6547$. Figure 4.1 demonstrates how the optimal inventory policy depends on the wealth level R. The numerical results are for period 3. It shows that when $R \geq ca_3^*$, $y_3^*(R) = a_3^*$; and when $R < ca_3^*$, $y_3^*(R) > R/c$. Note that the minimum of the dotted line and the solid curve is $\hat{y}_3^*(R)$.

It is interesting to observe that the optimal order-up-to level $y_3^*(R)$ is complicated and is not even monotone in R on the range $R \leq ca_3^*$. The optimal replenishment level $\hat{y}_n^*(R)$ is, however, always extremely simple, as we noted earlier.

Figure 4.2 presents the optimal policy in terms of salvage value γ on the range [−1, 1]. It demonstrates that a_n^* is decreasing in n and increasing in γ.

Figure 4.3 presents the optimal policy in terms of interest rate d on the range [0, 0.3]. It demonstrates that a_n^* is decreasing in n and d.

4.5 Concluding Remarks

In this chapter we study a dynamic inventory control problem with financial constraints. We derive the optimal inventory policy for each period, and characterize the dependence of the firm's optimal operational policy on its financial status. We also analyze the relationship between the optimal control parameters and system parameters.

Many interesting issues remain to be investigated. For example, if there is a holding cost rate h and shortage cost rate b for each period, then the optimality equation becomes

$$\max_{y_1,\ldots,y_N} E[S_{N+1}],$$

subject to

$$0 \leq y_n - x_n \leq S_n/c, \qquad n = 1, 2, \ldots, N,$$

where

$$S_{n+1} = p\min\{y_n, D_n\} - h\max\{y_n - D_n, 0\} - b\max\{D_n - y_n, 0\} \\ +(1+d)(S_n - c(y_n - x_n)),$$

and, as before, $x_{n+1} = (y_n - D_n)^+$. Note that Lemmas 4.3.1, 4.3.2 and 4.3.3 continue to hold, thus Theorem 4.3.4 also holds and the optimal inventory control policy is a capital dependent base-stock policy. As a matter of fact, Theorem 4.3.4, as well as Lemmas 4.3.1, 4.3.2 and 4.3.3 hold true under much more general settings, e.g., under general revenue function, and under utility function optimization, etc. However, for Theorems 4.3.6 and 4.3.7, the objective function can no longer be decomposed, and the control parameters of the optimal base-stock policy are complicated and state-dependent. The problem will be even more complicated if we allow the selling price p to be a decision variable when the demand D_n depends on the selling price in period n.

The setting used in this chapter assumes that the demands over periods are independent and identically distributed. We point out that the results Lemmas 4.3.1, 4.3.2 and 4.3.3 and Theorem 4.3.4 hold true as long as the demands over periods are independent and they do not need to be identically distributed.

Appendix

Proof of Theorem 4.3.6. The proof is by induction. By the definition of $G_n(y)$ and Lemma 4.3.5 we have

$$\pi_N(y,R) = pE\min\{y,D_N\}+(1+d)(R-cy)+\gamma E(y-D_N)^+ = (1+d)R+G_N(y).$$

Hence $y_N^*(R) = a_N^*$ for all R.

Assume that the results have been proved for $n+1$, i.e., when $R \geq ca_{n+2}^*$ and $y \leq R/c$, $\pi_{n+1}(y,R)$ can be decomposed as

$$\pi_{n+1}(y,R) = (1+d)^{N-n}R + G_{n+1}(y), \tag{4.10}$$

and that if $R \leq ca_{n+1}^*$, then $y_{n+1}^*(R) \geq R/c$; if $R \geq ca_{n+1}^*$, then $y_{n+1}^*(R) = a_{n+1}^*$.

To simplify the proof it is convenient to define a new function

$$\begin{aligned}\tilde{V}_n(x,R) &= V_n(x,S)\\ &= V_n(x,R-cx)\\ &= \max_{x\leq y\leq R/c} \pi_n(y,R)\\ &= \max_{x\leq y\leq R/c} E\big[\tilde{V}_{n+1}((y-D_n)^+,(p-c)\min\{y,D_n\}+(1+d)R-dcy)\big].\end{aligned}$$

From Lemma 4.3.3 it is straightforward to prove that $\tilde{V}_n(x,R)$ is jointly concave in x and R. From Theorem 4.3.4 we have

$$\tilde{V}_{n+1}(x,R) = \begin{cases}\pi_{n+1}(R/c,R), & R/c \leq y_{n+1}^*(R),\\ \pi_{n+1}(y_{n+1}^*(R),R), & x < y_{n+1}^*(R) < R/c,\\ \pi_{n+1}(x,R), & x \geq y_{n+1}^*(R).\end{cases} \tag{4.11}$$

On the other hand, by the induction assumption and Lemma 4.3.5, the following observations are made:

(a) If $R \le ca^*_{n+2} \le ca^*_{n+1}$, then $y^*_{n+1}(R) \ge R/c$, thus

$$\tilde{V}_{n+1}(x, R) = \pi_{n+1}(R/c, R).$$

(b) If $ca^*_{n+2} < R \le ca^*_{n+1}$, then $y^*_{n+1}(R) \ge R/c$ and $\pi_{n+1}(y, R)$ can be decomposed as (4.10), thus

$$\tilde{V}_{n+1}(x, R) = (1 + d)^{N-n} R + G_{n+1}(R/c).$$

(c) If $cx < ca^*_{n+1} < R$, then $y^*_{n+1}(R) = a^*_{n+1}$ and hence $x < y^*_{n+1}(R) < R/c$. In this case $\pi_{n+1}(y, R)$ can be decomposed as (4.10), thus

$$\tilde{V}_{n+1}(x, R) = (1 + d)^{N-n} R + G_{n+1}(a^*_{n+1}).$$

(d) If $a^*_{n+1} \le x \le R/c$, then $y^*_{n+1}(R) = a^*_{n+1}$ and hence $x \ge y^*_{n+1}(R)$. Further $\pi_{n+1}(y, R)$ can be decomposed as (4.10), thus

$$\tilde{V}_{n+1}(x, R) = (1 + d)^{N-n} R + G_{n+1}(x).$$

Therefore, we can rewrite (4.11) as

$$\tilde{V}_{n+1}(x, R) = \begin{cases} \pi_{n+1}(R/c, R), & R \le ca^*_{n+2}, \\ (1 + d)^{N-n} R + G_{n+1}(R/c), & ca^*_{n+2} < R \le ca^*_{n+1}, \\ (1 + d)^{N-n} R + G_{n+1}(a^*_{n+1}), & cx < ca^*_{n+1} < R, \\ (1 + d)^{N-n} R + G_{n+1}(x), & x \ge a^*_{n+1}. \end{cases} \tag{4.12}$$

The last two cases show that if $R \ge ca^*_{n+1}$, $\tilde{V}_{n+1}(x, R)$ can be rewritten as

$$\tilde{V}_{n+1}(x, R) = (1 + d)^{N-n} R + G_{n+1}(\max\{a^*_{n+1}, x\}).$$

As a result, if $R \ge ca^*_{n+1}$ and $y \le R/c$, then $(p-c)\min\{y, D_n\}+(1+d)R-dcy \ge ca^*_{n+1}$, and hence $\pi_n(y, R)$ can be expressed as

$$\begin{aligned} \pi_n(y, R) &= E[\tilde{V}_{n+1}((y - D_n)^+, (p - c)\min\{y, D_n\} + (1 + d)R - dcy)] \\ &= (1 + d)^{N+1-n} R + (1 + d)^{N-n}((p - c)E\min\{y, D_n\} - dcy) \\ &\quad + EG_{n+1}(\max\{a^*_{n+1}, (y - D_n)^+\}) \\ &= (1 + d)^{N+1-n} R + G_n(y). \end{aligned}$$

Therefore, when $R \geq ca_{n+1}^*$ and $y \leq R/c$, the maximizer of $\pi_n(y, R)$, $y_n^*(R)$, is equal to a_n^*, the maximizer of $G_n(y)$. Furthermore, when $R \geq ca_n^* \geq ca_{n+1}^*$, $y_n^*(R) = a_n^*$, and when $ca_{n+1}^* < R < ca_n^*$, $y_n^*(R) = a_n^* > R/c$.

We next prove that $y_n^*(R) \geq R/c$ on $R \leq ca_{n+1}^*$. For notational convenience in what follows we use $\tilde{V}_{n,1}(x, R)$ and $\tilde{V}_{n,2}(x, R)$ to represent the partial derivatives with respect to x and R respectively, and $\tilde{V}_{n,12}(x, R)$ the cross derivative. From (4.12), taking partial derivatives of $\tilde{V}_{n+1}(x, R)$ yields

$$\tilde{V}_{n+1,1}(x, R) = \begin{cases} 0, & x < a_{n+1}^*, \\ G'_{n+1}(x), & x \geq a_{n+1}^*, \end{cases} \tag{4.13}$$

and

$$\tilde{V}_{n+1,2}(x, R) = \begin{cases} \mathrm{d}\pi_{n+1}(R/c, R)/\mathrm{d}R, & R \leq ca_{n+2}^*, \\ (1+d)^{N-n} + G'_{n+1}(R/c)/c, & ca_{n+2}^* < R \leq ca_{n+1}^*, \\ (1+d)^{N-n}, & R > ca_{n+1}^*. \end{cases} \tag{4.14}$$

Note that $\tilde{V}_{n+1,2}(x, R)$ is independent of x, hence $\tilde{V}_{n+1,12}(x, R) = 0$. By

$$\pi_n(y, R) = E[\tilde{V}_{n+1}((y - D_n)^+, (p - c)\min\{y, D_n\} + (1+d)R - dcy)], \tag{4.15}$$

taking derivative of $\pi_n(y, R)$ with respect to y yields

$$\begin{aligned} \pi_{n,1}(y, R) = \int_0^y \big[&\tilde{V}_{n+1,1}(y - z, (p - c)z + (1+d)R - dcy) \\ &- dc\tilde{V}_{n+1,2}(y - z, (p - c)z + (1+d)R - dcy)\big]\mathrm{d}F(z) \\ &+ (p - (1+d)c)(1 - F(y))\tilde{V}_{n+1,2}(0, (p - (1+d)c)y + (1+d)R). \end{aligned}$$

Since $y_n^*(R)$ is the maximizer of $\pi_n(y, R)$, to prove $y_n^*(R) \geq R/c$ when on $R \leq ca_{n+1}^*$, it suffices to prove $\pi_{n,1}(R/c, R) \geq 0$ on $R \leq ca_{n+1}^*$. Noting $\tilde{V}_{n,1}(x, R) = 0$ when $x \leq R/c \leq a_{n+1}^*$, we have

$$\begin{aligned} \pi_{n,1}(R/c, R) &= (p - (1+d)c)(1 - F(R/c))\tilde{V}_{n+1,2}(0, pR/c) \\ &\quad - dc\int_0^{R/c} \tilde{V}_{n+1,2}(R/c - z, (p - c)z + R)\mathrm{d}F(z) \\ &\geq (p - (1+d)c)(1 - F(R/c))\tilde{V}_{n+1,2}(0, pR/c) \\ &\quad - dcF(R/c)\tilde{V}_{n+1,2}(R/c, R), \end{aligned} \tag{4.16}$$

where the inequality follows from the concavity of $\tilde{V}_{n+1}(x, R)$ in R:

$$\tilde{V}_{n+1,2}(R/c - z, (p-c)z + R) \leq \tilde{V}_{n+1,2}(R/c - z, R)$$

and that $\tilde{V}_{n+1,12}(x, R) = 0$ is independent of x:

$$\tilde{V}_{n+1,2}(R/c - z, R) = \tilde{V}_{n+1,2}(R/c, R).$$

Since $R \leq ca^*_{n+1} \leq cF^{-1}(\frac{p-(1+d)c}{p-c})$, we have

$$(p - (1+d)c)(1 - F(R/c)) \geq dcF(R/c). \tag{4.17}$$

To prove that the right hand side of (4.16) is nonnegative, we consider two ranges of R separately.

Case 1: $ca^*_{n+2} < R \leq ca^*_{n+1}$. By (4.7) we have

$$G'_{n+1}(y) \leq (1+d)^{N-n-1}[(p-c)(1-F(y)) - dc],$$

and since $\tilde{V}'_{n+1,2}(x, R)$ is decreasing in R, it follows from (4.14) that

$$\tilde{V}_{n+1,2}(0, pR/c) \geq \lim_{y\to\infty} \tilde{V}_{n+1,2}(0, y) \geq (1+d)^{N-n}.$$

Hence applying these inequalities and (4.14) on the interval $ca^*_{n+2} < R \leq ca^*_{n+1}$ yields

$$\begin{aligned}
&(p-(1+d)c)(1-F(R/c))\tilde{V}_{n+1,2}(0, pR/c) - dcF(R/c)\tilde{V}_{n+1,2}(R/c, R) \\
&\quad \geq (p-(1+d)c)(1-F(R/c))(1+d)^{N-n} - dcF(R/c) \\
&\qquad \big[(1+d)^{N-n} + G'_{n+1}(R/c)/c\big] \\
&\quad \geq \big[(p-(1+d)c)(1-F(R/c)) - dcF(R/c)\big](1+d)^{N-n} \\
&\qquad\quad - dcF(R/c)(1+d)^{N-n-1}\big[(p-c)(1-F(R/c)) - dc\big]/c \\
&\quad = (1+d)^{N-1-n}\big[1 + d(1-F(R/c))\big]\big[(p-(1+d)c)(1-F(R/c)) \\
&\qquad\qquad\qquad - dcF(R/c)\big] \\
&\quad \geq 0,
\end{aligned}$$

where the last inequality follows from (4.17). Therefore, $\pi_{n,1}(R/c, R) \geq 0$ in this case.

Case 2: $R \leq ca^*_{n+2}$. From (4.15), we have

$$\pi_{n+1}(R/c, R) = E[\tilde{V}_{n+2}((R/c - D_{n+1})^+, (p-c)\min\{R/c, D_{n+1}\} + R)], \tag{4.18}$$

hence

$$\begin{aligned}
\mathrm{d}\pi_{n+1}(R/c, R)/\mathrm{d}R &= \int_0^{R/c} [\tilde{V}_{n+2,1}(R/c - z, (p-c)z + R)]\mathrm{d}F(z) \\
&\quad + \frac{p}{c}(1 - F(R/c))\tilde{V}_{n+2,2}(0, pR/c) \\
&\quad + \int_0^{R/c} [\tilde{V}_{n+2,2}(R/c - z, (p-c)z + R)]\mathrm{d}F(z) \\
&= \frac{p}{c}(1 - F(R/c))\tilde{V}_{n+2,2}(0, pR/c) + \int_0^{R/c} [\tilde{V}_{n+2,2}(R/c - z, (p-c)z + R)]\mathrm{d}F(z) \\
&\le \frac{p}{c}(1 - F(R/c))\tilde{V}_{n+2,2}(0, pR/c) + \frac{p - (1+d)c}{dc}(1 - F(R/c))\tilde{V}_{n+2,2}(0, pR/c) \\
&= \frac{(p-c)(1+d)}{dc}(1 - F(R/c))\tilde{V}_{n+2,2}(0, pR/c), \qquad (4.19)
\end{aligned}$$

where the second equality follows from $\tilde{V}_{n+2,1}(R/c-z, (p-c)z+R)] = 0$ because of (4.13) and the induction assumptions for $n+2$, and the inequality is based on the following argument. When $R \le ca^*_{n+2} < ca^*_{n+1}$, from the induction assumption we have $y^*_{n+1}(R) \ge R/c$, and hence $\pi_{n+1,1}(R/c, R) \ge 0$, which implies, by (4.16) for $n+1$, that

$$\begin{aligned}
&(p - (1+d)c)(1 - F(R/c))\tilde{V}_{n+2,2}(0, pR/c) \\
&\qquad \ge dc \int_0^{R/c} \tilde{V}_{n+2,2}(R/c - z, (p-c)z + R)\mathrm{d}F(z).
\end{aligned}$$

For $R \le ca^*_{n+2}$, applying (4.16) and (4.19) we obtain

$$\begin{aligned}
&\pi_{n,1}(R/c, R) \\
&\quad \ge (p - (1+d)c)(1 - F(R/c))\tilde{V}_{n+1,2}(0, pR/c) - dcF(R/c)\tilde{V}_{n+1,2}(R/c, R) \\
&\quad = (p - (1+d)c)(1 - F(R/c))\tilde{V}_{n+1,2}(0, pR/c) - dcF(R/c)\mathrm{d}\pi_{n+1}(R/c, R)/\mathrm{d}R \\
&\quad \ge (1 - F(R/c))\big[(p - (1+d)c)\tilde{V}_{n+1,2}(0, pR/c) \\
&\qquad\qquad - (p-c)(1+d)F(R/c)\tilde{V}_{n+2,2}(0, pR/c)\big] \\
&\quad \ge (p - (1+d)c)(1 - F(R/c))\big[\tilde{V}_{n+1,2}(0, pR/c) - (1+d)\tilde{V}_{n+2,2}(0, pR/c)\big],
\end{aligned}$$

where the last inequality follows from $(p-c)F(R/c) \le p - (1+d)c$ because of $R \le ca^*_{n+1} < cF^{-1}(\frac{p-(1+d)c}{p-c})$.

Therefore, the desired result $\pi_{n,1}(R/c, R) \ge 0$ will follow if we can prove

$$\tilde{V}_{n+1,2}(0, R) - (1+d)\tilde{V}_{n+2,2}(0, R) \ge 0 \qquad (4.20)$$

for all R. This is again done by backward induction. First we have

$$\tilde{V}_{N,2}(0,R)-(1+d)\tilde{V}_{N+1,2}(0,R)\ge(1+d)-(1+d)=0.$$

Assume $\tilde{V}_{n+2,2}(0,R)-(1+d)\tilde{V}_{n+3,2}(0,R)\ge 0$, we proceed to prove (4.20). If $R>ca^*_{n+2}$, then by (4.14) for $n+2$,

$$\tilde{V}_{n+1,2}(0,R)-(1+d)\tilde{V}_{n+2,2}(0,R)\ge(1+d)^{N-n}-(1+d)(1+d)^{N-n-1}=0.$$

If $ca^*_{n+3}\le R\le ca^*_{n+2}$, then by (4.7) for $n+2$, we have

$$G'_{n+2}(R/c)\le(1+d)^{N-n-2}\big[(p-c)(1-F(R/c))-dc\big], \tag{4.21}$$

and by the concavity of $\tilde{V}_{n+2}(x,R)$ in R we have

$$\tilde{V}_{n+2,2}(R/c-z,(p-c)z+R)\ge\tilde{V}_{n+2,2}(R/c-z,pR/c), \tag{4.22}$$

and since $\tilde{V}_{n+2,2}(x,(p-c)z+R)$ is independent of x when $x\le ca^*_{n+2}$, we have

$$\tilde{V}_{n+2,2}(R/c-z,pR/c)=\tilde{V}_{n+2,2}(0,pR/c). \tag{4.23}$$

Applying (4.14) we obtain

$$\begin{aligned}
&\tilde{V}_{n+1,2}(0,R)-(1+d)\tilde{V}_{n+2,2}(0,R)\\
&\quad=\frac{\mathrm{d}\pi_{n+1}(R/c,R)}{\mathrm{d}R}-(1+d)^{N-n}-(1+d)G_{n+2}{}'(R/c)/c\\
&\quad\ge\frac{p}{c}(1-F(R/c))\tilde{V}_{n+2,2}(0,pR/c)+\int_0^{R/c}\tilde{V}_{n+2,2}(R/c-z,(p-c)z+R)\mathrm{d}F(z)\\
&\qquad-(1+d)^{N-n}-(1+d)^{N-n-1}\big[(p-c)(1-F(R/c))-dc\big]/c\\
&\quad\ge\left[\frac{p}{c}(1-F(R/c))+F(R/c)\right]\tilde{V}_{n+2,2}(0,pR/c)\\
&\qquad\quad-(1+d)^{N-n-1}\left[\frac{p}{c}(1-F(R/c))+F(R/c)\right]\\
&\quad=\left[\frac{p}{c}(1-F(R/c))+F(R/c)\right]\left(\tilde{V}_{n+2,2}(0,pR/c)-(1+d)^{N-n-1}\right)\\
&\quad\ge 0,
\end{aligned}$$

where the first inequality follows from (4.19) and (4.21), the second inequality is due to (4.22) and (4.23), and the last inequality follows from the observation that, by (4.14) for $n+2$, when $pR/c\ge R\ge ca^*_{n+3}$, we have $\tilde{V}_{n+2,2}(0,pR/c)\ge(1+d)^{N-n-1}$.

Finally, if $R < ca^*_{n+3}$, then

$$
\begin{aligned}
&\tilde{V}_{n+1,2}(0, R) - (1+d)\tilde{V}_{n+2,2}(0, R) \\
&\quad = \frac{\mathrm{d}\pi_{n+1}(R/c, R)}{\mathrm{d}R} - (1+d)\frac{\mathrm{d}\pi_{n+2}(R/c, R)}{\mathrm{d}R} \\
&\quad = \frac{p}{c}(1 - F(R/c))[\tilde{V}_{n+2,2}(0, pR/c) - (1+d)\tilde{V}_{n+3,2}(0, pR/c)] \\
&\qquad + \int_0^{R/c} [\tilde{V}_{n+2,2}(R/c - z, (p-c)z + R) \\
&\qquad -(1+d)\tilde{V}_{n+3,2}(R/c - z, (p-c)z + R)]\mathrm{d}F(z) \\
&\quad \geq 0,
\end{aligned}
$$

where the first equality follows from (4.14), the second equality follows from (4.19) for $n+2$ and $n+3$, and the inequality follows from the induction assumption.

Hence, we have proved $\pi_{n,1}(R/c, R) \geq 0$, implying $y^*_n(R) \geq R/c$ if $R \leq ca^*_{n+1}$. The proof of Theorem 4.3.6 is completed. □

Chapter 5
Dynamic Inventory Management with Short-Term Financing

5.1 Introduction

A large literature on corporate finance has paid sufficient attention on start-up and growing firms and tried to establish solution concepts for capital shortage problem.[1] While financial and operational decisions are usually studied separately. As one of the most fundamental results in corporate finance, Modigliani–Miller (MM) proposes that in perfect capital markets, the firm's capital structure and its financial decisions (e.g., the allocation between equity and debt) are independent of the firm's investment and its operational decisions (e.g., inputs and outputs, the levels of inventory and capital). However, real capital markets are imperfect: there are taxes, information asymmetry, accounting costs, bankruptcy costs, and so on. In many cases, start-up and growing firms with limited capital should seek help from banks or other lenders for more capital available to fund operations.

The firm considered in Chap. 4 is self-financed, a natural extension of the model is to allow the firm to borrow from the bank or other lenders. One can even impose an upper borrowing limit, which is typical in many applications.

In this chapter, we consider a classic dynamic inventory control problem of a retailer who periodically replenishes its stock from a supplier, sells it to the market, and is allowed to borrow from the bank or other lenders. Excess demand in each period is lost when insufficient inventory is available. The demands for different periods are i.i.d. random variables. Asset-based financing is allowed for the retailer being short of cash flow, which is updated periodically following the purchasing and the sales in each period. We seek to gain in understanding on how operational decisions interact with and are affected by cash flows and financial decisions in a dynamic setting. The objective of the firm is to maximize its expected wealth level at the end of the planning horizon. We obtain the explicit structure on how

[1]The following discussion in this chapter is largely based on the ideas and results presented in Chen (2008).

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the optimal inventory control strategy depends on the cash flows. Conditions are identified under which the retailer will choose either to borrow or to deposit in each period. The bankruptcy probability is also studied.

The rest of this chapter is organized as follows. Section 5.2 presents the model. Section 5.3 investigates the optimal inventory control strategy with short-term financing. Some numerical studies are included in Sect. 5.4. The chapter concludes in Sect. 5.5 with some remarks and some possible extensions. Throughout the chapter we use "increasing" and "decreasing" in non-strict sense, i.e., they represent "nondecreasing" and "nonincreasing", respectively. Also, for any real number x, $x^+ = \max\{x, 0\}$ and $x^- = \max\{-x, 0\}$.

5.2 Assumptions and Model Formulation

We consider the periodic-review inventory control problem with a single retailer who sells single product. The risk neutral retailer faces random demand and makes replenishment decisions over a finite time horizon of N periods. The successive periods' demands $D_n (1 \le n \le N)$ are independent and identically distributed nonnegative random variables, with $f(\cdot)$ and $F(\cdot)$ be their probability and cumulative distribution functions respectively.

Let p be the unit selling price, and c the unit ordering cost. Suppose that any inventory left at the end of the planning horizon has a salvage value γ per unit. To avoid triviality we assume

$$-\infty < \gamma \le c < p, \tag{5.1}$$

with a negative value of γ representing disposal cost. Furthermore, the lead time is assumed to be zero and hence an order placed at the beginning of period n arrives immediately before demand is realized. The unsatisfied demand is assumed lost. Without loss of generality, we assume no holding cost and shortage cost.

The sequence of events in each period is as follows. At the beginning of each period, the retailer borrows from the lender with fixed interest rate b and then places an order with his capital borrowed and on hand. Then the retailer deposits the surplus capital in a saving account with fixed interest rate d, where $d < b$. During the period demand is realized. At the end of the period the retailer receives the selling income and repay the interest to the lender (or receive the saving interest).

We further assume that $p > (1 + b)c$. If this condition is not satisfied, then the firm will never borrow.

Let S_n be the capital level at the beginning of period n, let x_n and y_n be the inventory levels, before and after the replenishment decisions respectively, at the beginning of period n, and let S_{N+1} be the terminal wealth at the end of the planning horizon.

Because the borrowing interest is higher than the saving interest, if the retailer borrows, then no deposit will be issued, and the vice versa. Specifically, if the retailer needs to borrow, then $c(y_n - x_n) - S_n$ is borrowed and $b(c(y_n - x_n) - S_n)$ should be repayed as the borrowing interest. And if the retailer has enough capital such that $c(y_n - x_n) \leq S_n$, then the remaining capital in period n, $S_n - c(y_n - x_n)$, is deposited in the saving account to generate an interest of $d(S_n - c(y_n - x_n))$. The revenue from sales in period n is $p \min\{y_n, D_n\}$. Hence, the total capital level at the end of period n, which is also the capital level at the beginning of period $n + 1$, is

$$
\begin{aligned}
S_{n+1} = &\ p \min\{y_n, D_n\} + (1 + d)(S_n - c(y_n - x_n))^+ \\
&-(1 + b)(S_n - c(y_n - x_n))^-, \text{ for } n = 1, 2, \ldots, N
\end{aligned} \tag{5.2}
$$

Since we consider lost-sales model, the inventory level at the beginning of period $n + 1$ is

$$x_{n+1} = (y_n - D_n)^+, \text{ for } n = 1, 2, \ldots, N \tag{5.3}$$

Therefore, the decision problem of the retailer is to decide an ordering policy to maximize the final capital level, given initial inventory level x_1 and capital level S_1, and subject to a capital constraint for each period. That is, the decision problem is

$$\max_{y_1,\ldots,y_N} E[S_{N+1}], \tag{5.4}$$

subject to (5.2) and (5.3), and $y_n \geq x_n, n = 1, 2, \ldots, N$.

Next we assume that the lender will not set a loan limit, i.e., the retailer can borrow as much as he wants.

Denote by $V_n(x, S)$ the maximum expected terminal wealth given that the inventory level and capital level at the beginning of period n are x and s. Then the following dynamic program can be employed to solve decision problem (5.4). The optimality equation is

$$
\begin{aligned}
V_n(x, S) = \max_{y \geq x} E[V_{n+1}((y - D_n)^+, &\ p \min\{y, D_n\} + (1 + d)(S - c(y - x))^+ \\
&-(1 + b)(S - c(y - x))^-)].
\end{aligned} \tag{5.5}
$$

with boundary condition

$$V_{N+1}(x, S) = S + \gamma x,$$

The trade-off in the dynamic programming equation above is between ordering inventory (and therefore earning profit from sales) and borrowing (and paying interests) or putting cash in savings account (and earning interests). When inventory is ordered, the retailer runs the risk of not selling the inventory and therefore pays more in the form of borrowing interests or loses the opportunity of earning an interest. Note that the problem in the last period is effectively a newsvendor problem.

5.3 The Optimal Inventory Control Strategy with Short-Term Financing

In this section, we investigate the optimal inventory control strategy with short-term financing.

To derive the optimal control strategy, several lemmas are needed. The first lemma follows immediately from induction.

Lemma 5.3.1. *For any period n and fixed x, $V_n(x, S)$ is increasing in S.*

Lemma 5.3.1 is intuitively clear: More the initial capital level, is better for the final objective. To establish the second order property of the value function V_n, we need the following result.

Lemma 5.3.2. *For any n, $V_n(A - z, B + pz)$ is increasing in z for fixed A and B.*

Proof. Note that

$$\begin{aligned} &V_n(A - z, B + pz) \\ &\quad = \max_{y \geq A - z} E\Big[V_{n+1}((y - D_n)^+, p\min\{y, D_n\} \\ &\qquad\qquad + (1 + d)(cA + B + (p - c)z - cy)^+ \\ &\qquad\qquad - (1 + d)(cA + B + (p - c)z - cy)^-\Big]. \end{aligned}$$

It follows from Lemma 5.3.1 that the function being maximized above is increasing in z. Since the feasible region $y \geq A - z$ is also increasing in z, $V_n(A - z, B + pz)$ is increasing in z. □

Lemma 5.3.2 is essential in proving the second order property of the value function. The lemma says that it is better to keep cash than having inventory in stock at the beginning of the period. This can be intuitively explained as follows: Capital at the beginning of a period is more flexible than inventory in stock since the firm can always convert it to inventory by placing an order. However, the reverse is not true. In particular, if the on-hand inventory is higher than necessary, it would have been better to have part of that inventory in the form of cash to earn interest.

Lemma 5.3.3. *For any n, $V_n(x, S)$ is jointly concave in x and S.*

With unlimited borrowing allowed, in any period if the loan level is high enough while the selling income is not as expected, then the retailer may find itself still in arrears after repaying the borrowing interest, i.e., $S < 0$. Assume that the inventory value can also be accounted by salvage, define a retailer as bankrupt for period n if $S_{n+1} + \gamma x_{n+1} < 0$, the following theorem presents the probability of retailer bankruptcy.

Theorem 5.3.4. *For period n, if the retailer borrows more than* $\frac{\gamma(S_n+cx_n)}{(1+b)c-\gamma}$, *then the probability of retailer bankruptcy in the end of the period is* $F\left(\frac{((1+b)c-\gamma)y_n-(1+b)(S_n+cx_n)}{p-\gamma}\right)$.

Proof. First notice that only if the retailer borrows, the bankruptcy will occur. If $y_n \le D_n$, then $S_{n+1} = (p-(1+b)c)y + (1+b)(S+cx) > 0$. Hence $y_n > D_n$. Thus

$$S_{n+1} + \gamma x_{n+1} = (p-\gamma)D_n - ((1+b)c-\gamma)y_n + (1+b)(S_n + cx_n) \le 0$$

yields

$$D_n \le \frac{((1+b)c-\gamma)y_n - (1+b)(S_n+cx_n)}{p-\gamma}.$$

Finally we obtain the probability of retailer bankruptcy in the end of period n. □

Without loss of generality, we assume that once the retailer is bankrupt for any period, it is allowed that productions can be proceeded in the following periods. But clearly borrowing has to be issued in the next period. Specifically, if $S \le 0$, then no deposit will occur.

We find it convenient to study the value function in terms of x and $R = S + cx$. Define

$$\pi_n(y,R) = E[V_{n+1}((y-D_n)^+, p\min\{y, D_n\} + (1+d)(R-cy)^+ - (1+b)(R-cy)^-)].$$

Then the optimality equation (5.5) can be rewritten, after introducing a new function $\tilde{V}_n$, as

$$\tilde{V}_n(x,R) = V_n(x, R-cx) = \max_{y\ge x} \pi_n(y,R).$$

Note that $\pi_n(y,R)$ is jointly concave in (y,R). For given R, let $y_n^*(R)$ be the maximizer of the unconstrained optimization problem $\max_y \pi_n(y,R)$. Then the optimal inventory policy is given in the following result. Its proof follows directly from Lemma 5.3.3. Hence, it is omitted here.

Theorem 5.3.5. *When the state is* (x,S) *at the beginning of period n, a capital-dependent base stock inventory policy* $y_n^*(R)$, *where* $R = S + cx$, *is optimal. More specifically,*

(i) If $x \le y_n^*(R)$, *it is optimal to order up to* $y_n^*(R)$.
(ii) If $x \ge y_n^*(R)$, *then it is optimal not to order.*

We refer to $y_n^*(R)$ as the optimal *base-stock level* for period n. Hence, for each state (x,S) with $R = S + cx$ there is an order-up-to level $y_n^*(R)$.

The optimal strategy can be messy if $y_n^*(R)$ is a complicated function of R. In the following we study $y_n^*(R)$ in terms of different R. Define

$$\begin{aligned}\pi_n^d(y,R) &= \pi_n(y,R)|R \geq cy\\ &= E[V_{n+1}((y-D_n)^+, p\min\{y,D_n\} + (1+d)(R-cy))]\end{aligned}$$

and

$$\begin{aligned}\pi_n^b(y,R) &= \pi_n(y,R)|R \leq cy\\ &= E[V_{n+1}((y-D_n)^+, p\min\{y,D_n\} + (1+b)(R-cy))]\end{aligned}$$

and $y_n^d(R)$ and $y_n^b(R)$ as the maximizers of concave functions $\pi_n^d(y,R)$ and $\pi_n^b(y,R)$ respectively. The following lemma shows the relationship between $y_n^*(R)$ and $y_n^d(R)$ and $y_n^b(R)$, with the proof by noting that $\pi_n(y,R)$ is concave.

Lemma 5.3.6. *The following relationship is satisfied*:

$$y_n^*(R) = \begin{cases} y_n^b(R), & R \leq cy_n^b(R),\\ R/c, & cy_n^b(R) < R < cy_n^d(R),\\ y_n^d(R), & R \geq y_n^d(R).\end{cases} \tag{5.6}$$

Lemma 5.3.6 shows that $y_n^*(R)$ depends on R in following ways.

(i) If $R \leq cy_n^b(R)$ is at a low level, then the retailer borrows and replenishes the inventory to $y_n^b(R)$.
(ii) If $cy_n^b(R) < R < cy_n^d(R)$, the retailer will use up the capital on hand while not borrow nor deposit.
(iii) If $R \geq cy_n^d(R)$ is at a high level, then the retailer deposits and replenishes the inventory to $y_n^d(R)$.

The optimal strategy can still be messy if $y_n^b(R)$ or $y_n^d(R)$ is complicated function of R. We further need to characterize the properties of $y_n^b(R)$ and $y_n^d(R)$. Firstly, let

$$\hat{y}_n^d(R) = \min\{y_n^d(R), R/c\}.$$

Then $y_n^*(R) = \max\{y_n^b(R), \hat{y}_n^d(R)\}$. Fortunately, in the following we are able to present a extremely simple form for $\hat{y}_n^d(R)$. More specifically, we show that $\hat{y}_n^d(R)$ can be completely determined by a single parameter a_n^d. To that end, we introduce a sequence of concave functions $G_n^d(y)$ as follows: $G_{N+1}^d(y) = (\gamma - c)y$ and for $n = 1, 2, \ldots, N$,

$$\begin{aligned}G_n^d(y) = (1+d)^{N-n}((p-c)E[\min\{y,D_n\}] - dcy)\\ +E[G_{n+1}^d(\max\{a_{n+1}^d, (y-D_n)^+\})],\end{aligned} \tag{5.7}$$

where $a_{N+1}^d = 0$ and for $n = 1, 2, \ldots, N$, a_n^d is the maximizer of $G_n^d(y)$.

Lemma 5.3.7. *The following relationship is satisfied*:

$$F^{-1}\left(\frac{p-(1+d)c}{p-c}\right) \geq a_1^d \geq a_2^d \geq \cdots \geq a_N^d = F^{-1}\left(\frac{p-(1+d)c}{p-\gamma}\right),$$

where F^{-1} is the inverse function of F.

Proof. We prove the result by induction. Clearly

$$G_N^d(y) = (p-c)E\min\{y, D_N\} - dcy + (\gamma - c)(y - D_N)^+$$

is concave in y, its maximizer

$$a_N^d = F^{-1}\left(\frac{p-(1+d)c}{p-\gamma}\right) \leq F^{-1}\left(\frac{p-(1+d)c}{p-c}\right)$$

is the newsvendor solution. Assume that we have proved the result for $n+1$, i.e., $G_{n+1}, \ldots, G_N$ are concave and

$$F^{-1}\left(\frac{p-(1+d)c}{p-c}\right) \geq a_{n+1}^d \geq a_{n+2}^d \geq \cdots \geq a_N^d = F^{-1}\left(\frac{p-(1+d)c}{p-\gamma}\right),$$

we need to prove n. Take derivative of $G_n^d(y)$ with respect to y yields

$$\begin{aligned} G_n^{d\prime}(y) &= (1+d)^{N-n}[(p-c)(1-F(y)) - dc] \\ &\quad + E\big[G_{n+1}^{d\;\prime}(y-D_n)1[D_n \leq y - a_{n+1}^d]\big], \end{aligned} \tag{5.8}$$

$$G_n^{d\prime\prime}(y) = -(1+d)^{N-n}(p-c)f(y) + E\left[G_{n+1}^{d\;\prime\prime}(y-D_n)1[D_n \leq y - a_{n+1}^d]\right]. \tag{5.9}$$

Hence, it follows from the induction assumption that (5.9) is non-positive and $G_n^d(y)$ is concave in y.

Substituting $y = a_{n+1}^d$ in (5.8), the second term on the right hand side of (5.8) is 0, and the first term is non-negative. Thus $G_n^{d\prime}(a_{n+1}^d) \geq 0$ and $a_n^d \geq a_{n+1}^d$. Furthermore, note that the first term on the right hand side of (5.8) vanishes at $y = F^{-1}\left(\frac{p-(1+d)c}{p-c}\right)$, while the second term

$$G_{n+1}^{d\;\prime}(y-D_n)1[D_n \leq y - a_{n+1}^d] = G_{n+1}^{d\;\prime}(y-D_n)1[y - D_n \geq a_{n+1}^d]$$

is always non-positive because a_{n+1}^d is the maximizer of G_{n+1}^d. This shows that $a_n^d \leq F^{-1}\left(\frac{p-(1+d)c}{p-c}\right)$. □

The following decomposition result allows us to characterize $y_n^d(R)$. Its proof is lengthy and is provided in the Appendix.

Theorem 5.3.8. *(i) For any period n, when $R \geq ca_{n+1}^d$ and $y \leq R/c$, the objective function $\pi_n^d(y, R)$ can be decomposed as*

$$\pi_n^d(y, R) = (1 + d)^{N+1-n} R + G_n^d(y).$$

(ii) If $R \geq ca_n^d$, then $y_n^d(R) = a_n^d$, and if $R < ca_n^d$, then $y_n^d(R) > R/c$.

Theorem 5.3.8 states that, for large R and small inventory level y, the value function $\pi_n^d(y, R)$ can be decomposed to concave functions of R and y alone. This is not true, however, for small R or large y. Indeed, in general we would expect the value function $\pi_n^d(y, R)$ to be a complicated function of (y, R), and the separability comes as a surprise. It is this separability result that enables us to significantly simplify the optimal inventory control strategy. Note that $y_n^d(R)$ is the optimal solution for $\max_y \pi_n^d(y, R)$, which is the desired inventory level for stage n without the capital constraint, while a_n^d is a constant, that is, the maximizer of concave function $G_n^d(\cdot)$. Part (ii) of Theorem 5.3.8 states that $y_n^d(R)$ becomes flat and equal to a_n^d on $R \geq ca_n^d$.

Following directly from part (ii) of Theorem 5.3.8 and $\hat{y}_n^d(R) = \min\{y_n^d(R), R/c\}$, we find that

$$\hat{y}_n^d(R) = \begin{cases} a_n^d, & R \geq ca_n^d, \\ R/c, & R \leq ca_n^d. \end{cases}$$

and moreover

$$y_n^*(R) = \max\{y_n^b(R), \hat{y}_n^d(R)\} = \begin{cases} a_n^d, & R \geq ca_n^d, \\ \max\{y_n^b(R), R/c\}, & R \leq ca_n^d. \end{cases}$$

So far we have shown when the retailer will deposit, next we will try to figure out the properties for the retailer borrowing. To that end, we introduce a decreasing sequence of constants $F^{-1}\left(\frac{p-(1+b)c}{p-c}\right) > a_1^b > \cdots > a_N^b = F^{-1}\left(\frac{p-(1+b)c)}{p-\gamma}\right)$, where a_n^b satisfies

$$\int_0^{a_n^b - a_{n+1}^d} {G_{n+1}^d}'(a_n^b - z)\mathrm{d}F(z) + [p - (1 + b)c - (p - c)F(a_n^b)](1 + d)^{N-n} = 0 \tag{5.10}$$

and $a_{n+1}^d < a_n^b < a_n^d$. Then the following is the main result of this chapter.

Theorem 5.3.9. *Suppose the state at the beginning of period n is* (x, S) *and let* $R = S + cx$. *The optimal base stock level for period n is*

$$y_n^*(R) = \begin{cases} y_n^b(R), & R < ca_n^b, \\ R/c, & ca_n^b \le R \le ca_n^d, \\ a_n^d, & R > ca_n^d. \end{cases}$$

That is, for any period n with state (x, R), *the optimal inventory control policy is to,*

(i) Replenish the inventory level to $y_n^b(R)$ *if* $R < ca_n^b$ *and* $x < y_n^b(R)$
(ii) Replenish the inventory level to R/c *if* $ca_n^b \le R \le ca_n^d$ *and* $R/c \ge x$
(iii) Replenish the inventory level to a_n^d *if* $R > ca_n^d > cx$
(iv) Do not order anything if $R < ca_n^b \le cx$ *or* $R > cx \ge ca_n^d$

Theorem 5.3.9 further shows that the retailer will have to borrow when $R \le ca_n^b$ in period n. However, we point out that the optimal ordering quantity, $q_n^b(R)$, can be quite complicated when $R \le ca_n^b$. As a matter, $q_n^b(R)$ may not be even monotone on $R \in [-\infty, ca_n^b]$. See the numerical example in Sect. 5.4.

5.4 Numerical Examples

We present numerical examples to demonstrate the optimal inventory policy and its dependency on wealth level R. The model parameters in all these numerical examples are $p = 1.3$ and $c = 1$. The interest rates are set at $b = 0.1$ and $d = 0.05$, and the salvage level is $\gamma = 0.5$. The demand has truncated normal distribution with mean 10 and variance 10. Assume that there are totally $N = 4$ periods.

First compute the optimal policy parameters a_n^d and a_n^b for $n = 1, 2, 3$ and 4 via optimizing concave functions $G_n^d(y)$ and solving (5.10). Note that the upper and lower bounds for a_n^d are given by $F^{-1}\left(\frac{p-(1+d)c}{p-c}\right) = 20.7931$ and $F^{-1}\left(\frac{p-(1+d)c}{p-\gamma}\right) = 8.0214$, and for a_n^b are given by $F^{-1}\left(\frac{p-(1+b)c}{p-c}\right) = 15.8151$ and $F^{-1}\left(\frac{p-(1+b)c}{p-\gamma}\right) = 6.6547$.

Figure 5.1 demonstrates how the optimal solutions $y_n^b(R)$ and $y_n^d(R)$ depend on the wealth level R. The numerical results are for period 3. It shows that $y_3^b(R) < y_3^d(R)$ and they are both quite complicated. However, we have the following findings:

(1) When $R \ge ca_3^d$, $y_3^d(R) = a_3^d \le R/c$, and when $R < ca_3^d$, $y_3^d(R) > R/c$
(2) When $R > ca_3^b$, $y_3^b(R) < R/c$, and when $R \le ca_3^d$, $y_3^d(R) \ge R/c$

Next Fig. 5.2 demonstrates how the optimal inventory policy depends on the wealth level R. It shows that when $R \ge ca_3^d$, $y_3^*(R) = a_3^d$; and when $ca_3^b < R < ca_3^d$, $y_3^*(R) = R/c$; and when $R \le ca_3^b$, $y_3^*(R) = y_3^b(R)$.

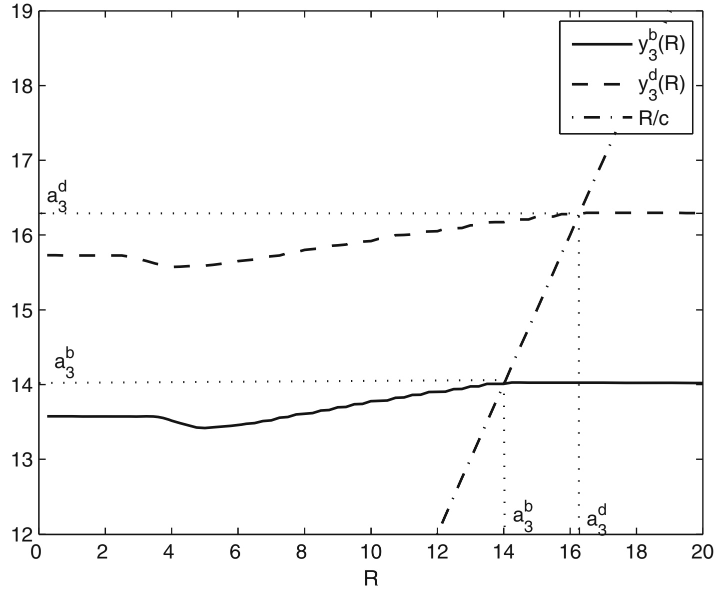

Fig. 5.1 Optimal solutions $y_3^b(R)$ and $y_3^d(R)$ for period 3

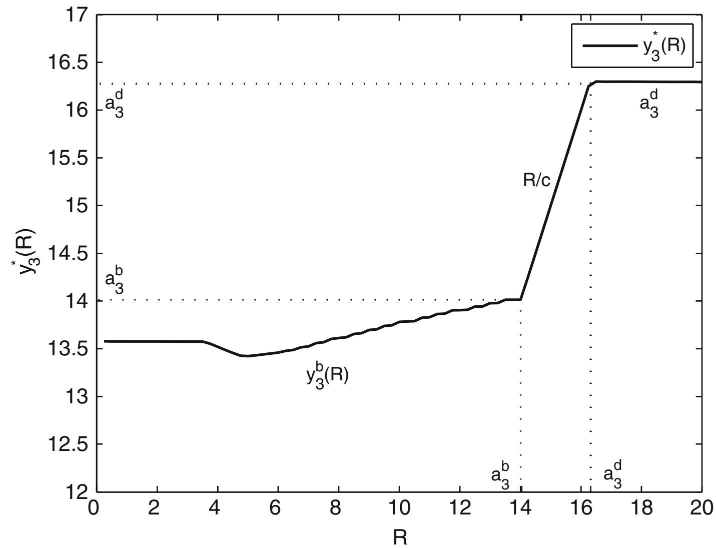

Fig. 5.2 Optimal base stock policy for period 3

5.5 Concluding Remarks

In this chapter we study a dynamic inventory control problem with asset-based financing. We derive the optimal inventory policy for each period, and characterize the dependence of the firm's optimal operational policy on its financial status. We also identify when the Retailer will borrow or deposit. The model further refer to the probability of firm bankruptcy as well. We find that for most of these models, the results in Lemmas 4.3.1, 4.3.2, and 4.3.3 and Theorem 4.3.4 in Chap. 4 can still be obtained. However, more precise structure of the optimal control policies beyond the "capital-dependent base-stock", such as Theorem 4.3.7 in Chap. 4, is difficult to obtain without imposing further structure in the model. These are just a few possible extensions and it appears that each of these variations will lead to different optimal solution structure that is worthy of study.

Many interesting issues remain to be investigated. For example, if there is a holding cost rate h and shortage cost rate b for each period, then the optimality equation becomes

$$\max_{y_1,\ldots,y_N} E[S_{N+1}],$$

subject to

$$y_n \geq x_n, \qquad n = 1, 2, \ldots, N,$$

where

$$\begin{aligned} S_{n+1} = {} & p\min\{y_n, D_n\} - h\max\{y_n - D_n, 0\} - b\max\{D_n - y_n, 0\} \\ & +(1+d)(S_n - c(y_n - x_n))^+ - (1+b)(S_n - c(y_n - x_n))^-, \end{aligned}$$

and, as before, $x_{n+1} = (y_n - D_n)^+$. Note that Lemmas 5.3.1, 5.3.2, and 5.3.3 continue to hold, thus Theorem 5.3.5 also holds and the optimal inventory control policy is a capital dependent base-stock policy. As a matter of fact, Theorem 5.3.5, as well as Lemmas 5.3.1, 5.3.2, and 5.3.3 hold true under much more general settings, e.g., under general revenue function, and under utility function optimization, etc. However, for Theorems 5.3.8 and 5.3.9, the objective function can no longer be decomposed, and the control parameters of the optimal base-stock policy are complicated and state-dependent. The problem will be even more complicated if we allow the selling price p to be a decision variable when the demand D_n depends on the selling price in period n.

The setting used in this chapter assumes that the demands over periods are independent and identically distributed. We pointed out that the results, Lemmas 5.3.1, 5.3.2, and 5.3.3 and Theorem 5.3.5 hold true as long as the demands over periods are independent and they do not need to be identically distributed.

Appendix

Proof of Lemma 5.3.3. By induction. Clearly, $V_{N+1}(x, S) = S + \gamma x$ is jointly concave in x and S. Assume that $V_{n+1}(x, S)$ is jointly concave in x and S. We now prove the property for n.

We firstly prove $V_{n+1}((y-D_n)^+, p\min\{y, D_n\}+(1+d)(S-c(y-x))^+-(1+b)(S-c(y-x))^-)$ is jointly concave in (y, x, S). Then for any (y_1, x_1, S_1) and (y_2, x_2, S_2) and $0 \leq \lambda \leq 1$, we need to prove

$$\begin{aligned}
V_{n+1}\Big(&(\lambda y_1 + (1-\lambda)y_2 - D_n)^+, p\min\{\lambda y_1 + (1-\lambda)y_2, D_n\} \\
&+(1+d)\big(\lambda S_1 + (1-\lambda)S_2 - c(\lambda y_1 + (1-\lambda)y_2 - \lambda x_1 - (1-\lambda)x_2)\big)^+ \\
&-(1+b)\big(\lambda S_1 + (1-\lambda)S_2 - c(\lambda y_1 + (1-\lambda)y_2 - \lambda x_1 - (1-\lambda)x_2)\big)^-\Big) \\
\geq{}& \lambda V_{n+1}\Big((y_1 - D_n)^+, p\min\{y_1, D_n\} + (1+d)\big(S_1 - c(y_1 - x_1)\big)^+ \\
&\quad -(1+b)\big(S_1 - c(y_1 - x_1)\big)^-\Big) \\
&+(1-\lambda)V_{n+1}\Big((y_2 - D_n)^+, p\min\{y_2, D_n\} + (1+d)\big(S_2 - c(y_2 - x_2)\big)^+ \\
&\quad -(1+b)\big(S_2 - c(y_2 - x_2)\big)^-\Big)
\end{aligned}$$

Notice the relationships $(y - D_n)^+ = y - \min\{y, D_n\}$ and $(1+d)(S-c(y-x))^+ - (1+b)(S-c(y-x))^- = (b-d)\min\{cy, S + cx\} - (1+b)cy + (1+d)(S + cx)$. For convenience let

$$\begin{aligned}
\bar{y} &= \lambda y_1 + (1-\lambda)y_2, \\
\tilde{y} &= \min\{\lambda y_1 + (1-\lambda)y_2, D_n\}, \\
\hat{y} &= \lambda\min\{y_1, D_n\} + (1-\lambda)\min\{y_2, D_n\}, \\
\tilde{S} &= \min\{c(\lambda y_1 + (1-\lambda)y_2), \lambda S_1 + (1-\lambda)S_2 + c(\lambda x_1 + (1-\lambda)x_2)\}, \\
\hat{S} &= \lambda\min\{cy_1, S_1 + cx_1\} + (1-\lambda)\min\{cy_2, S_2 + cx_2\}.
\end{aligned}$$

Then by $\tilde{y} \geq \hat{y}$ and $\tilde{S} \geq \hat{S}$, we have

$$\begin{aligned}
V_{n+1}\Big(&(\lambda y_1 + (1-\lambda)y_2 - D_n)^+, p\min\{\lambda y_1 + (1-\lambda)y_2, D_n\} \\
&+(1+d)\big(\lambda S_1 + (1-\lambda)S_2 - c(\lambda y_1 + (1-\lambda)y_2 - \lambda x_1 - (1-\lambda)x_2)\big)^+ \\
&-(1+b)\big(\lambda S_1 + (1-\lambda)S_2 - c(\lambda y_1 + (1-\lambda)y_2 - \lambda x_1 - (1-\lambda)x_2)\big)^-\Big)
\end{aligned}$$

$$
\begin{aligned}
&= V_{n+1}\Big((\lambda y_1 + (1-\lambda)y_2 - D_n)^+, p\min\{\lambda y_1 + (1-\lambda)y_2, D_n\} \\
&\qquad +(b-d)\min\{c(\lambda y_1 + (1-\lambda)y_2), \lambda S_1 + (1-\lambda)S_2 \\
&\qquad +c(\lambda x_1 + (1-\lambda)x_2)\} - (1+b)c(\lambda y_1 + (1-\lambda)y_2) \\
&\qquad +(1+d)(\lambda S_1 + (1-\lambda)S_2 + c(\lambda x_1 + (1-\lambda)x_2))\Big) \\
&= V_{n+1}\Big(\bar{y} - \tilde{y}, p\tilde{y} + (b-d)\tilde{S} - (1+b)c(\lambda y_1 + (1-\lambda)y_2) \\
&\qquad +(1+d)(\lambda S_1 + (1-\lambda)S_2 + c(\lambda x_1 + (1-\lambda)x_2))\Big) \\
&\geq V_{n+1}\Big(\bar{y} - \hat{y}, p\hat{y} + (b-d)\hat{S} - (1+b)c(\lambda y_1 + (1-\lambda)y_2) \\
&\qquad +(1+d)(\lambda S_1 + (1-\lambda)S_2 + c(\lambda x_1 + (1-\lambda)x_2))\Big) \\
&= V_{n+1}\Big(\lambda(y_1 - D_n)^+ + (1-\lambda)(y_2 - D_n)^+, \lambda p\min\{y_1, D_n\} + (1-\lambda)p\min\{y_2, D_n\} \\
&\qquad +\lambda(b-d)\min\{cy_1, S_1 + cx_1\} + (1-\lambda)(b-d)\min\{cy_2, S_2 + cx_2\} \\
&\qquad -(1+b)c(\lambda y_1 + (1-\lambda)y_2) + (1+d)(\lambda S_1 + (1-\lambda)S_2 + c(\lambda x_1 + (1-\lambda)x_2))\Big) \\
&\geq \lambda V_{n+1}\Big((y_1 - D_n)^+, p\min\{y_1, D_n\} + (b-d)\min\{cy_1, S_1 + cx_1\}) - (1+b)cy_1 \\
&\qquad + (1+d)(S_1 + cx_1)\Big) \\
&\quad +(1-\lambda)V_{n+1}\Big((y_2 - D_n)^+, p\min\{y_2, D_n\} + (b-d)\min\{cy_2, S_2 + cx_2\} \\
&\qquad - (1+b)cy_2 + (1+d)(S_2 + cx_2)\Big) \\
&= \lambda V_{n+1}\Big((y_1 - D_n)^+, p\min\{y_1, D_n\} + (1+d)\big(S_1 - c(y_1 - x_1)\big)^+ \\
&\qquad -(1+b)\big(S_1 - c(y_1 - x_1)\big)^-\Big) \\
&\quad + (1-\lambda)V_{n+1}\Big((y_2 - D_n)^+, p\min\{y_2, D_n\} + (1+d)\big(S_2 - c(y_2 - x_2)\big)^+ \\
&\qquad - (1+b)\big(S_2 - c(y_2 - x_2)\big)^-\Big),
\end{aligned}
$$

where the first inequality follows from Lemma 5.3.2 and the second inequality follows from the concavity of $V_{n+1}(x, S)$. Thus $V_{n+1}((y - D_n)^+, p\min\{y, D_n\} + (1+d)(S - c(y-x))^+ - (1+b)(S - c(y-x))^-)$ is jointly concave in (y, x, S) and so is the expected value. Finally since

$$\mathbb{C} = \{y : y \geq x\}$$

is a convex set, applying Proposition B-4 of Heyman and Sobel (1984) we conclude that $V_n(x, S)$ is jointly concave in x and S. □

Proof of Theorem 5.3.8. The proof is by induction. By the definition of $G_n^d(y)$ and Lemma 5.3.7 we have

$$\begin{aligned}\pi_N^d(y,R) &= pE\min\{y,D_N\} + (1+d)(R-cy) + \gamma E(y-D_N)^+ \\ &= (1+d)R + G_N^d(y).\end{aligned}$$

Hence $y_N^d(R) = a_N^d$ for all R.

Assume that the results have been proved for $n+1$, i.e., when $R \geq ca_{n+2}^d$ and $y \leq R/c$, $\pi_{n+1}^d(y,R)$ can be decomposed as

$$\pi_{n+1}^d(y,R) = (1+d)^{N-n}R + G_{n+1}^d(y)$$

and that if $R \geq ca_{n+1}^d$, then $y_{n+1}^d(R) = a_{n+1}^d$, and if $R < ca_{n+1}^d$, then $y_{n+1}^d(R) > R/c$. We will now prove the statement for n.

From Theorem 5.3.5 we have

$$\tilde{V}_{n+1}(x,R) = \begin{cases} \pi_{n+1}(R/c,R), & R/c \leq y_{n+1}^*(R), \\ \pi_{n+1}(y_{n+1}^*(R),R), & x < y_{n+1}^*(R) < R/c, \\ \pi_{n+1}(x,R), & x \geq y_{n+1}^*(R). \end{cases} \tag{5.11}$$

By the inductive assumption and Lemma 5.3.7 we can rewrite (5.11) as

$$\tilde{V}_{n+1}(x,R) = \begin{cases} \pi_{n+1}^b(y_{n+1}^b(R),R), & R \leq cy_{n+1}^b(R)\ \&\ x \leq y_{n+1}^b(R), \\ \pi_{n+1}^d(R/c,R), & cy_{n+1}^b(R) < R < ca_{n+2}^d\ \&\ x \leq R/c, \\ (1+d)^{N-n}R + G_{n+1}^d(R/c), & ca_{n+2}^d \leq R \leq ca_{n+1}^d\ \&\ x \leq R/c, \\ (1+d)^{N-n}R + G_{n+1}^d(a_{n+1}^d), & x < a_{n+1}^d < R/c, \\ (1+d)^{N-n}R + G_{n+1}^d(x), & a_{n+1}^d \leq x \leq R/c, \\ \pi_{n+1}^b(x,R), & y_{n+1}^b(R) < x\ \&\ R/c < x, \end{cases} \tag{5.12}$$

where & denotes "and".

The last two shows that if $R \geq ca_{n+1}^d$ and $R \geq cx$, then $\tilde{V}_{n+1}(x,R)$ can be rewritten as

$$\tilde{V}_{n+1}(x,R) = (1+d)^{N-n}R + G_{n+1}^d(\max\{a_{n+1}^d, x\}).$$

As a result, if $R \geq ca_{n+1}^d$ and $y \leq R/c$, then $(p-c)\min\{y, D_n\} + (1+d)R - dcy$ is larger than ca_{n+1}^d and $c(y - D_n)^+$, and hence $\pi_n^d(y, R)$ can be expressed as

$$\begin{aligned}\pi_n^d(y, R) &= E[\tilde{V}_{n+1}((y - D_n)^+, (p-c)\min\{y, D_n\} + (1+d)R - dcy)] \\ &= (1+d)^{N+1-n}R + (1+d)^{N-n}((p-c)E\min\{y, D_n\} - dcy) \\ &\quad + EG_{n+1}^d(\max\{a_{n+1}^d, (y - D_n)^+\}) \\ &= (1+d)^{N+1-n}R + G_n^d(y).\end{aligned}$$

Therefore, when $R \geq ca_{n+1}^d$ and $y \leq R/c$, the maximizer of $\pi_n^d(y, R)$, $y_n^d(R)$, is equals to a_n^d, the maximizer of $G_n^d(y)$. Furthermore, when $R \geq ca_n^d > ca_{n+1}^d$, $y_n^d(R) = a_n^d$, and when $ca_{n+1}^d < R < ca_n^d$, $y_n^d(R) > R/c$.

We next prove that $y_n^d(R) \geq R/c$ on $R \leq ca_{n+1}^d$. For notational convenience in what follows we use $\tilde{V}_{n,1}(x, R)$ and $\tilde{V}_{n,2}(x, R)$ to represent the partial derivatives with respect to x and R respectively, and $\tilde{V}_{n,12}(x, R)$ the cross derivative. From (5.12), taking derivatives of $\tilde{V}_{n+1}(x, R)$ yields

$$\tilde{V}_{n+1,1}(x, R) = \begin{cases} 0, & R > cx \,\&\, x \leq a_{n+1}^d \; or \; R \leq cx \leq cy_{n+1}^b(R), \\ G_{n+1}^d{}'(x), & a_{n+1}^d \leq x \leq R/c, \\ \pi_{n+1,1}^b(x, R), & y_{n+1}^b(R) < x \,\&\, R/c < x. \end{cases} \tag{5.13}$$

and

$$\tilde{V}_{n+1,2}(x, R) = \begin{cases} \mathrm{d}\pi_{n+1}^b(y_{n+1}^b(R), R)/\mathrm{d}R, & R \leq cy_{n+1}^b(R) \,\&\, x \leq y_{n+1}^b(R), \\ \mathrm{d}\pi_{n+1}^d(R/c, R)/\mathrm{d}R, & cy_{n+1}^b(R) < R < ca_{n+2}^d \,\&\, \leq R/c, \\ (1+d)^{N-n} + G_{n+1}^d{}'(R/c)/c, & ca_{n+2}^d \leq R \leq ca_{n+1}^d \,\&\, x \leq R/c, \\ (1+d)^{N-n}, & R/c > a_{n+1}^d \,\&\, x \leq R/c, \\ \pi_{n+1,2}^b(x, R), & y_{n+1}^b(R) < x \,\&\, R/c < x. \end{cases} \tag{5.14}$$

Note that $\tilde{V}_{n+1,2}(x, R)$ is independent of x when $x \leq R/c$, hence $\tilde{V}_{n+1,12}(x, R) = 0$ when $x \leq R/c$. By

$$\pi_n^d(y, R) = E[\tilde{V}_{n+1}((y - D_n)^+, (p-c)\min\{y, D_n\} + (1+d)R - dcy)],$$

taking derivative of $\pi_n^d(y, R)$ with respect to y yields

$$\pi_{n,1}^d(y,R) = \int_0^y [\tilde{V}_{n+1,1}(y-z,(p-c)z+(1+d)R-dcy)$$
$$-dc\tilde{V}_{n+1,2}(y-z,(p-c)z+(1+d)R-dcy)]\mathrm{d}F(z)$$
$$+(p-(1+d)c)(1-F(y))\tilde{V}_{n+1,2}(0,(p-(1+d)c)y+(1+d)R).$$

Since $y_n^d(R)$ is the maximizer of $\pi_n^d(y, R)$, to prove $y_n^d(R) \geq R/c$ on $R \leq ca_{n+1}^d$, it suffices to prove $\pi_{n,1}^d(R/c, R) \geq 0$ on $R \leq ca_{n+1}^d$. Noting that $\tilde{V}_{n+1,1}(x, R) = 0$ when $x \leq R/c \leq a_{n+1}^d$, We have

$$\begin{aligned}
&\pi_{n,1}^d(R/c, R)\\
&= (p-(1+d)c)(1-F(R/c))\tilde{V}_{n+1,2}(0, pR/c)\\
&\quad -dc\int_0^{R/c}\tilde{V}_{n+1,2}(R/c-z,(p-c)z+R)\mathrm{d}F(z)\\
&> (p-(1+d)c)(1-F(R/c))\tilde{V}_{n+1,2}(0, pR/c)-dcF(R/c)\tilde{V}_{n+1,2}(R/c, R),
\end{aligned} \tag{5.15}$$

where the inequality follows from the concavity of $\tilde{V}_{n+1}(x, R)$ in R and $\tilde{V}_{n+1,12}(x, R) = 0$ when $R > cx$. Since $R \leq ca_{n+1}^d < F^{-1}(\frac{p-(1+d)c}{p-c})$, we have

$$(p-(1+d)c)(1-F(R/c)) > dcF(R/c). \tag{5.16}$$

To prove the right hand side of (5.15) is non-negative, we consider two ranges of R separately.

Case 1: $ca_{n+2}^d < R \leq ca_{n+1}^d$. By (5.8) we have

$$G_n^{d\prime}(y) \leq (1+d)^{N-n}[(p-c)(1-F(y))-dc], \tag{5.17}$$

and since $\tilde{V}_{n+1,2}(x, R)$ is decreasing in R, it follows from (5.14) that

$$\tilde{V}_{n+1,2}(0, pR/c) \geq \lim_{y\to\infty}\tilde{V}_{n+1,2}(0, y) \geq (1+d)^{N-n}.$$

Hence, applying these inequalities and (5.14) on the interval $ca_{n+2}^d < R \leq ca_{n+1}^d$ yields

$$\begin{aligned}
&(p-(1+d)c)(1-F(R/c))\tilde{V}_{n+1,2}(0, pR/c) - dcF(R/c)\tilde{V}_{n+1,2}(R/c, R)\\
&\quad\geq (p-(1+d)c)(1-F(R/c))(1+d)^{N-n}-dcF(R/c)\\
&\quad[(1+d)^{N-n} + G_{n+1}^{d}{}'(R/c)/c]
\end{aligned}$$

$$
\begin{aligned}
&\geq [(p-(1+d)c)(1-F(R/c)) - dcF(R/c)](1+d)^{N-n} \\
&\quad - dcF(R/c)(1+d)^{N-1-n}[(p-c)(1-F(R/c)) - dc]/c \\
&= (1+d)^{N-1-n}\{[(p-(1+d)c)(1-F(R/c)) - dcF(R/c)] \\
&\quad + d(1-F(R/c))[(p-(1+d)c) - (p-c)F(R/c)]\} \\
&\geq 0.
\end{aligned}
$$

Where the last inequality follows from (5.16). Therefore, $\pi^d_{n,1}(R/c, R) \geq 0$ in this case.

Case 2: $R \leq ca^d_{n+2}$. From

$$\pi_{n+1}(R/c, R) = E[\tilde{V}_{n+2}((R/c - D_{n+1})^+, (p-c)\min\{R/c, D_{n+1}\} + R)]$$

We have

$$
\begin{aligned}
&\mathrm{d}\pi^d_{n+1}(R/c, R)/\mathrm{d}R \\
&= \int_0^{R/c} [\tilde{V}_{n+2,1}(R/c-z, (p-c)z+R)]\mathrm{d}F(z) \\
&\quad + \frac{p}{c}(1-F(R/c))\tilde{V}_{n+2,2}(0, pR/c) + \int_0^{R/c} [\tilde{V}_{n+2,2}(R/c-z, (p-c)z+R)]\mathrm{d}F(z) \\
&= \frac{p}{c}(1-F(R/c))\tilde{V}_{n+2,2}(0, pR/c) + \int_0^{R/c} [\tilde{V}_{n+2,2}(R/c-z, (p-c)z+R)]\mathrm{d}F(z) \\
&\leq \frac{p}{c}(1-F(R/c))\tilde{V}_{n+2,2}(0, pR/c) + (p-(1+d)c) \\
&\quad \times(1-F(R/c))\tilde{V}_{n+2,2}(0, pR/c)/dc \\
&= \frac{(p-c)(1+d)}{dc}(1-F(R/c))\tilde{V}_{n+2,2}(0, pR/c).
\end{aligned}
\tag{5.18}
$$

Where the first equalities follow from $\tilde{V}_{n+2,1}(R/c - z, (p-c)z + R) = 0$ because of (5.13) and the inductive assumption for $n + 2$. The inequality can be obtained as follows. When $R \leq ca^d_{n+2} < ca^d_{n+1}$, according to the inductive assumption we have $y^d_{n+1}(R) \geq R/c$ and hence $\pi^d_{n+1,1}(R/c, R) \geq 0$, which implies, by (5.15) for $n + 1$, that

$$
\begin{aligned}
&p-(1+d)c)(1-F(R/c))\tilde{V}_{n+2,2}(0, pR/c) \\
&\quad > dc\int_0^{R/c} \tilde{V}_{n+2,2}(R/c-z, (p-c)z+R)\mathrm{d}F(z).
\end{aligned}
$$

For $R \le ca^d_{n+2}$, applying (5.15) and (5.18) we obtain

$$\begin{aligned}
&\pi^d_{n,1}(R/c, R)\\
&\quad\ge (p-(1+d)c)(1-F(R/c))\tilde{V}_{n+1,2}(0, pR/c) - dcF(R/c)\tilde{V}_{n+1,2}(R/c, R)\\
&\quad= (p-(1+d)c)(1-F(R/c))\tilde{V}_{n+1,2}(0, pR/c) - dcF(R/c)\mathrm{d}\pi^d_{n+1}(R/c, R)/\mathrm{d}R\\
&\quad\ge (1-F(R/c))[(p-(1+d)c)\tilde{V}_{n+1,2}(0, pR/c)\\
&\qquad -(p-c)(1+d)F(R/c)\tilde{V}_{n+2,2}(0, pR/c)]\\
&\quad\ge (p-(1+d)c)(1-F(R/c))[\tilde{V}_{n+1,2}(0, pR/c) - (1+d)\tilde{V}_{n+2,2}(0, pR/c)].
\end{aligned}$$

Where the last inequality follows from $(p-c)F(R/c) \le p-(1+d)c$ because of $R \le ca^d_{n+1} < F^{-1}(\frac{p-(1+d)c}{p-c})$.

Next $\pi^d_{n_1}(R/c, R) > 0$ will be true if we can prove

$$\tilde{V}_{n+1,2}(0, R) - (1+d)\tilde{V}_{n+2,2}(0, R) \ge 0 \tag{5.19}$$

for all R. This is again proved by backward induction. First we have

$$\tilde{V}_{N,2}(0, R) - (1+d)\tilde{V}_{N+1,2}(0, R) \ge (1+d) - (1+d) = 0.$$

Assume $\tilde{V}_{n+2,2}(0, R) - (1+d)\tilde{V}_{n+3,2}(0, R) \ge 0$, we proceed to prove (5.19). If $R > ca^d_{n+2}$, then by (5.14) for $n+2$,

$$\tilde{V}_{n+1,2}(0, R) - (1+d)\tilde{V}_{n+2,2}(0, R) \ge (1+d)^{N-n} - (1+d)(1+d)^{N-1-n} = 0.$$

If $ca^d_{n+3} \le R \le ca^d_{n+2}$, then by (5.8) for $n+2$, we have

$$G^d_{n+2}{}'(R/c) \le (1+d)^{N-n-2}\big[(p-c)(1-F(R/c)) - dc\big], \tag{5.20}$$

and by the concavity of $\tilde{V}_{n+2}(x, R)$ in R we have

$$\tilde{V}_{n+2,2}(R/c - z, (p-c)z + R) \ge \tilde{V}_{n+2,2}(R/c - z, pR/c), \tag{5.21}$$

and since $\tilde{V}_{n+2,2}(x, (p-c)z + R)$ is independent of x when $x \le ca^d_{n+2}$, we have

$$\tilde{V}_{n+2,2}(R/c - z, pR/c) = \tilde{V}_{n+2,2}(0, pR/c). \tag{5.22}$$

Applying (5.14) we obtain

$$\begin{aligned}
&\tilde{V}_{n+1,2}(0, R) - (1+d)\tilde{V}_{n+2,2}(0, R)\\
&\quad= \frac{\mathrm{d}\pi^d_{n+1}(R/c, R)}{\mathrm{d}R} - (1+d)^{N-n} - (1+d)G_{n+2}{}^{d\prime}(R/c)/c
\end{aligned}$$

$$
\begin{aligned}
&\geq \frac{p}{c}(1-F(R/c))\tilde{V}_{n+2,2}(0,pR/c)+\int_0^{R/c}\tilde{V}_{n+2,2}(R/c-z,(p-c)z+R)\mathrm{d}F(z)\\
&\quad -(1+d)^{N-n}-(1+d)^{N-n-1}\big[(p-c)(1-F(R/c))-dc\big]/c\\
&\geq \left[\frac{p}{c}(1-F(R/c))+F(R/c)\right]\tilde{V}_{n+2,2}(0,pR/c)\\
&\quad -(1+d)^{N-n-1}\left[\frac{p}{c}(1-F(R/c))+F(R/c)\right]\\
&= \left[\frac{p}{c}(1-F(R/c))+F(R/c)\right]\left(\tilde{V}_{n+2,2}(0,pR/c)-(1+d)^{N-n-1}\right)\\
&\geq 0,
\end{aligned}
$$

where the first inequality follows from (5.18) and (5.20), the second inequality is due to (5.21) and (5.22), and the last inequality follows from the observation that, by (5.14) for $n+2$, when $pR/c \geq R \geq ca^d_{n+3}$, we have $\tilde{V}_{n+2,2}(0,pR/c) \geq (1+d)^{N-n-1}$.

Finally, if $R \leq ca^d_{n+2}$, then

$$
\begin{aligned}
&\tilde{V}_{n+1,2}(0,R)-(1+d)\tilde{V}_{n+2,2}(0,R)\\
&\quad = \frac{\mathrm{d}\pi^d_{n+1}(R/c,R)}{\mathrm{d}R}-(1+d)\frac{\mathrm{d}\pi^d_{n+2}(R/c,R)}{\mathrm{d}R}\\
&\quad = \frac{p}{c}(1-F(R/c))[\tilde{V}_{n+2,2}(0,pR/c)-(1+d)\tilde{V}_{n+3,2}(0,pR/c)]\\
&\qquad +\int_0^{R/c}[\tilde{V}_{n+2,2}(R/c-z,(p-c)z+R)\\
&\qquad\qquad -(1+d)\tilde{V}_{n+3,2}(R/c-z,(p-c)z+R)]\mathrm{d}F(z)\\
&\quad \geq 0,
\end{aligned}
$$

where the first equality follows from (5.14), the second equality follows from (5.18) for $n+2$ and $n+3$, and the inequality follows from the inductive assumption.

Hence, we have proved $\pi^d_{n,1}(R/c,R) \geq 0$, implying $y^d_n(R) \geq R/c$ if $R \leq ca^d_{n+1}$. The proof of Theorem 5.3.8 is completed. □

Proof of Theorem 5.3.9. The proof is by induction. First notice that

$$
\begin{aligned}
\pi^b_N(y,R) &= pE\min\{y,D_N\}+(1+b)(R-cy)+\gamma E(y-D_N)^+\\
&= (1+b)R+(p-\gamma)\min\{y,D_N\}-((1+b)c-\gamma)y.
\end{aligned}
$$

Hence,

$$
y^b_N(R)=a^b_N=F^{-1}\left(\frac{p-(1+b)c}{p-\gamma}\right)
$$

and

$$y_N^*(R) = \begin{cases} y_N^b(R), & R < ca_N^b, \\ R/c, & ca_N^b \le R \le ca_N^d, \\ a_N^d, & R > ca_N^d. \end{cases}$$

Assume that the results have been proved for $n + 1$, i.e.,

$$y_{n+1}^*(R) = \begin{cases} y_{n+1}^b(R), & R < ca_{n+1}^b, \\ R/c, & ca_{n+1}^b \le R \le ca_{n+1}^d, \\ a_{n+1}^d, & R > ca_{n+1}^d. \end{cases}$$

From Theorem 5.3.5 we have

$$\tilde{V}_{n+1}(x, R) = \begin{cases} \pi_{n+1}(R/c, R), & R/c \le y_{n+1}^*(R), \\ \pi_{n+1}(y_{n+1}^*(R), R), & x < y_{n+1}^*(R) < R/c, \\ \pi_{n+1}(x, R), & x \ge y_{n+1}^*(R). \end{cases} \tag{5.23}$$

Furthermore, by the inductive assumption and Lemma 5.3.8, we can rewrite (5.23) as

$$\tilde{V}_{n+1}(x, R) = \begin{cases} \pi_{n+1}^b(y_{n+1}^b(R), R), & R \le ca_{n+1}^b \,\&\, x \le y_{n+1}^b(R), \\ \pi_{n+1}^d(R/c, R), & ca_{n+1}^b \le R \le ca_{n+1}^d \,\&\, R \ge cx, \\ (1+d)^{N-n}R + G_{n+1}^d(a_{n+1}^d), & x < a_{n+1}^d < R/c, \\ (1+d)^{N-n}R + G_{n+1}^d(x), & R/c > x \ge a_{n+1}^d, \\ \pi_{n+1}^b(x, R), & x > y_{n+1}^b(R) \,\&\, x > R/c. \end{cases} \tag{5.24}$$

For notational convenience, in what follows, we use $\tilde{V}_{n,1}(x, R)$ and $\tilde{V}_{n,2}(x, R)$ to represent the partial derivatives with respect to x and R respectively, and $\tilde{V}_{n,12}(x, R)$ the cross derivative. From (5.24), taking partial derivatives of $\tilde{V}_{n+1}(x, R)$ yields

$$\tilde{V}_{n+1,1}(x, R) = \begin{cases} 0, & R > cx \,\&\, x \le a_{n+1}^d \text{ or } R \le cx \le cy_{n+1}^b(R), \\ G_{n+1}^d{}'(x), & R/c > x > a_{n+1}^d, \\ \pi_{n+1,1}^b(x, R), & x > y_{n+1}^b(R) \,\&\, x > R/c. \end{cases} \tag{5.25}$$

and

$$\tilde{V}_{n+1,2}(x,R) = \begin{cases} \pi^b_{n+1_2}(y^b_{n+1}(R),R), & R \le cy^b_{n+1}(R) \,\&\, x \le a^b_{n+1}, \\ \mathrm{d}\pi^d_{n+1}(R/c,R)/\mathrm{d}R, & ca^b_{n+1} < R < ca^d_{n+1}, R \ge cx, \\ (1+d)^{N-n}, & R/c \ge x \,\&\, R \ge ca^d_{n+1}, \\ \pi^b_{n+1,2}(x,R), & x > y^b_{n+1}(R) \,\&\, x > R/c. \end{cases} \tag{5.26}$$

Note that $\tilde{V}_{n+1,2}(x,R)$ is independent of x when $x \le R/c$, hence $\tilde{V}_{n+1,12}(x,R) = 0$ when $x \le R/c$. By

$$\pi^b_n(y,R) = E[\tilde{V}_{n+1}((y-D_n)^+, (p-c)\min\{y,D_n\} + (1+b)R - bcy)],$$

taking derivative of $\pi^b_n(y,R)$ with respect to y yields

$$\begin{aligned}\pi^b_{n,1}(y,R) = \int_0^y & [\tilde{V}_{n+1,1}(y-z,(p-c)z+(1+b)R-bcy) \\ & - bc\tilde{V}_{n+1,2}(y-z,(p-c)z+(1+b)R-bcy)]\mathrm{d}F(z) \\ & + (p-(1+b)c)(1-F(y))\tilde{V}_{n+1,2}(0,(p-(1+b)c)y+(1+b)R).\end{aligned}$$

Since $y^b_n(R)$ is the maximizer of $\pi^b_n(y,R)$, to study the relationship between $y^b_n(R)$ and R/c, it suffices to examine the sign of $\pi^b_{n,1}(R/c,R)$.

If $R \ge ca^d_{n+1}$, following from (5.25) and (5.26) and noting that $R/c - z \le (p-c)z + R$, then

$$\begin{aligned}\pi^b_{n,1}(R/c,R) &= \int_0^{R/c} [\tilde{V}_{n+1,1}(R/c-z,(p-c)z+R) \\ &\qquad -bc\tilde{V}_{n+1,2}(R/c-z,(p-c)z+R)]\mathrm{d}F(z) \\ &\quad + (p-(1+b)c)(1-F(R/c))\tilde{V}_{n+1,2}(0,pR/c) \\ &= \int_0^{R/c-a^d_{n+1}} {G^d_{n+1}}'(R/c-z)\mathrm{d}F(z) \\ &\quad + (p-(1+b)c-(p-c)F(R/c))(1+d)^{N-n}.\end{aligned}$$

Further notice that

$$\begin{aligned}\frac{\mathrm{d}\pi^b_{n,1}(R/c,R)}{\mathrm{d}R} = \int_0^{R/c-a^d_{n+1}} & {G^d_{n+1}}''(R/c-z)\mathrm{d}F(z) \\ & -(p-c)(1+d)^{N-n} f(R/)/c \le 0.\end{aligned}$$

and $\pi^b_{n,1}(R/c, R)|\{R = ca^d_{n+1}\} > 0$ and $\pi^b_{n,1}(R/c, R)|\{R = cF^{-1}(\frac{p-(1+b)c}{p-c})\}<0$. There exists a^b_n, $a^d_{n+1} < a^b_n < F^{-1}\big(\frac{p-(1+b)c}{p-c}\big)$, such that

(i) $\pi^b_{n,1}(R/c, R) = 0$ hence $y^b_n(R) = R/c$ when $R = a^b_n$
(ii) $\pi^b_{n,1}(R/c, R) > 0$ hence $y^b_n(R) > R/c$ when $ca^d_{n+1} \leq R < ca^b_n$
(iii) $\pi^b_{n,1}(R/c, R) < 0$ hence $y^b_n(R) < R/c$ when $ca^b_n < R < cF^{-1}\big(\frac{p-(1+b)c}{p-c}\big)$

Finally we prove that $y^b_n(R) > R/c$ when $R < ca^d_{n+1}$. We have

$$\begin{aligned}\pi^b_{n,1}(R/c, R) &= (p - (1 + b)c)(1 - F(R/c))\tilde{V}_{n+1,2}(0, pR/c)\\ &\quad -bc\int_0^{R/c} \tilde{V}_{n+1,2}(R/c - z, (p - c)z + R)]\mathrm{d}F(z)\\ &\geq (p - (1 + b)c)(1 - F(R/c))\tilde{V}_{n+1,2}(0, pR/c)\\ &\quad -bcF(R/c)\tilde{V}_{n+1,2}(R/c, R),\end{aligned}$$

where the inequality follows from the concavity of $\tilde{V}_{n+1}(x, R)$ in R and $\tilde{V}_{n+1,12}(x, R) = 0$ when $x \leq R/c$. Next through similar proof to Lemma 5.3.8 we find that $\pi^b_{n,1}(R/c, R) \geq 0$ when $R < ca^d_{n+1}$.

Therefore, together with the results from Lemma 5.3.8, the theorem is proved.

□

Chapter 6
Delayed Cash Payment, Receivable and Inventory Management

6.1 Introduction

Chapters 4 and 5 incorporates the financial issue into inventory management. More specifically, the cash on hand is characterized as the financial constraint. Besides cash, another state of the firm is receivable, which is mainly due to the delayed cash payment. In practice of a supply chain, it is common that downstream firms pay for upstream firms with certain delay. Actually, powerful retailers (e.g., Wal-Mart, Carrefour) usually delay up to 50% of their payments for several months. On the other hand, firms also offer potential customers some preferential choice to delay their payment. Installment plan is a common case for it.

Although receivable plays an important roles in corporate finance and accounting, it is almost ignored by most of the supply chain management literature. A rare exception is Arcelus and Srinivasan (1993) who considered the problem of a vendor who attempts to dispose of unanticipated inventory levels through an offer to a prospective buyer of a credit-period within which no payment is required. They derived the feasible range beyond which the offer is not accepted, and analyzed the trade-offs between the credit-period and extra-stock accepted to both parties. Finally the optimal inventory policy are derived.

Here we study the simple finite horizon inventory system and attempt to derive the optimal operational policy of the firm with the incorporations of capital and receivable constraints. We will try to figure out the question that how should the firm's inventory replenishment decisions depend on both the cash and receivable.

The rest of this chapter is organized as follows. Section 6.2 gives assumptions and formulates the basic model of this chapter. Then Sect. 6.3 finds the optimal operational policy for inventory replenishment. In Sect. 6.4, the influences of delayed cash payment on the optimal inventory policy and achievable profit are studied. The model is also compared with traditional inventory model without consideration of delayed cash payment. Section 6.5 proposes some numerical examples which demonstrate the main results. Finally Sect. 6.6 concludes the chapter and discusses some extensions.

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6.2 Assumptions and Model Formulation

In this chapter, we consider the fundamental single-retailer and single-product case, where the risk neutral retailer facing random demands makes replenishment decisions over a finite time horizon of N periods. The inventory is reviewed periodically and demand D_n in different periods are independent of each other. The cumulative and probability distributions of stochastic demand are assumed to be $F(\cdot)$ and $f(\cdot)$ respectively. Let x_n be the inventory level and y_n be the inventory position for period n.

The cost function is assumed linear with variable costs. Then denote the selling price by p and the ordering cost by c. Furthermore, the lead time is assumed to be zero and hence an order placed at the beginning of period n arrives immediately before demand is realized. The unsatisfied demand is assumed lost. Without loss of generality, we assume no holding cost and shortage cost.

The capital on hand is denoted by S_n at the beginning of period n. Then the operational decision will be constrained by S_n in following way: $c(y_n - x_n) \leq S_n$. Assume that the cash payment is delayed by k period. Let R_n be the receivable in the end of period n. Then R_n implies the selling income of period n which will arrive in the end of period $n + k$.

Based on above discussion, the sequence of events in period $n(1 \leq n \leq N)$ is as follows. At the beginning of each period, the retailer places an order with his capital on hand. Then the demand is satisfied. At the end of the period, the retailer receives the selling income of period $n - k$.

Therefore, the decision problem of the retailer is to decide an ordering policy to maximize the final capital and receivable, given an initial inventory level x_1, a capital level S_1 and receivable R_1, subject to a capital constraint for each period. That is, the decision problem is:

$$\max_{y_1,\ldots,y_N} E\left[S_{N+1} + \sum_{i=1}^{k} R_{N+2-i}\right], \tag{6.1}$$

subject to

$$0 \leq y_n - x_n \leq \frac{S_n}{c}, n = 1, 2, \ldots, N,$$

where $x_{n+1} = (y_n - D_n)^+$, $S_{n+1} = S_n + R_{n+1-k} - c(y_n - x_n)$ and $R_{n+1} = p\min\{y_n, D_n\}$, n=1,2, …, N.

Denote by $V_n(x, S, R_1, \ldots, R_k)$ the maximum achievable capital starting at the beginning of period n with an initial inventory level x, accumulated capital S and receivables from R_1 to R_k, where R_i is the selling income i period ago. Then the following dynamic program can be employed to solve decision problem (6.1).

$$V_{N+1}(x, S, R_1, \ldots, R_k) = S + \sum_{k=1}^{k} R_i,$$

and

$$V_n(x, S, R_1, \ldots, R_k) = \max_{x \le y \le x + \frac{S}{c}} E[V_{n+1}(x_+, S_+, R_{1+}, \cdots, R_{m+})], \quad (6.2)$$

where $x_+ = (y - D_n)^+$, $S_+ = S + R_k - c(y - x)$, $R_{1+} = p\min\{y, D_n\}$ and $R_{i+} = R_{i-1}, i = 2, \ldots, k$. Note that we assume $V_{N+1}(x, S, R_1, \ldots, R_k)$ is independent of x here, which implies zero salvage value.

6.3 Delayed Cash Payment Offered by the Retailer

Note that here only the retailer is considered and the delay happens in the payment process from customer to retailer. Our focus in this chapter is the influence of delayed cash payment offered by the retailer on the optimal inventory policy and achievable profit.

For simplicity and without loss of generality, assume that only one-period delay is allowed, i.e., $k = 1$. The model with k periods delayed is similar. Therefore, dynamic program in (6.2) can be simplified into

$$V_{N+1}(x, S, R) = S + R,$$

and

$$V_n(x, S, R) = \max_{x \le y \le x + \frac{S}{c}} E[V_{n+1}(x_+, S_+, R_+)], \quad (6.3)$$

where $x_+ = (y - D_n)^+$, $S_+ = S + R - c(y - x)$ and $R_+ = p\min\{y, D_n\}$.

Next before deriving the optimal inventory policy under delayed cash payment, some lemmas are needed.

Lemma 6.3.1. *For any period n and fixed (x, R), $V_n(x, S, R)$ is increasing in S.*

The proof is straightforward by simple induction.

Lemma 6.3.2. *For any period n,*

(a) $V_n(A - z, B, pz)$ *is increasing in z for fixed A and B.*
(b) $V_n((A - z + B/c - D)^+, pz, p\min\{A - z + B/c, D\})$ *is increasing in z for fixed A and B.*
(c) $V_n(x, S, R)$ *is jointly concave in* (x, S, R).

Proof. The proof is by induction. The statement is trivially true for $n = N + 1$. Assume that the statement is true for some $n + 1$. We will now prove the statement for n.

For part (c), we first prove $V_{n+1}(x_+, S_+, R_+) = V_{n+1}((y - D_n)^+, S + R - c(y - x), p\min\{y, D_n\})$ is jointly concave in (y, x, S, R). Since the linear part is trivial, we can ignore it. Then for any $y_1, y_2 \in \mathbb{R}^+$ and $\lambda \in (0, 1)$, we only need to prove

$$\begin{aligned}
&V_{n+1}((\lambda y_1 + (1-\lambda)y_2 - D_n)^+, \ldots, p\min\{\lambda y_1 + (1-\lambda)y_2, D_n\}) \\
&\quad \geq \lambda V_{n+1}((y_1 - D_n)^+, \ldots, p\min\{y_1, D_n\}) \\
&\qquad + (1-\lambda)V_{n+1}((y_2 - D_n)^+, \ldots, p\min\{y_2, D_n\}).
\end{aligned}$$

With the inductive assumption of part (c), we have

$$\begin{aligned}
&V_{n+1}(\lambda(y_1 - D_n)^+ + (1-\lambda)(y_2 - D_n)^+, \ldots, p\lambda\min\{y_1, D_n\} \\
&\qquad + p(1-\lambda)\min\{y_2, D_n\}) \\
&\quad \geq \lambda V_{n+1}V((y_1 - D_n)^+, \ldots, p\min\{y_1, D_n\}) \\
&\qquad + (1-\lambda)V_{n+1}((y_2 - D_n)^+, \ldots, p\min\{y_2, D_n\}).
\end{aligned}$$

Since $(y - D_n)^+ = y - \min\{y, D_n\}$ and $\min\{\lambda y_1 + (1-\lambda)y_2, D_n\} \geq \lambda\min\{y_1, D_n\} + (1-\lambda)\min\{y_2, D_n\}$, the inductive assumption of part (a) allows us to show that

$$\begin{aligned}
&V_{n+1}((\lambda y_1 + (1-\lambda)y_2 - D_n)^+, \ldots, p\min\{\lambda y_1 + (1-\lambda)y_2, D_n\}) \\
&\quad \geq V_{n+1}(\lambda(y_1 - D_n)^+ + (1-\lambda)(y_2 - D_n)^+, \ldots, \\
&\qquad p\lambda\min\{y_1, D_n\} + p(1-\lambda)\min\{y_2, D_n\}).
\end{aligned}$$

Hence, $V_{n+1}((y - D_n)^+, S + R - c(y - x), p\min\{y, D_n\})$ is jointly concave in (y, x, S, R). This implies that $E[V_{n+1}(x_+, S_+, R_+)]$ is jointly concave in (y, x, S, R).

Then, since $\mathbb{C} = \{(y, x, S) | x \leq y \leq x + S/c\}$ is a convex set, we find that $V_n(x, S, R)$ is jointly concave in (x, S, R).

Next we prove part (a) for period n. First

$$\begin{aligned}
V_n(A - z, B, pz) = \max_{A - z \leq y \leq A - z + B/c} & E[V_{n+1}((y - D_n)^+, B + cA \\
& + (p - c)z - cy, p\min\{y, D_n\})].
\end{aligned}$$

From Lemma 6.3.1 and the inductive assumption of (c), $E[V_{n+1}((y - D_n)^+, B + cA + (p - c)z - cy, p\min\{y, D_n\})]$ is increasing in z and concave in y. To prove $V_n(A - z, B, pz)$ is increasing in z, we only need to prove $E[V_{n+1}((A - z + B/c - D_n)^+, B + cA + (p - c)z - c(A - z + B/c), p\min\{A - z + B/c, D_n\})]$ is increasing in z. It follows from the inductive assumption of part (b).

Finally, we prove part (b) for period n. We have

$$V_n\left(\left(A-z+\frac{B}{c}-D\right)^+, pz, p\min\left\{A-z+\frac{B}{c}, D\right\}\right)$$

$$= \max_{(A-z+\frac{B}{c}-D)^+\le y\le\left(A-z+\frac{B}{c}-D\right)^+ + \frac{zp}{c}} E\left[V_{n+1}\left((y-D_n)^+, (p-c)\min\left\{A-z+\frac{B}{c}, D\right\}\right.\right.$$

$$\left.\left. + (p-c)z+B+cA-cy, p\min\{y, D_n\}\right)\right].$$

Then the result follows directly from Lemma 6.3.1 and that $(A-z+B/c-D)^+ + zp/c$ is increasing in z. □

Then we will show that when delayed cash payment exists for the retailer, the optimal operational policy is a capital dependent base stock inventory policy.

Theorem 6.3.3. *A capital dependent base stock inventory policy is optimal.*

Proof. Let $\hat{y}_n(S + R + cx)$ be an optimal solution for the problem

$$\max_y E[V_{n+1}(x_+, S_+, R_+)].$$

It has been shown in the proof for part (c) of Lemma 6.3.2 that $E[V_{n+1}(x_+, S_+, R_+)]$ is concave in y for any fixed $S + R + cx$. Then it is optimal to order-up-to $\min\{\hat{y}_n(S+cx, R), x+S/c\}$ when $x < \hat{y}_n(S+cx, R)$ and not to order otherwise. In other words, a state dependent base stock policy is optimal. □

Accordingly, the optimal operational policy is shown in following theorem.

Theorem 6.3.4. *The optimal operational policy* $y_n^*(x, S, R)$ *is given by*

$$y_n^*(x, S, R) = \begin{cases} x + S/c, & x + S/c \le \hat{y}_n(S + R + cx), \\ \hat{y}_n(S + R + cx), & x < \hat{y}_n(S + R + cx) < x + S/c, \\ x, & x \ge \hat{y}_n(S + R + cx). \end{cases}$$

Note that the optimal inventory policy depends on initial capital on hand and receivable in each period. Theorem 6.3.4 allows us to show that:

(i) Retailers with low wealth level $x + S/c$ will have insufficient capital or inventory. Therefore, they will use all the cash they have to finance their inventory but can not carry out their optimal inventory policy. More wealth will lead to more ordering.
(ii) Retailers with high enough wealth level $x + S/c$ will have more surplus cash or inventory for their operations. Therefore they can make their best choice to order.

Since the theoretical discussion about how optimal operational policy depends on cash on hand S and receivable R is quite complex, we will show the properties by some numerical examples in Sect. 6.5.

6.4 Influence of Delayed Cash Payment

In this section the influence of delayed cash payment on retailer's optimal profit is studied. Firstly how the optimal profit depends on delayed cash payment will be derived, then a comparison between the model in this chapter and traditional inventory model without delayed cash payment will be employed.

Lemma 6.4.1. *For any period n, $V_n(x, S, R)$ will increase as: (1) S increases; (2) R increases; (3) the ratio of S in total wealth increases, i.e. $S/(S + R)$ increases with $S + R$ unchanged.*

The proof follows from Lemma 6.3.1 and dynamic program equation (6.3).

Lemma 6.4.1 implies that the achievable capital will be more if the initial receivable is less while the initial capital on hand is more. As a straightforward result, we have $V_n(x, S, R) \leq V_n(x, S + R, 0)$, which is under the condition there is no initial delayed cash payment.

Then a natural thing to do next is to compare the optimal achievable capital with the traditional multi-period inventory system without delayed cash payment. The optimization problem of traditional case is as follows:

$$W_{N+1}(x', S') = S$$

and

$$W_n(x', S') = \max_{x' \leq y \leq x' + S'/c} E[W_{n+1}((y - D_n)^+, S' + p \min\{y, D_n\} - c(y - x'))].$$

Furthermore, the optimal operational policy is given by $y_n^*(x', S')$.

Theorem 6.4.2. *Assume $x = x'$ and $S + R = S'$, for any period n,*

(a) $V_n(x, S, R) \leq W_n(x', S')$.
(b) $y_n^*(x, S, R) \leq y_n^*(x', S')$.

Proof. The proof is by induction. We first prove part (a). The statement is trivially true for $n = N + 1$. Assume that the statement is true for some $n + 1$. We will now prove the statement for n as follows.

$$\begin{aligned} V_n(x, S, R) &\leq V_n(x', S', 0) \\ &= \max_{x' \leq y \leq x' + S'/c} E[V_{n+1}((y - D_n)^+, S' - c(y - x'), p \min\{y, D_n\})] \end{aligned}$$

$$
\begin{aligned}
&\le \max_{x' \le y \le x' + S'/c} E[W_{n+1}((y - D_n)^+, S' - c(y - x') + p \min\{y, D_n\})] \\
&= W_n(x', S'),
\end{aligned}
$$

where the first inequality follows from Lemma 6.4.1, and the second inequality follows from inductive assumption of part (a).

For part (b), first noting that

$$
\begin{aligned}
V_N(x, S, R) &= \max_{x \le y \le x + S/c} E[S + R + p \min\{y, D_n\} - c(y - x)] \\
&= W_N(x', S'),
\end{aligned}
$$

then $y_N^*(x, S, R) = y_N^*(x', S')$. Assume that the statement is true for some $n + 1$. We will now prove the statement for n.

Notice that

$$
\begin{aligned}
&V_n(x, S, R) \\
&= \max_{x \le y_n \le x + S/c} E[V_{n+1}((y_n - D_n)^+, S' - c(y_n - x), p \min\{y_n, D_n\})] \\
&= \max_{x \le y_n \le x + S/c} E \Bigg\{ \max_{(y_n - D_n)^+ \le y_{n+1} \le S'/c + x - \min\{y_n, D_n\}} E \big[V_{n+2}((y_{n+1} - D_{n+1})^+, \\
&\qquad S' + cx + (p - c) \min\{y_n, D_n\} - c y_{n+1}, p \min\{y_{n+1}, D_{n+1}\})\big] \Bigg\},
\end{aligned}
$$

and

$$
\begin{aligned}
&W_n(x', S') \\
&= \max_{x \le y_n \le x + S'/c} E[W_{n+1}((y_n - D_n)^+, S' + p \min\{y_n, D_n\} - c(y_n - x))] \\
&= \max_{x \le y_n \le x + S'/c} E \Bigg\{ \max_{(y_n - D_n)^+ \le y_{n+1} \le S'/c + x + (p/c - 1) \min\{y_n, D_n\}} E[W_{n+2}((y_{n+1} - D_{n+1})^+, \\
&\qquad S' + cx + (p - c) \min\{y_n, D_n\} + p \min\{y_{n+1}, D_{n+1}\} - c y_{n+1})] \Bigg\}.
\end{aligned}
$$

As y_n increases, $S'/c + x - \min\{y_n, D_n\}$ will decrease and $S'/c + x + (p/c - 1) \min\{y_n, D_n\}$ will increase. Furthermore, $S'/c + x - \min\{y_n, D_n\} \le S'/c + x + (p/c - 1) \min\{y_n, D_n\}$. Therefore, the inductive assumption of part (b) allows us to show that $\hat{y}_n(x, S, R) \le \hat{y}_n'(x', S')$. □

Theorem 6.4.2 allows us to show that if the retailer offers customer an option that payment can be delayed, then the retailer will order less because the capital available on hand is less. It consequently results in a lower profit. Therefore, to the retailer the essential of this offer is a trade-off between the benefits from demand increase and the loss from limited operations.

6.5 Numerical Examples

In this section we propose some numerical examples to show the dependencies of $y_n^*(x, S, R)$ and $V_n(x, S, R)$ on S and R. More specifically, only two recursions are considered for computational simplicity. Or in other words, only $y_{N-1}^*(x, S, R)$ and $V_{N-1}(x, S, R)$ are studied. Furthermore, it will be shown that offering delayed cash payment leads to lower replenishment level and profit.

Assumptions of parameters are given as follows: $p = 1.3$, $c = 1$, and exponential demand with mean 10. Without loss of generality, we also assume the initial inventory level $x = 0$.

First, according to formula of dynamic program (6.3), the optimal solution for the last period is given by $\hat{y}_N(S + R + cx) = F^{-1}((p - c)/p) = 2.6236$. We then compute the optimal operational policy for period $N - 1$.

Figure 6.1 shows the dependence of $y_{N-1}^*(x, S, R)$ on S, where the receivable R takes value of 0, 1, 2, 3 respectively. It demonstrates that:

- When capital on hand S is small enough, the operation will be limited and the retailer will use up all the cash to order. Hence, $y_{N-1}^*(x, S, R)$ is linear increasing in S.

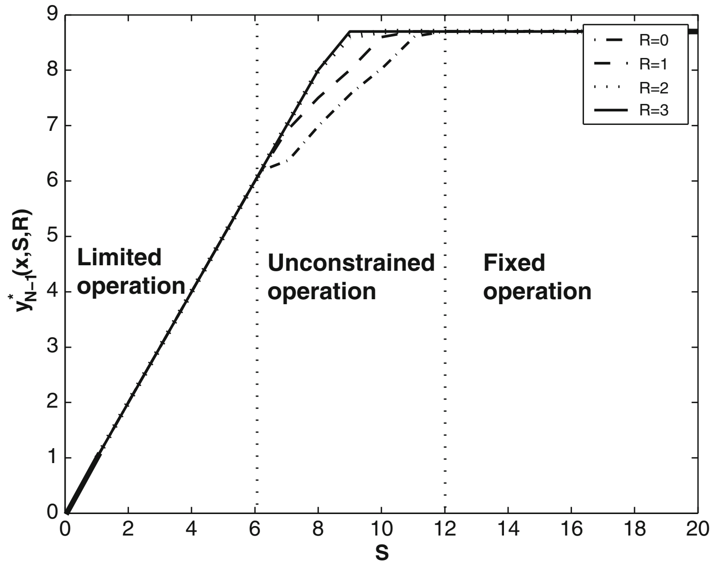

Fig. 6.1 Influence of S and R on the optimal operational policy

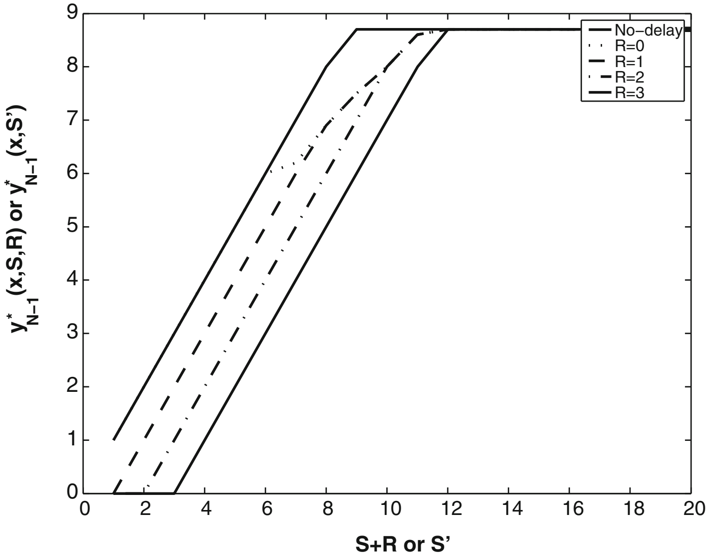

Fig. 6.2 Comparison of optimal operational policy between models with and without delayed cash payment

- When capital on hand is sufficient, the retailer can employ an unconstrained operation. As S increases, the order-up-to level will increase correspondingly.
- When the cash level is high enough, the retailer will order a sufficient quantity and hence the operations will remain fixed.
- If the capital on hand S remain unchanged while the receivable R increases, the optimal replenishment level will increase.

Furthermore, Fig. 6.1 also implies that the range of unconstrained operation becomes shorter when the receivable R increases. In fact, if R is large enough so that $R \geq 4$, the optimal operational policy of the retailer will almost remain unchanged.

Next, we compute the optimal operational policy for traditional multi-period inventory model without delayed cash payment, and then compare it with our model. Here the same assumptions for the basic inventory model are made as mentioned above. The optimal base-stock level for period N is also given by: $y_N^*(x, S') = F^{-1}((p-c)/p) = 2.6236$.

As shown in Theorem 6.4.2, the comparison between two models is based on the condition that $S + R = S'$. Hence relative to Fig. 6.1, the influence of wealth $S + R$ or S' on the optimal operational policies will be studied in Fig. 6.2.

Figure 6.2 finally demonstrates that:

- Higher ratio of receivable in wealth $S + R$ leads to lower replenishment level.
- Operational policy under the condition that there is not delayed cash payment is larger than that with delayed cash payment. However, when capital on hand is small enough or large enough, the optimal operational policies in two models are equivalent.

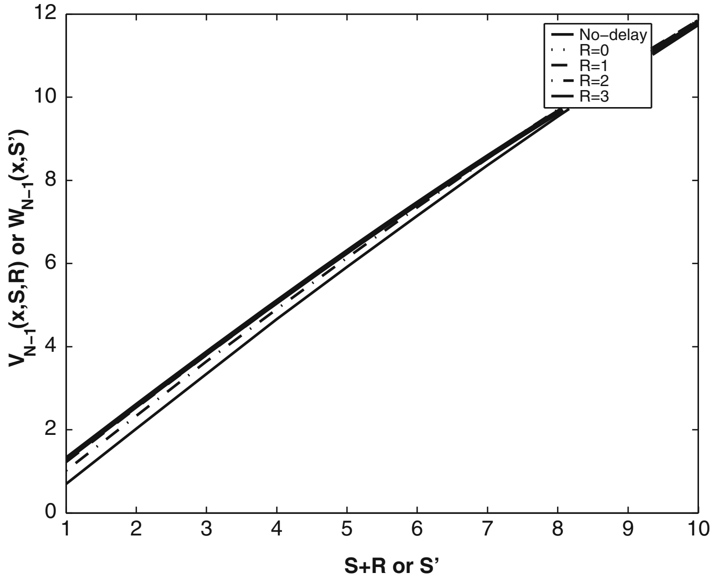

Fig. 6.3 Comparison of optimal achievable profit between models with and without delayed cash payment

Finally we compare the influence of wealth $S+R$ or S' on the optimal achievable profit $V_{N-1}(x, S, R)$ or $W_{N-1}(x, S')$ of the two models. As shown in Fig. 6.3, the retailer obtains more profit if no delayed cash payment is offered. If the retailer offers delayed cash payment, then the higher the ratio of receivable in wealth $S+R$, the lower the level of the profit achieved.

Therefore, the numerical examples demonstrate the dependences, relationships, and influence which have been shown in Lemma 6.4.1 and Theorem 6.4.2.

6.6 Concluding Remarks

In this chapter, a framework is proposed for incorporating financial considerations into multi-period inventory models. More specifically, delayed cash payment, a common existing phenomenon in industry, is studied by introducing two financial states of firms: capital on hand and receivable. Then we considered the simple one-retailer and one-item case, where the firm's operational decisions will be constrained by limited capital.

For the model, the optimal inventory policy is derived, which turns out to be a capital dependent base stock policy. Furthermore, we studied the influences of capital on hand and receivable on the optimal achievable profit and find that reducing the ratio of receivable in the total wealth will increase the profit.

In addition, traditional inventory model without consideration of delayed cash payment is used to compare with our model. It allows us to show that without offering delayed cash payment, the retailer will order higher level quantity and obtain higher level profit.

Finally computational results demonstrate the dependences of delayed cash payment on the retailer's optimal operational policy and achievable profit. It validates the essentiality of retailer considering the influence of offering delayed cash payment.

We find that for most of these models, the results in Lemmas 5.3.1, 5.3.2 and 5.3.3 and Theorem 5.3.5 in Chap. 5 can still be obtained. However, more precise structure of the optimal control policies beyond the "capital-dependent base-stock", such as Theorem 5.3.9 in Chap. 5, is difficult to obtain without imposing further structure in the model. These are just a few possible extensions and it appears that each of these variations will lead to different optimal solution structure that is worthy of study.

Consequently a straightforward extension to our model is to consider the trade-off between cost of offering delayed cash payment and benefits from demand increase.

Chapter 7
Game Analysis in Negotiation of Iron Ore Price

7.1 Introduction

The international iron ore market determines prices through yearly negotiations, using certain long-term trade agreements as its main price-setting mechanism. According to convention, the new fiscal year's iron ore prices are decided before April of every year. During the process, the largest iron and steel enterprises, acting as industry representatives, negotiate with iron ore suppliers to form the basic prices for European and Asian importers. Australia's BHP Billiton Ltd, Rio Tinto Group, and Brazil's Companhia Vale do Rio Doce are the three major suppliers of iron ore across the world. While for a long time, Japan sets the standard for Asia.

The long-term supply contracts and yearly price determination enable the industry to maintain relative stability, and allow both suppliers and buyers to make rational decisions. However, Chinese mills and iron ore traders had to accept in 2005 a 71.5% rise in iron ore price, which was set by Japanese companies. Since it means $3 billion more to pay for imports, China's iron and steel industry has deeply understood the importance of the "price-decision right".

Further in 2006, at the beginning of the negotiations, enterprises from Europe, Japan and South Korea all sought a reduction in iron ore prices. However, China did not grasp the chance to form alliances with other countries and lost the opportunity to set up its own "price-decision right". On the contrary, those suppliers grasped the chance and put China in a plight. Noticing that most European steel plants use iron ore pellets, suppliers took the lead in making an agreement that the price of iron ore shavings would increase 19% while the price of iron ore pellets would decrease 3%. It finally left China in a passive situation.

Iron ore prices have a significant influence on China's steel industry, which is currently faced with soaring production costs and declining profits. Continuous jump in prices of iron ore will put an even larger strain on the industry, and eventually reduce the growth of the national economy.

To pursue the reason for China's failure in iron ore price negotiation in the past 2 years, it is worth noting that many foreign iron and steel companies have

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their own mines. Mittal Steel, the world's largest iron and steel enterprise, for example, owns a sufficient number of mines to meet its raw material needs, forming a vertically integrated production system. Japanese enterprises have invested in all 24 major mines in Australia so that the rise in iron ore prices has no influence on Japanese iron and steel enterprises by and large. Although China's Baosteel Group and Shougang Group have also participated in exploiting iron mines in Australia, Mongolia, Vietnam, South America and some countries in Africa, this is not enough for Chinese iron and steel enterprises to have a strong say in the negotiation.

Another factor undermining China's negotiating basis is that importers are not united. Because of the large number of steel-producing enterprises and the reform of the foreign trade system, the number of iron ore importing enterprises in China has sharply increased, reaching 523 in 2004 from 173 in 2001. However, the three major suppliers (Brazil's Companhia Vale do Rio Doce, Australia's BHP Billiton and Rio Tinto) control 70–80% of global iron ore production and ocean shipping trade. In this situation the buyers are overly decentralized and sellers relatively centralized, is unfavorable to China's iron and steel enterprises.

During the negotiation of iron ore price for 2007, the consumers seem to be beating the same drum as previous years, obviously. Prices have gone up enough over the last 2 years, and the consumers believed that they should have a stronger say in the price negotiations. Their stance is summed up by the commentary below, taken from a Chinese business website (Ministry of Commerce of P.R. China).

> *The world top three iron ore providers declined to comment on the iron prices next year, only saying that they are optimistic about China's steel market and iron ore imports. They also predicted a gap between demand and supply in China. Lu Jianhua, Director of China's Ministry of Commerce's Foreign Trade Department, did not think imported iron ore prices would continue to rise, saying that past four straight years of increase had made mines gain huge profits but left steel businesses earning little or even suffering losses. "It is not in the interests of the two sides of demand and supply," he said.*

As the biggest buyer of iron ore in the world, China has the capability and right to influence iron ore prices. Figures from the customs show that in 2005, China imported iron ore of 275 million tons, up 32.3% year-on-year and accounting for 43% of the world's total ore shipment.

Under this background, this chapter seeks to provide some insights for the negotiation of iron ore price. We establish mathematical and economical models to answer following questions: Why Japanese steel producers accepted a price increase of 71.5% in 2005; what inspiration can we expose for China's steel manufactures to have a stronger say in the future bargaining.

Many solution concepts and methods have been established and employed to find solutions for negotiation. According to Nash (1950), the axiomatic approach requires the solution to satisfy a certain set of axioms. Besides the Nash bargaining solution proposed in Nash (1950), the most common axiomatic solutions are then the non-symmetric Nash solution by Harsanyi and Selten (1972), the Kalai–Smorodinsky solution by Kalai and Smorodinsky (1975), the reference function solution by Anbarci (1995), the egalitarian solution by Kalai (1977), and so on. Finally, it is recommended that Thomson (1994) made comprehensive review of cooperative models on bargaining.

According to the mechanism of iron ore price negotiation, we extend the Nash bargaining framework to a Nash game between two bargaining groups. Specifically, each group including two players negotiates under the Nash bargaining rule, and the disagreement payoff is the payoff under the price which may be stricken by the other group.

The rest of this chapter is organized as follows. Section 7.2 presents notations and assumptions for the use throughout this study. In Sect. 7.3, we establish an economic game model and derive the optimal prices for each manufacturer to obtain his maximum revenue respectively. Section 7.4 proposes a Nash bargaining-based game model to derive the equilibrium price, i.e., the negotiated price. Some numerical examples are provided in Sect. 7.5 for illustration of the theoretical results. Finally we conclude the chapter and suggest some future directions in Sect. 7.6.

7.2 Notation and Assumptions

Based on whether to invest on iron ore mines or not, without loss of generality we assume only two types of steel producers in the market. One is the producer who has no investment on the iron ore, such as China, denoted by manufacturer 1. The other type of producer who has certain investment on the iron ore, such as Japan, denoted by manufacturer 2. Each manufacturer has domestic market for its steel sales. Also they compete in the international steel market. For simplicity we assume only one supplier. Then the two steel manufacturers procure iron ore from the supplier to produce steel. The final iron ore price is determined via simultaneous negotiations between the supplier and each manufacturer.

In the international steel market, let p_0 be the steel selling price, and M_0 be the market size. Then according to economic theories the demand of steel q_0 can be given by

$$q_0 = M_0 - \beta p_0,$$

where β is the influence of fluctuation of price on demand. Similarly for the domestic markets of manufacturers 1 and 2, the demand curve can be given by

$$q_1 = M_1 - \beta p_1$$

and

$$q_2 = M_2 - \beta p_2.$$

The rest definitions of the notation are presented below.

c_0 unit variable production cost of iron ore
w unit wholesale price of iron ore

α the production ratio from iron ore to steel, $\alpha \geq 1$
μ the investment ratio of manufacturer 2 on the iron ore mines

Since in this chapter only two manufacturers are assumed to compete in the international steel market, then $q_0 = q_{10} + q_{20}$, where q_{10} and q_{20} are the international steel supply of manufacturers 1 and 2 respectively. Therefore, the response functions of steel price on production quantities in the different markets can be written as

$$p_0(q_{10}, q_{20}) = (M_0 - q_{10} - q_{20})/\beta,$$
$$p_1(q_1) = (M_1 - q_1)/\beta,$$
$$p_2(q_2) = (M_2 - q_2)/\beta.$$

Further to make sure that the manufacturers as well as the supplier can obtain positive profit, the following conditions need be satisfied

$$w \geq c_0,$$
$$p_0, p_1, p_2 \geq \alpha w,$$

and hence

$$M_i \geq \alpha\beta w, \quad i = 0, 1, 2. \tag{7.1}$$

In addition, we assume that the steel production quantities of the two manufacturers are nonnegative. And noting that when $\mu \geq 1/2$, manufacturer 2 will act as a supplier. Then to avoid trivial cases, we assume $\mu < 1/2$.

7.3 Economic Model of the Supply Chain

In this section, a quantity game model is established to derive the optimal quantities of each manufacturer under any given wholesale price w. Then under the optimal production quantity, the optimal wholesale prices are derived for the three parties to obtain their maximum revenue, respectively.

7.3.1 The Optimal Production Quantity

Since manufacturer 1 is assumed to have no investment on iron ore, its revenue function is as follows for any given q_{20} and w:

$$\Pi_1(q_1, q_{10}, q_{20}, w) = q_1(M_1 - q_1)/\beta + q_{10}(M_0 - q_{10} - q_{20})/\beta - \alpha w(q_1 + q_{10}).$$

Manufacturer 2 has μ ratio of investment on iron ore. Thus its revenue contains another part from sales of materials. That is,

$$\Pi_2(q_2, q_{20}, q_{10}, w) = q_2(M_2 - q_2)/\beta + q_{20}(M_0 - q_{20} - q_{10})/\beta - \alpha w(q_2 + q_{20})$$
$$+\mu\alpha(w - c_0)(q_1 + q_2 + q_{10} + q_{20}).$$

For any given wholesale price w, manufactures 1 and 2 will choose their production quantity for both international and domestic markets simultaneously to maximum their respective revenue. Straightforwardly notice that the optimal production quantity for domestic market can be given by

$$q_1^* = \frac{M_1 - \alpha\beta w}{2}$$

and

$$q_2^* = \frac{M_2 - (1-\mu)\alpha\beta w - \mu\alpha\beta c_0}{2}.$$

Notice that following from (7.1) we find that q_1^* and q_2^* are both positive. Then the revenue functions can be rewritten as

$$\Pi_1(q_{10}, q_{20}, w) = \frac{(M_1 - \alpha\beta w)^2}{4\beta} + q_{10}(M_0 - q_{10} - q_{20})/\beta - \alpha w q_{10}$$

and

$$\Pi_2(q_{20}, q_{10}, w) = \frac{(M_2 - (1-\mu)\alpha\beta w - \mu\alpha\beta c_0)^2}{4\beta} + \frac{\mu\alpha(w - c_0)(M_1 - \alpha\beta w)}{2}$$
$$+ q_{20}(M_0 - q_{20} - q_{10})/\beta - \alpha w q_{20} + \mu\alpha(w - c_0)(q_{10} + q_{20}).$$

Thus the competition in the international steel market can be described as the following problem.

$$\begin{cases} \max_{q_{10}\geq 0} \Pi_1(q_{10}, q_{20}, w), \\ \max_{q_{20}\geq 0} \Pi_2(q_{20}, q_{10}, w). \end{cases} \tag{7.2}$$

Here the game between manufacturer 1 and manufacturer 2 is the quantity game. According to traditional game theory, we have the following theorem.

Theorem 7.3.1. *There is a unique pure-strategy Nash equilibrium (hereafter NE) (q_1^*, q_2^*) in the game between the two manufacturers. Specifically, the solution for the equilibrium, i.e., the production quantities of the two manufacturers are as follows:*

$$q_{10}^*(w) = \frac{M_0 - (1+\mu)\alpha\beta w + \mu\alpha\beta c_0}{3} \tag{7.3}$$

and

$$q_{20}^*(w) = \frac{M_0 - (1-2\mu)\alpha\beta w - 2\mu\alpha\beta c_0}{3}. \tag{7.4}$$

Furthermore, there is $q_{20}^*(w) \geq q_{10}^*(w)$.

Proof. First we prove the existence of the NE of the game model (7.2). It is straightforward by noting that the payoff function $\Pi_1(q_{10}, q_{20}, w)$ is concave in q_{10} and $\Pi_2(q_{20}, q_{10}, w)$ is concave in q_{20}.

Next we prove the uniqueness of NE. Given the price of iron ore and the production quantity of manufacturer 2, the optimal production quantity of manufacturer 1 can be obtained as follows:

$$\frac{\partial \Pi_1(q_{10}, q_{20}, w)}{\partial q_{10}} = 0 \Rightarrow q_{10}(q_{20}, w) = \frac{M_0 - q_{20} - \alpha\beta w}{2}.$$

And in the same way we can get the optimal production quantity of manufacturer 2.

$$\frac{\partial \Pi_2(q_{20}, q_{10}, w)}{\partial q_{20}} = 0 \Rightarrow q_{20}(q_{10}, w) = \frac{M_0 - q_{10} - (1-\mu)\alpha\beta w - \mu\alpha\beta c_0}{2}.$$

Since

$$\left|\frac{\partial q_{10}(q_{20}, w)}{\partial q_{20}}\right| = \frac{1}{2}$$

and

$$\left|\frac{\partial q_{20}(q_{10}, w)}{\partial q_{10}}\right| = \frac{1}{2},$$

the best response mapping is a contraction. Therefore, the NE is unique.

Furthermore, let $q_{10} = q_{10}(q_{20}(q_{10}, w), w)$ and $q_{20} = q_{20}(q_{10}(q_{20}, w), w)$, we then derive the unique NE solution of the game as follows:

$$q_{10}^*(w) = \frac{M_0 - (1+\mu)\alpha\beta w + \mu\alpha\beta c_0}{3}$$

and

$$q_{20}^*(w) = \frac{M_0 - (1-2\mu)\alpha\beta w - 2\mu\alpha\beta c_0}{3}.$$

Finally note that $q_{20}^*(w) - q_{10}^*(w) = \mu\alpha\beta(w - c_0) \geq 0$. Thus we have $q_{20}^*(w) \geq q_{10}^*(w)$. □

Let $\hat{w}_1$, $\hat{w}_2$, $\hat{w}_{10}$ and $\hat{w}_{20}$ be respectively the iron ore price which make the corresponding optimal ordering quantity equal to zero. We find that

$$\hat{w}_1 = \frac{M_1}{\alpha\beta},\ \hat{w}_2 = \frac{M_2 - \mu\alpha\beta c_0}{(1-\mu)\alpha\beta},\ \hat{w}_{10} = \frac{M_0 + \mu\alpha\beta c_0}{(1+\mu)\alpha\beta},\ \hat{w}_{20} = \frac{M_0 - 2\mu\alpha\beta c_0}{(1-2\mu)\alpha\beta}.$$

They reflect the maximal acceptable iron ore price for each manufacturer in both their domestic market and international market.

Furthermore, we find that the maximal acceptable iron ore price of each manufacturer depends on the investment ratio μ in following ways.

Remark 7.3.2. $\hat{w}_2$ and $\hat{w}_{20}$ are increasing in μ, while $\hat{w}_{10}$ is decreasing in μ.

The following assumption is given in terms of the survey of industrial practice.

Assumption 7.3.3. *There exists $\hat{\mu}_1$ and $\hat{\mu}_2$ such that $\hat{w}_1 = \hat{w}_{10}|_{\mu=\hat{\mu}_1}$ and $\hat{w}_2 = \hat{w}_{20}|_{\mu=\hat{\mu}_2}$, and furthermore, the following inequalities are satisfied:*

$$\begin{cases} \hat{w}_1 \le \hat{w}_{10}, & 0 \le \mu \le \hat{\mu}_1, \\ \hat{w}_1 > \hat{w}_{10}, & \hat{\mu}_1 < \mu < 1/2. \end{cases}$$

and

$$\begin{cases} \hat{w}_2 \ge \hat{w}_{20}, & 0 \le \mu \le \hat{\mu}_2, \\ \hat{w}_2 < \hat{w}_{20}, & \hat{\mu}_2 < \mu < 1/2. \end{cases}$$

The following lemma is then proposed straightforward from Assumption 1.

Lemma 7.3.4. *There are $M_1 \le M_0 \le M_2$, $3M_1 > 2M_0 + \alpha\beta c_0$, and furthermore, $\hat{\mu}_1 = \frac{M_0 - M_1}{M_1 - \alpha\beta c_0}$ and $\hat{\mu}_2 = \frac{M_2 - M_0}{2M_2 - M_0 - \alpha\beta c_0}$.*

The proof for $M_1 \le M_0 \le M_2$ follows by examining the case of $\mu = 0$ from Assumption 1. And $3M_1 > 2M_0 + \alpha\beta c_0$ follows from $\hat{w}_1 > \hat{w}_{10}$ with the case of $\mu = 1/2$ in Assumption 1.

Remark 7.3.2 shows that:

(i) Manufacturer 2 can accept higher iron ore price in both international and domestic markets when μ increases
(ii) The increase of μ by manufacturer 1 will not affect manufacturer 1's maximal acceptable price in the domestic market, but decreases the price in international market

In the following analysis to assure that the production quantity is positive, some boundary conditions can easily be obtained:

$$w \le \min\{\hat{w}_1, \hat{w}_{10}\}. \tag{7.5}$$

7.3.2 The Optimal Wholesale Price

Noting down that with the optimal ordering quantity $q_{10}^*(w)$ and $q_{20}^*(w)$, the revenue functions of the two manufacturers are given by

$$\Pi_1(w) = \frac{(M_1 - \alpha\beta w)^2}{4\beta} + \frac{[M_0 + \mu\alpha\beta c_0 - (1+\mu)\alpha\beta w]^2}{9\beta} \tag{7.6}$$

and

$$\Pi_2(w) = \frac{(M_2 - (1-\mu)\alpha\beta w - \mu\alpha\beta c_0)^2}{4\beta} + \frac{(M_0 - (1-2\mu)\alpha\beta w - 2\mu\alpha\beta c_0)^2}{9\beta} + \frac{\mu\alpha(w - c_0)(M_1 - \alpha\beta w)}{2} + \frac{\mu\alpha(w-c_0)(M_0 - (1+\mu)\alpha\beta w + \mu\alpha\beta c_0)}{3}. \tag{7.7}$$

Note that the revenue function of the supplier is

$$\Pi_s(w, q_1, q_2, q_{10}, q_{20}) = (1-\mu)\alpha(w - c_0)(q_1 + q_2 + q_{10} + q_{20}). \tag{7.8}$$

Then the revenue function under the equilibrium quantity is

$$\Pi_s(w) = \frac{(1-\mu)\alpha(w-c_0)}{6}[3M_1 + 3M_2 + 4M_0 - 5\mu\alpha\beta c_0 - 5(2-\mu)\alpha\beta w].$$

To understand exactly the behavior of supplier and manufacturers in the negotiation, it is important to know what wholesale prices can satisfy them, or in other words, what wholesale price do they expect the most. Next based on the equilibrium production quantities presented in Theorem 7.3.1, we derive the optimal iron ore prices for the two manufacturers as well as the supplier respectively. The proof can be found in Appendix.

Theorem 7.3.5. *The optimal iron ore prices for the two manufacturers are given as follows:*

(a) For manufacturer 1, there is $w_1^* = c_0$.
(b) For manufacturer 2, if $3M_1 - 3M_2 + 2M_0 - 2\alpha\beta c_0 > 0$, *then*

$$w_2^* = \begin{cases} c_0, & 0 \le \mu \le \dfrac{9M_2 + 4M_0 - 13\alpha\beta c_0}{9M_1 + 9M_2 + 14M_0 - 32\alpha\beta c_0}, \\ \dfrac{9\mu M_1 - 9(1-\mu)M_2 + 2(7\mu - 2)M_0 + \mu(32 - 13\mu)\alpha\beta c_0}{(-13\mu^2 + 64\mu - 13)\alpha\beta}, & \dfrac{9M_2 + 4M_0 - 13\alpha\beta c_0}{9M_1 + 9M_2 + 14M_0 - 32\alpha\beta c_0} < \mu < \dfrac{1}{2}; \end{cases}$$

and if $3M_1 - 3M_2 + 2M_0 - 2\alpha\beta c_0 \le 0$, *then* $w_2^* = c_0$.

Theorem 7.3.5 allows us to show that in terms of the investment ratio μ, there are two cases.

(i) Without investment on materials, the lower price of iron ore is, the better for manufacturer 1. So it is the case for manufacturer 2 when its investment ratio μ is less than the threshold $\frac{9M_2+4M_0-13\alpha\beta c_0}{9M_1+9M_2+14M_0-32\alpha\beta c_0}$.

(ii) However, once the investment ratio is beyond the threshold, the above case for manufacturer 2 will not suit again. And there exists an optimal positive price of iron ore for manufacturer 2. But the optimal price for manufacturer 2 is always less than that for the supplier.

The following remark characterizes how the optimal prices depend on the investment ratio μ as well as the size of each market.

Remark 7.3.6. (a) w_2^* is increasing in μ. (b) w_2^* is increasing in M_1 and M_0, while decreasing in M_2.

Proof. For (a), notice that

$$\begin{aligned}
\frac{\partial w_2^*(\mu)}{\mu} &= \frac{1}{(-13\mu^2+64\mu-13)^2\alpha\beta}\{[9M_1+9M_2+14M_0 \\
&\quad +(32-26\mu)\alpha\beta c_0](-13\mu^2+64\mu-13)-[9\mu M_1-9(1-\mu)M_2 \\
&\quad +2(7\mu-2)M_0+\mu(32-13\mu)\alpha\beta c_0](-26\mu+64)\} \\
&= \frac{1}{(-13\mu^2+64\mu-13)^2\alpha\beta}[9(13\mu^2-13)M_1+9(13\mu^2-26\mu+51)M_2 \\
&\quad +2(91\mu^2-52\mu+37)M_0-26(16\mu^2-13\mu+16)\alpha\beta c_0] \\
&\geq \frac{26(16\mu^2-13\mu+16)(M_0-\alpha\beta c_0)}{(-13\mu^2+64\mu-13)^2\alpha^2\beta^2} \\
&\geq 0,
\end{aligned}$$

where the first inequality follows from Lemma 7.3.4 by noting that $13\mu^2 - 26\mu + 51 > 0$ when $0 < \mu < 1/2$, and the second inequality follows from $16\mu^2 - 13\mu + 16 > 0$. We find that w_2^* is increasing in μ.

For (b), we first prove the dependence of w_2^* on M_1. By noting that $w_2^* = c_0$ when $M_1 \leq (3M_2 - 2M_0 + 2\alpha\beta c_0)/3$ and $w_2^* \geq c_0$ is increasing in M_1 when $M_1 > (3M_2 - 2M_0 + 2\alpha\beta c_0)/3$, we find that w_2^* is increasing in M_1. The proof for the dependence on M_2 and M_0 is similar. □

Note that when the market size M_1 or M_0 is low, or M_2 is high such that $3M_1 - 3M_2 + 2M_0 - 2\alpha\beta c_0 \leq 0$, the manufacturer with investment on iron ore prefers the same wholesale price to the other manufacturer. In other words, the two manufacturers will have the same preference in the iron ore price negotiation.

Theorem 7.3.7. *If $2M_0 \geq M_1 + M_2$, then the optimal iron ore price for the supplier is*

$$w_0^* = \frac{3M_1 + 3M_2 + 4M_0 + 10(1-\mu)\alpha\beta c_0}{10(2-\mu)\alpha\beta}.$$

And if $2M_0 < M_1 + M_2$, then the optimal iron ore price for the supplier is

$$w_0^* = \begin{cases} \frac{3M_1 + 3M_2 + 4M_0 + 10(1-\mu)\alpha\beta c_0}{10(2-\mu)\alpha\beta}, & 0 \leq \mu \leq \min\left\{\frac{-17M_1 + 3M_2 + 4M_0 + 10\alpha\beta c_0}{10M_1 - 10\alpha\beta c_0}, \frac{-3M_1 - 3M_2 + 16M_0 - 10\alpha\beta c_0}{3M_1 + 3M_2 + 14M_0 - 20\alpha\beta c_0}\right\}, \\ \min\{\hat{w}_1, \hat{w}_{10}\}, & \min\left\{\frac{-17M_1 + 3M_2 + 4M_0 + 10\alpha\beta c_0}{10M_1 - 10\alpha\beta c_0}, \frac{-3M_1 - 3M_2 + 16M_0 - 10\alpha\beta c_0}{3M_1 + 3M_2 + 14M_0 - 20\alpha\beta c_0}\right\} < \mu < \frac{1}{2}. \end{cases}$$

Proof. Taking derivative of $\Pi_0(w)$ on w yields

$$\begin{aligned} \frac{\partial \Pi_s(w)}{\partial w} &= -\frac{5(1-\mu)(2-\mu)\alpha^2\beta}{3} w \\ &\quad + \frac{(1-\mu)\alpha[3M_1 + 3M_2 + 4M_0 + 10(1-\mu)\alpha\beta c_0]}{6}. \end{aligned}$$

Clearly $\Pi_s(w)$ is concave in w and the unconstrained maximum point is

$$\frac{3M_1 + 3M_2 + 4M_0 + 10(1-\mu)\alpha\beta c_0}{10(2-\mu)\alpha\beta}.$$

If $2M_0 \geq M_1 + M_2$, then $\frac{-3M_1-3M_2+16M_0-10\alpha\beta c_0}{3M_1+3M_2+14M_0-20\alpha\beta c_0} \geq \frac{1}{2}$. Thus when $0 \leq \mu < \frac{1}{2} \leq \frac{-3M_1-3M_2+16M_0-10\alpha\beta c_0}{3M_1+3M_2+14M_0-20\alpha\beta c_0}$, we prove

$$c_0 \leq \frac{3M_1 + 3M_2 + 4M_0 + 10(1-\mu)\alpha\beta c_0}{10(2-\mu)\alpha\beta} \leq \min\{\hat{w}_1, \hat{w}_{10}\}$$

by

$$\begin{aligned} &[3M_1 + 3M_2 + 4M_0 + 10(1-\mu)\alpha\beta c_0](1+\mu) - 10(2-\mu)(M_0 + \mu\alpha\beta c_0) \\ &\quad = 3(1+\mu)M_1 + 3(1+\mu)M_2 + (14\mu - 16)M_0 + 10(1-2\mu)\alpha\beta c_0 \\ &\quad \leq 0 \end{aligned}$$

and

$$\begin{aligned} &3M_1 + 3M_2 + 4M_0 + 10(1-\mu)\alpha\beta c_0 - 10(2-\mu)M_1 \\ &\quad = (10\mu - 17)M_1 + 3M_2 + 4M_0 + 10(1-\mu)\alpha\beta c_0 \\ &\quad \leq (10\mu - 17)M_1 + (6M_0 - 3M_1) + 4M_0 + 10(1-\mu)\alpha\beta c_0 \\ &\quad \leq (10\mu - 20)M_1 + (15M_1 - 5\alpha\beta c_0) + 10(1-\mu)\alpha\beta c_0 \\ &\quad = 5(1-2\mu)(\alpha\beta c_0 - M_1) \\ &\quad \leq 0, \end{aligned}$$

where the first inequality follows from $2M_0 \geq M_1 + M_2$, and the second inequality follows from Lemma 7.3.4. Therefore, the optimal price for the supplier is $w_0^* = \frac{3M_1+3M_2+4M_0+10(1-\mu)\alpha\beta c_0}{10(2-\mu)\alpha\beta}$.

If $2M_0 < M_1 + M_2$, then $\frac{-3M_1-3M_2+16M_0-10\alpha\beta c_0}{3M_1+3M_2+14M_0-20\alpha\beta c_0} < \frac{1}{2}$. Next when

$$0 \leq \mu \leq \min\left\{\frac{-17M_1+3M_2+4M_0+10\alpha\beta c_0}{10M_1-10\alpha\beta c_0}, \frac{-3M_1-3M_2+16M_0-10\alpha\beta c_0}{3M_1+3M_2+14M_0-20\alpha\beta c_0}\right\},$$

similarly we find that $\frac{3M_1+3M_2+4M_0+10(1-\mu)\alpha\beta c_0}{10(2-\mu)\alpha\beta} \leq \min\{\hat{w}_1, \hat{w}_{10}\}$. Thus the optimal price for the supplier is $w_0^* = \frac{3M_1+3M_2+4M_0+10(1-\mu)\alpha\beta c_0}{10(2-\mu)\alpha\beta}$. When

$$\min\left\{\frac{-17M_1+3M_2+4M_0+10\alpha\beta c_0}{10M_1-10\alpha\beta c_0}, \frac{-3M_1-3M_2+16M_0-10\alpha\beta c_0}{3M_1+3M_2+14M_0-20\alpha\beta c_0}\right\} < \mu < \frac{1}{2},$$

we find that $\frac{3M_1+3M_2+4M_0+10(1-\mu)\alpha\beta c_0}{10(2-\mu)\alpha\beta} > \min\{\hat{w}_1, \hat{w}_{10}\}$. Thus the revenue function is increasing in w when $c_0 \leq w \leq \min\{\hat{w}_1, \hat{w}_{10}\}$, and the optimal price for the supplier is $w_0^* = \min\{\hat{w}_1, \hat{w}_{10}\}$. □

Remark 7.3.8. (a) If $2M_0 \geq M_1 + M_2$, then w_0^* are increasing in μ; if $2M_0 < M_1 + M_2$, then w_0^* is quasi-concave in μ. (b) w_0^* is increasing in M_1, M_2 and M_0.

Proof. For (a), if $2M_0 \geq M_1 + M_2$, note that

$$\begin{aligned}\frac{\partial w_0^*(\mu)}{\mu} &= \frac{-10\alpha\beta c_0(2-\mu) - [3M_1+3M_2+4M_0+10(1-\mu)\alpha\beta c_0](-1)}{10(2-\mu)^2\alpha\beta} \\ &= \frac{3M_1+3M_2+4M_0-10\alpha\beta c_0}{10(2-\mu)^2\alpha\beta} \\ &\geq 0.\end{aligned}$$

We find that w_0^* is increasing in μ. If $2M_0 < M_1 + M_2$, when $\mu > \frac{-3M_1-3M_2+16M_0-10\alpha\beta c_0}{3M_1+3M_2+14M_0-20\alpha\beta c_0}$, $w_0^* = \hat{w}_{10}$ is decreasing in μ. Thus w_0^* is quasi-concave in μ.

The proof for (b) is straightforward by noting that $\frac{3M_1+3M_2+4M_0+10(1-\mu)\alpha\beta c_0}{10(2-\mu)\alpha\beta}$ is increasing in M_1, M_2 and M_0, $\hat{w}_1$ is increasing in M_1, and $\hat{w}_{10}$ is increasing in M_0. □

Theorem 7.3.9. *There is $w_1^* \leq w_2^* < w_0^*$.*

Proof. First we prove

$$\begin{aligned}&\frac{9\mu M_1 - 9(1-\mu)M_2 + 2(7\mu-2)M_0 + \mu(32-13\mu)\alpha\beta c_0}{(-13\mu^2+64\mu-13)\alpha\beta} \\ &\leq \frac{3M_1+3M_2+4M_0+10(1-\mu)\alpha\beta c_0}{10(2-\mu)\alpha\beta}\end{aligned}$$

by noting that

$$
\begin{aligned}
&10(2-\mu)[9\mu M_1 - 9(1-\mu)M_2 + 2(7\mu-2)M_0 + \mu(32-13\mu)\alpha\beta c_0] \\
&\quad -(-13\mu^2+64\mu-13)[3M_1+3M_2+4M_0+10(1-\mu)\alpha\beta c_0] \\
&= (-51\mu^2-12\mu+39)M_1 + (-51\mu^2+78\mu-141)M_2 \\
&\quad +(-88\mu^2+64\mu-28)M_0 + (190\mu^2-130\mu+130)\alpha\beta c_0 \\
&\le (190\mu^2-130\mu+130)(\alpha\beta c_0 - M_0) \\
&\le 0,
\end{aligned}
$$

where the first inequality follows from Lemma 7.3.4 by noting that $-51\mu^2 - 12\mu + 39 \ge 0$ and $-51\mu^2 + 78\mu - 141 \le 0$ when $0 \le \mu < 1/2$.

Further from (7.10), if $\mu > \frac{9M_2+4M_0-13\alpha\beta c_0}{9M_1+9M_2+14M_0-32\alpha\beta c_0}$, then

$$
\frac{9\mu M_1 - 9(1-\mu)M_2 + 2(7\mu-2)M_0 + \mu(32-13\mu)\alpha\beta c_0}{(-13\mu^2+64\mu-13)\alpha\beta} \le \min\{\hat{w}_1, \hat{w}_{10}\}.
$$

Finally since $w_0^* \ge c_0$, we find that $w_2^* \le w_0^*$. And furthermore, there is $w_1^* \le w_2^* \le w_0^*$. □

In Sect. 7.4 we will present the main result of this work, i.e., the wholesale price negotiation. Notice that $2M_0 \ge M_1 + M_2$ implies $3M_1 - 3M_2 + 2M_0 - 2\alpha\beta c_0 > 0$ in terms of Lemma 7.3.4. Without loss of generality, we only consider the case $2M_0 \ge M_1 + M2$ in Sect. 7.4.

7.4 Wholesale Price Negotiation

Since the supplier and manufacturers have different interest on the iron ore price, they have to negotiate the final wholesale price. In this section we will characterize the bargaining process in the negotiation of iron ore price. The rule of negotiation follows from iterative procedure. In the first round of negotiation, the supplier first negotiates with one manufacturer. If both of the two players agree with certain price, then the price will be the final iron ore price. If no price is achieved, then the supplier will negotiate with the other manufacturer. If any price is achieved, then the bargaining is over and the price will be used as the final iron ore price. If the negotiation fails, then the supplier has to negotiate with the first manufacturer again and starts the second round of negotiation. The negotiation will go on following this procedure. Specifically, we assume that there are N rounds of negotiation.

We apply Nash bargaining game to characterize the negotiation between the supplier and any manufacturer. The Nash bargaining game requires us to identify a feasible set of payoffs and a disagreement point. Each player's payoff is the profit. And note that if the negotiation fails, then the final iron ore price can be the negotiated price of the following bargaining. Thus we assume the disagreement payoff allocation to be defined as the profit under the negotiated price got from the following negotiation. For example, for the bargaining between the supplier and manufacturer 1, the disagreement payoff for the two players are $r_1(w_2)$ and $r_0(w_2)$ respectively, where w_2 is the negotiated price during the following bargaining between the supplier and manufacturer 2. However, during the last round of negotiation, if still no price can be achieved, then the disagreement payoff is 0 for any participant.

Thus for any given wholesale price w as the disagreement price, the objective function of the Nash bargaining between manufacturer 1 and the supplier is given as follows.

$$\Pi_{1s}(w_1|w) = [\Pi_1(w_1) - \Pi_1(w)][\Pi_s(w_1) - \Pi_s(w)].$$

Note that $\Pi_1(w_1) \geq \Pi_1(w)$ and $\Pi_s(w_1) \geq \Pi_s(w)$, otherwise there is no need for the negotiation. In addition, recall that w_1^*, w_2^* and w_0^* are the optimal prices for the two manufacturer and the supplier respectively. In terms of the discussion in Theorem 7.3.5, the final negotiated price w_1^{NB} and w_2^{NB} should satisfy $w_1^* \leq w_1^{\text{NB}} \leq w_0^*$ and $w_2^* \leq w_2^{\text{NB}} \leq w_0^*$ such that the results would make sense. Let $\Omega_1(w_1|w)$ be the feasible set for the negotiation with disagreement price w, then $\Omega_1(w_1|w) = \{w_1|\Pi_1(w_1) \geq \Pi_1(w), \Pi_s(w_1) \geq \Pi_s(w), w_1^* \leq w_1^{\text{NB}} \leq w_0^*\}$.

Similarly, given w the objective function of the Nash bargaining between manufacturer 2 and the supplier is proposed as follows:

$$\Pi_{2s}(w_2|w) = [\Pi_2(w_2) - \Pi_2(w)][\Pi_s(w_2) - \Pi_s(w)].$$

And the feasible set is defined as $\Omega_2(w_2|w) = \{w_2|\Pi_2(w_2) \geq \Pi_2(w), \Pi_s(w_2) \geq \Pi_s(w), w_2^* \leq w_2^{\text{NB}} \leq w_0^*\}$.

Furthermore, in the last bargaining, if no agreement is achieved, then both the bargainer can earn nothing. Thus the objective function is

$$\Pi_{is}(w) = \Pi_i(w)\Pi_s(w).$$

Next we derive the Nash bargaining solutions backwardly, i.e., we first study the Nash bargaining of the last round of negotiation. There are two kinds of sequence for the negotiation process, with one starting from the negotiation between the supplier and manufacturer 1 and the other starting from the negotiation between manufacturer 2 and the supplier.

7.4.1 The Manufacturer Without Investment First Negotiates

If manufacturer 1 first joins the negotiation with the supplier, then in the last round of negotiation, the last bargaining is between the supplier and manufacturer 2. The optimization problem can be formulated as

$$\max_{w_2^* \le w \le w_0^*} \Pi_{2s}(w) = \max_{w_2^* \le w \le w_0^*} \Pi_2(w)\Pi_s(w).$$

Theorem 7.4.1. *In the last round of negotiation and the last bargaining between the supplier and manufacturer 2, there is unique Nash bargaining solution* $w_2^{\rm NB}$ *given by the first order condition*

$$\frac{\partial \Pi_{2s}(w_2^{\rm NB})}{\partial w} = \Pi_2'(w_2^{\rm NB})\Pi_s(w_2^{\rm NB}) + \Pi_s'(w_2^{\rm NB})\Pi_2(w_2^{\rm NB}) = 0,$$

where $w_2^* \le w_2^{\rm NB} \le w_0^*$. *Specifically,*

(i) If $0 \le \mu \le \frac{32-3\sqrt{95}}{13}$, *then* $w_2^{\rm NB}$ *is the smallest real root.*
(ii) If $\frac{32-3\sqrt{95}}{13} < \mu < \frac{1}{2}$, *then* $w_2^{\rm NB}$ *is the middle real root.*

Proof. Taking derivatives of $f_2(w)$ on w yields

$$\frac{\partial \Pi_{2s}(w)}{\partial w} = \Pi_2'(w)\Pi_s(w) + \Pi_s'(w)\Pi_2(w)$$
$$\frac{\partial^2 \Pi_{2s}(w)}{\partial w^2} = \Pi_2''(w)\Pi_s(w) + \Pi_s''(w)\Pi_2(w) + 2\Pi_2'(w)\Pi_s'(w).$$

Quadratic function $\Pi_s(w) \ge 0$ is increasing and concave in w when $w_2^* \le w \le w_0^*$, and has two zero point c_0 and $\frac{3M_1+3M_2+4M_0-5\mu\alpha\beta c_0}{5(2-\mu)\alpha\beta c_0}$. Further it is straightforward to show that $\Pi_2(w)$ also has two zero points. Thus there are four zero points for quartic function $\Pi_{2s}(w)$. And the first order condition $\frac{\partial \Pi_{2s}(w)}{\partial w} = 0$ yields three real roots. Next note that the properties of manufacturer 2's profit function depends on the investment ratio μ.

If $0 \le \mu < \frac{32-3\sqrt{95}}{13}$, then it has been shown that $\Pi_2(w)$ is decreasing and convex in w when $c_0 = w_2^* \le w \le w_0^* \le \min\{w_1^0, w_{10}^0\}$. Since $\Pi_2(w) > 0$ when $c_0 \le w \le w_0^*$, the two zero points of quadratic function $\Pi_2(w)$ are larger than w_0^*. Further since the zero point of $\Pi_s(w)$, $\frac{3M_1+3M_2+4M_0-5\mu\alpha\beta c_0}{5(2-\mu)\alpha\beta c_0}$, is larger than w_0^*, there is only one zero point c_0 for $\Pi_{2s}(w) = \Pi_2(w)\Pi_s(w)$ when $c_0 \le w \le w_0^*$. Next, note that $\frac{\partial \Pi_{2s}(c_0)}{\partial w} = \Pi_2(c_0)\Pi_s'(c_0) > 0$ and $\frac{\partial \Pi_{2s}(w_0^*)}{\partial w} = \Pi_2'(w_0^*)\Pi_s(w_0^*) < 0$. Thus the quadratic function $\Pi_{2s}(w) \ge 0$ is quasiconcave in w when $c_0 \le w \le w_0^*$. Finally we find that the optimal wholesale price $w_2^{\rm NB}$ is given by the first order condition $\Pi_2'(w)\Pi_s(w) + \Pi_s'(w)\Pi_2(w) = 0$. And based on above analysis we find that $c_0 \le w_2^{\rm NB} \le w_0^*$ is the smallest real root.

If $\mu = \frac{32-3\sqrt{95}}{13}$, then $\Pi_2(w) \geq 0$ is linearly decreasing in w when $c_0 = w_2^* \leq w \leq w_0^*$. Since $\Pi_s(w) \geq 0$ is increasing and concave in w, we find that $\Pi_{2s}(w)$ is concave in w. Further due to $\frac{\partial \Pi_{2s}(c_0)}{\partial w} = \Pi_2(c_0)\Pi_s'(c_0) > 0$ and $\frac{\partial \Pi_{2s}(w_0^*)}{\partial w} = \Pi_2'(w_0^*)\Pi_s(w_0^*) < 0$, we find that the optimal wholesale price is given by the first order condition, which is a quadratic function with two solutions. Since the zero point of $\Pi_2(w)$ is larger than w_0^*, the optimal solution $c_0 \leq w_2^{\mathrm{NB}} \leq w_0^*$ is the smaller real root.

If $\frac{32-3\sqrt{95}}{13} < \mu \leq \min\{\frac{9M_2+4M_0-13\alpha\beta c_0}{9M_1+9M_2+14M_0-32\alpha\beta c_0}, \frac{1}{2}\}$, then $\Pi_2(w) \geq 0$ is decreasing and concave in w when $c_0 = w_2^* \leq w \leq w_0^*$. Since $\Pi_s(w) \geq 0$ is increasing and concave in w, we find that $\Pi_{2s}(w)$ is concave in w when $c_0 \leq w \leq w_0^*$. Further due to $\frac{\partial \Pi_{2s}(c_0)}{\partial w} = \Pi_2(c_0)\Pi_s'(c_0) > 0$ and $\frac{\partial \Pi_{2s}(w_0^*)}{\partial w} = \Pi_2'(w_0^*)\Pi_s(w_0^*) < 0$, we find that the optimal wholesale price can be computed by the first order condition, which is a cubic function with three solutions. Since c_0 is between the two zero points of $\Pi_2(w)$, c_0 is the second zero point of $\Pi_{2s}(w)$. Thus $c_0 \leq w_2^{\mathrm{NB}} \leq w_0^*$ is the middle real root of the first order condition.

If $\min\{\frac{9M_2+4M_0-13\alpha\beta c_0}{9M_1+9M_2+14M_0-32\alpha\beta c_0}, \frac{1}{2}\} < \mu < \frac{1}{2}$, then $\Pi_2(w) \geq 0$ is concave in w, and when $w_2^* \leq w \leq w_0^*$, $\Pi_2(w)$ is decreasing and concave in w. Since $\Pi_s(w) \geq 0$ is increasing and concave in w, we find that $\Pi_{2s}(w)$ is concave in w when $w_2^* \leq w \leq w_0^*$. Finally due to $\frac{\partial \Pi_{2s}(w_2^*)}{\partial w} = \Pi_s'(w_2^*)\Pi_2(w_2^*) > 0$ and $\frac{\partial \Pi_{2s}(w_0^*)}{\partial w} = \Pi_2'(w_0^*)\Pi_s(w_0^*) < 0$, the optimal wholesale price is given by the first order condition. Notice that c_0 is still between the two zero points of $r_2(w)$. Hence, c_0 is the second zero point of $\Pi_{2s}(w)$. Further since the larger zero points of $\Pi_2(w)$ and $\Pi_s(w)$ are larger than w_0^*, $w_2^* \leq w_2^{\mathrm{NB}} \leq w_0^*$ is the middle real root of the first order condition. □

Next we study the negotiation between manufacturer 1 and the supplier. The disagreement payoff is the profit with iron ore price w_2^{NB} obtained in Theorem 7.4.1. Thus the optimization problem is

$$\max_{w_1 \in \Omega_1(w_1|w_2^{\mathrm{NB}})} \Pi_{1s}(w_1|w_2^{\mathrm{NB}}) = \max_{w_1 \in \Omega_1(w_1|w_2^{\mathrm{NB}})} [\Pi_1(w_1) - \Pi_1(w_2^{\mathrm{NB}})][\Pi_s(w_1) - \Pi_s(w_2^{\mathrm{NB}})].$$

Lemma 7.4.2. *In the last round of negotiation between the supplier and manufacturer 1, there is unique Nash bargaining solution* $w_1^*(w_2^{\mathrm{NB}}) = w_2^{\mathrm{NB}}$.

Proof. Since $\Pi_1(w)$ is decreasing in w and $\Pi_s(w)$ is increasing in w when $c_0 = w_1^* \leq w \leq w_0^*$, we find that $\Omega_1(w_1|w_2^{\mathrm{NB}}) = \{w_1|w_1 = w_2^{\mathrm{NB}}\}$. Thus the Nash bargaining solution for the negotiation between manufacturer 1 and the supplier is $w_1^*(w_2^{\mathrm{NB}}) = w_2^{\mathrm{NB}}$. □

Next we continue discussing the bargaining backwardly, i.e., the negotiation between manufacturer 2 and the supplier in the second last round. The disagreement payoff is the profit with iron ore price $w_1^*(w_2^{\mathrm{NB}}) = w_2^{\mathrm{NB}}$. Thus the optimization problem is

$$\max_{w_2 \in \Omega_2(w_2|w_2^{\mathrm{NB}})} \Pi_{2s}(w_2|w_2^{\mathrm{NB}}) = \max_{w_2 \in \Omega_2(w_2|w_2^{\mathrm{NB}})} [\Pi_2(w_2) - \Pi_2(w_2^{\mathrm{NB}})][\Pi_s(w_2) - \Pi_s(w_2^{\mathrm{NB}})].$$

Lemma 7.4.3. *In the second last round of negotiation between the supplier and manufacturer 2, there is unique Nash bargaining solution* $w_2^*(w_2^{NB}) = w_2^{NB}$.

Proof. We also discuss the Nash bargaining solution in terms of different μ. First note that $w_2^* \le w_2^{NB} \le w_0^*$.

If $0 \le \mu \le \min\left\{\frac{9M_2+4M_0-13\alpha\beta c_0}{9M_1+9M_2+14M_0-32\alpha\beta c_0}, \frac{1}{2}\right\}$, then $\Pi_2(w)$ is decreasing in w when $c_0 = w_2^* \le w \le w_0^*$. Further since $\Pi_s(w)$ is increasing in w when $w_2^* \le w \le w_0^*$, we find that $\Omega_2(w_2|w_2^{NB}) = \{w_2|w_2 = w_2^{NB}\}$. Thus the Nash bargaining solution is $w_2^*(w_2^{NB}) = w_2^{NB}$.

If $\min\left\{\frac{9M_2+4M_0-13\alpha\beta c_0}{9M_1+9M_2+14M_0-32\alpha\beta c_0}, \frac{1}{2}\right\} < \mu < \frac{1}{2}$, then $\Pi_2(w)$ is concave in w, and decreasing in w when $w_2^* \le w \le w_0^*$. Further since $\Pi_s(w)$ is increasing in w when $w_2^* \le w \le w_0^*$, we find that $\Omega_2(w_2|w_2^{NB}) = \{w_2|w_2 = w_2^{NB}\}$. Thus the Nash bargaining solution is $w_2^*(w_2^{NB}) = w_2^{NB}$. □

From Lemmas 7.4.2 and 7.4.3, the Nash bargaining solution will remain unchanged if we proceed the analysis backwardly. Therefore, for the price negotiation starting from the bargaining between manufacturer 1 and the supplier, we find the final negotiated price as follows.

Theorem 7.4.4. *If the iron ore price negotiation starts from the bargaining between the supplier and manufacturer 1 who has no investment on the iron ore, then the final negotiated price is* w_2^{NB}.

7.4.2 The Manufacturer with Investment First Negotiates

If manufacturer 2 first joins the negotiation with the supplier, then in the last round of negotiation, the last bargaining is between the supplier and manufacturer 1. The optimization problem can be formulated as

$$\begin{aligned}\max_{w_1^* \le w \le w_0^*} \Pi_{1s}(w) &= \max_{w_1^* \le w \le w_0^*} \Pi_1(w)\Pi_s(w)\\ &= \max_{w_1^* \le w \le w_0^*}\left[\frac{(M_1-\alpha\beta w)^2}{4\beta} + \frac{[M_0+\mu\alpha\beta c_0-(1+\mu)\alpha\beta w]^2}{9\beta}\right]\\ &\quad\times\frac{(1-\mu)\alpha(w-c_0)}{6}[3M_1+3M_2+4M_0\\ &\qquad -5\mu\alpha\beta c_0-5(2-\mu)\alpha\beta w].\end{aligned}$$

Theorem 7.4.5. *In the last round of negotiation and the last bargaining between the supplier and manufacturer 1, there is unique Nash bargaining solution* w_1^{NB} *given by the first order condition*

$$\frac{\partial\Pi_{1s}(w_1^{NB})}{\partial w} = \Pi_1'(w_1^{NB})\Pi_s(w_1^{NB}) + \Pi_s'(w_1^{NB})\Pi_1(w_1^{NB}) = 0,$$

where $c_0 \le w_1^{NB} \le w_0^*$ *is the only real root.*

Proof. Taking derivatives of $\Pi_{1s}(w)$ on w yields

$$\frac{\partial \Pi_{1s}(w)}{\partial w} = \Pi_1'(w)\Pi_s(w) + \Pi_s'(w)\Pi_1(w).$$

When $c_0 = w_1^* \leq w \leq w_0^*$, $\Pi_s(w)$ is increasing and concave in w, and $\Pi_1(w)$ is decreasing and convex in w. Since $\Pi_1(w) > 0$ for all w, there are two zero points for $\Pi_{1s}(w)$ and only one real root for $\frac{\partial \Pi_{1s}(w)}{\partial w} = 0$. Since $\frac{\partial \Pi_{1s}(c_0)}{\partial w} = \Pi_1(c_0)\Pi_s'(c_0) > 0$ and $\frac{\partial \Pi_{1s}(w_0^*)}{\partial w} = \Pi_1'(w_0^*)\Pi_s(w_0^*) < 0$, $\Pi_{1s}(w)$ is unimodal when $c_0 \leq w \leq w_0^*$. Thus the optimal iron ore price is the only real root of the first order condition. □

Next we study the negotiation between manufacturer 2 and the supplier. The disagreement payoff is the profit with iron ore price w_1^{NB} obtained in Theorem 7.4.5. Thus the optimization problem is

$$\max_{w_2 \in \Omega_2(w_2|w_1^{\text{NB}})} \Pi_{2s}(w_2|w_1^{\text{NB}}) = \max_{w_2 \in \Omega_2(w_2|w_1^{\text{NB}})} [\Pi_2(w_2) - \Pi_2(w_1^{\text{NB}})][\Pi_0(w_2) - \Pi_0(w_1^{\text{NB}})].$$

Lemma 7.4.6. *In the last round of negotiation between the supplier and manufacturer 2, there is unique Nash bargaining solution* $w_2^*(w_1^{\text{NB}}) = w_1^{\text{NB}}$.

Proof. The proof is also in terms of different μ.

If $0 \leq \mu \leq \min\left\{\frac{9M_2+4M_0-13\alpha\beta c_0}{9M_1+9M_2+14M_0-32\alpha\beta c_0}, \frac{1}{2}\right\}$, then $\Pi_2(w)$ is decreasing in w when $c_0 = w_2^* \leq w \leq w_0^*$. Further, since $\Pi_s(w)$ is increasing in w when $c_0 \leq w \leq w_0^*$, we find that $\Omega_2(w_2|w_1^{\text{NB}}) = \{w_2|w_2 = w_1^{\text{NB}}\}$. Thus the Nash bargaining solution is $w_2^*(w_1^{\text{NB}}) = w_1^{\text{NB}}$.

If $\min\left\{\frac{9M_2+4M_0-13\alpha\beta c_0}{9M_1+9M_2+14M_0-32\alpha\beta c_0}, \frac{1}{2}\right\} < \mu < \frac{1}{2}$, we first show

$$w_1^{\text{NB}} > w_2^* = \frac{9\mu M_1 - 9(1-\mu)M_2 + 2(7\mu - 2)M_0 + \mu(32 - 13\mu)\alpha\beta c_0}{(-13\mu^2 + 64\mu - 13)\alpha\beta}$$

by

$$\frac{\partial \Pi_{1s}(w_2^*)}{\partial w} > 0.$$

When $\min\left\{\frac{9M_2+4M_0-13\alpha\beta c_0}{9M_1+9M_2+14M_0-32\alpha\beta c_0}, \frac{1}{2}\right\} < \mu < \frac{1}{2}$, $\Pi_2(w)$ is concave in w, and hence decreasing in w when $w_2^* \leq w \leq w_0^*$. Further, since $\Pi_0(w)$ is increasing in w when $w_2^* \leq w \leq w_0^*$, we find that $\Omega_2(w_2|w_1^{\text{NB}}) = \{w_2|w_2 = w_1^{\text{NB}}\}$. Thus the Nash bargaining solution is $w_2^*(w_1^{\text{NB}}) = w_1^{\text{NB}}$. □

Next we continue discussing the bargaining backwardly, i.e., the negotiation between manufacturer 1 and the supplier in the second last round. The disagreement payoff is the profit with iron ore price $w_2^*(w_1^{\text{NB}}) = w_1^{\text{NB}}$. Thus the optimization problem is

$$\max_{w_1 \in \Omega_1(w_1|w_1^{\text{NB}})} \Pi_{1s}(w_1|w_1^{\text{NB}}) = \max_{w_1 \in \Omega_1(w_1|w_1^{\text{NB}})} [\Pi_1(w_1) - \Pi_1(w_1^{\text{NB}})][\Pi_s(w_1) - \Pi_s(w_1^{\text{NB}})].$$

Lemma 7.4.7. *In the second last round of negotiation between the supplier and manufacturer 1, there is unique Nash bargaining solution* $w_1^*(w_1^{NB}) = w_1^{NB}$.

Proof. Since $\Pi_1(w)$ is decreasing in w and $\Pi_s(w)$ is increasing in w when $c_0 = w_1^* \leq w \leq w_0^*$, we find that $\Omega_1(w_1|w_1^{NB}) = \{w_1|w_1 = w_1^{NB}\}$. Thus the Nash bargaining solution for the negotiation between manufacturer 1 and the supplier is $w_1^*(w_1^{NB}) = w_1^{NB}$. □

From Lemmas 7.4.6 and 7.4.7, the Nash bargaining solution will remain unchanged if we proceed the analysis backwardly. Therefore, for the price negotiation starting from the bargaining between manufacturer 2 and the supplier, we find the final negotiated price as follows.

Theorem 7.4.8. *If the iron ore price negotiation starts from the bargaining between the supplier and manufacturer 2 who has investment on the iron ore, then the final negotiated price is* w_1^{NB}.

7.5 Numerical Examples

In this section some numerical examples are given to illustrate the theoretical results of this chapter. The computational analysis are conducted from two folds. First, we investigate the impact of investment ratio μ on the optimal wholesale price of the three players and on the final bargaining price. Second, we explore the impact of the unit steel production cost c on optimal wholesale price of the three players and on the final bargaining price.

Based on the theoretical analysis above, the investment ratio of player 2 is assumed as $\mu \in [0, 0.5)$. Let $M_1 = 900$, $M_2 = 1{,}100$ and $M_0 = 1{,}000$ (dollar) be the market size respectively. The other parameters in the example are given as follows: $c_0 = 100$, $\alpha = 2$ and $\beta = 1$.

Firstly we examine the impact of manufacturer 2's investment ratio on the optimal wholesale price of each player. μ gets values in the interval $[0, 0.5]$. Figure 7.1 illustrate the results derived in Sect. 7.3.2. Specifically,

- The optimal iron ore price for player 2, w_2^*, is equal to c_0 when $\mu \leq \frac{9M_2+4M_0-13\alpha\beta c_0}{9M_1+9M_2+14M_0-32\alpha\beta c_0}$.
- The optimal iron ore price for player 2, w_2^*, is larger than c_0 and increasing in μ when $\frac{9M_2+4M_0-13\alpha\beta c_0}{9M_1+9M_2+14M_0-32\alpha\beta c_0} \leq \mu < \frac{1}{2}$.
- The optimal iron ore price for the supplier, w_0^*, is increasing in μ.

Then we study the impact of manufacturer 2's investment ratio on the negotiated iron ore price for each of the bargaining sequence. Figure 7.2 illustrates that $w_1^{NB} < w_2^{NB}$.

Furthermore, the optimal profits under different bargaining sequences are examined. Specifically, we first show the impact of manufacturer 2's investment ratio on

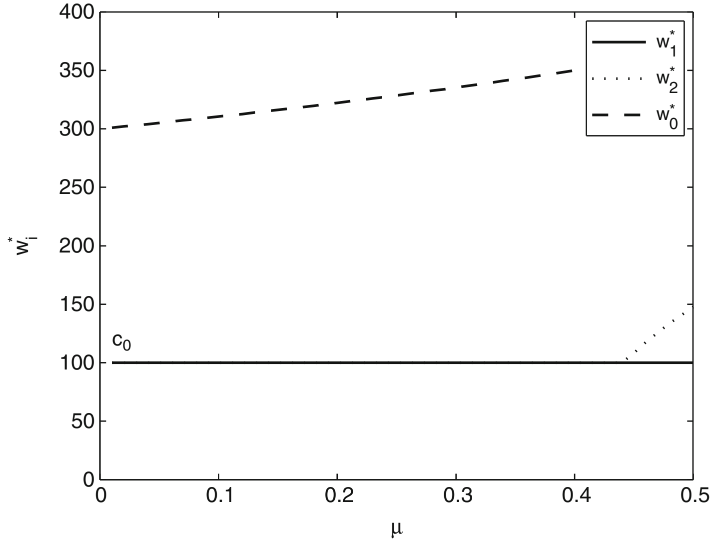

Fig. 7.1 The impact of investment ratio on optimal wholesale prices of three players

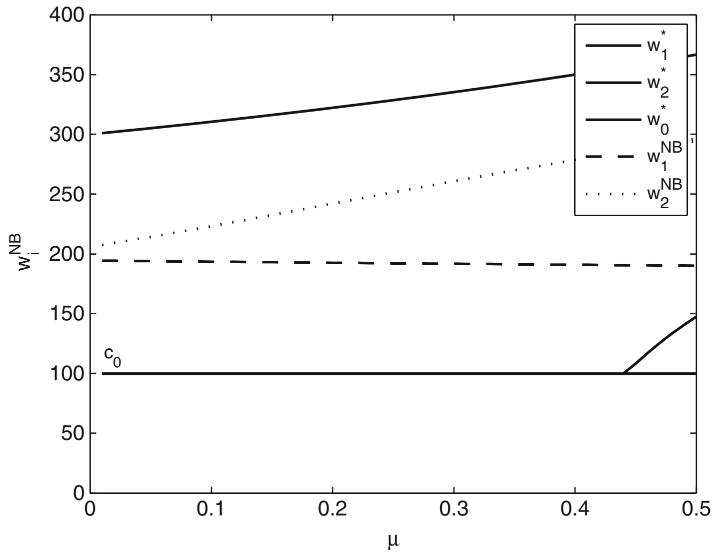

Fig. 7.2 The impact of investment ratio on negotiated iron ore price

the optimal profit with negotiated iron ore price w_1^{NB}. Figure 7.3 illustrates that if the negotiation starts between the supplier and manufacturer 2, then

- The optimal profit of manufacturer 1 is slightly decreasing in μ.
- The optimal profit of the supplier is decreasing in μ.

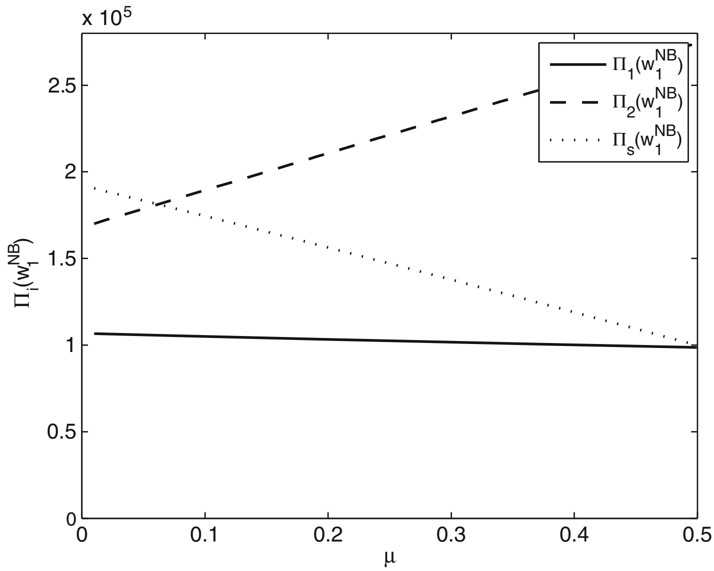

Fig. 7.3 The optimal profit with negotiated iron ore price w_1^{NB}

- The optimal profit of manufacturer 2 is increasing in μ, and will be larger than that of the supplier when μ becomes high enough.

Then we show the impact of manufacturer 2's investment ratio on the optimal profit with negotiated iron ore price w_2^{NB}. Figure 7.4 illustrates that if the negotiation starts between the supplier and manufacturer 1, then:

- The optimal profit of manufacturer 1 is decreasing in μ.
- The optimal profit of the supplier is first slightly increasing and then decreasing in μ.
- The optimal profit of manufacturer 2 is increasing in μ, and will be larger than that of the supplier when μ becomes high enough.

Finally we examine all the bargaining sequences each manufacturer or the supplier prefers. The results are shown in Figs. 7.5–7.7. We find that:

- Manufacturers 1 and 2 prefer the negotiation starting between the supplier and manufacturer 2
- The supplier prefers the negotiation starting between the supplier and manufacturer 1

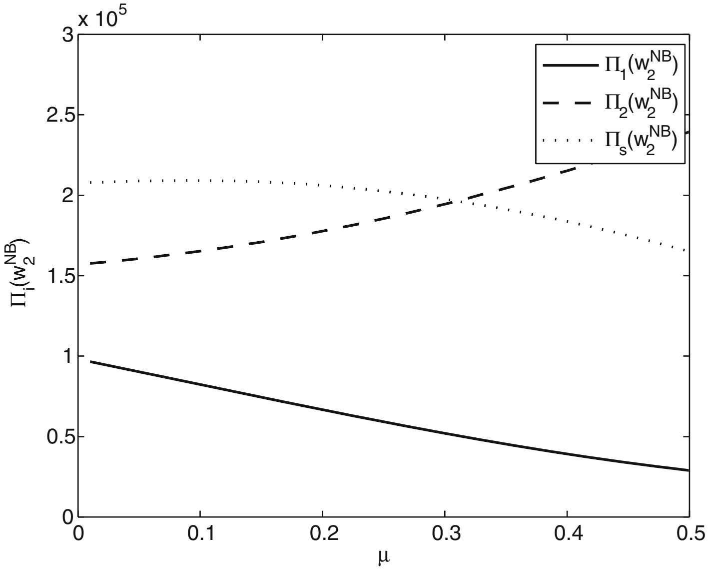

Fig. 7.4 The optimal profit with negotiated iron ore price w_2^{NB}

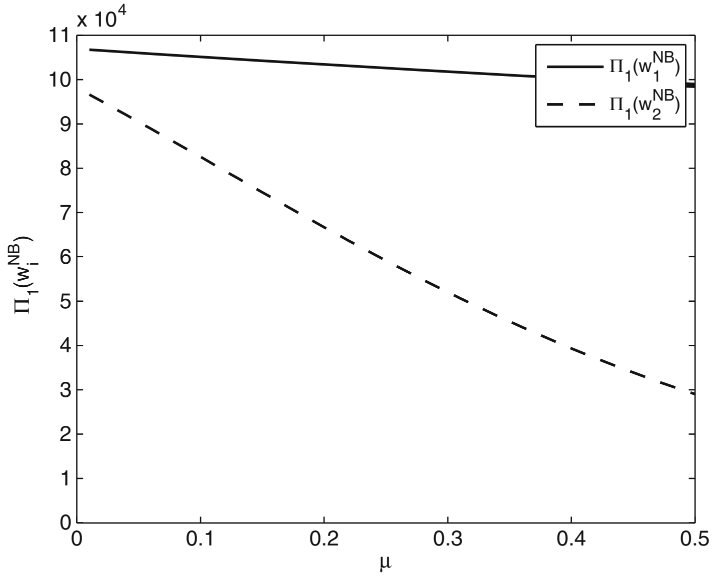

Fig. 7.5 The optimal profit of manufacturer 1 with different negotiated iron ore price

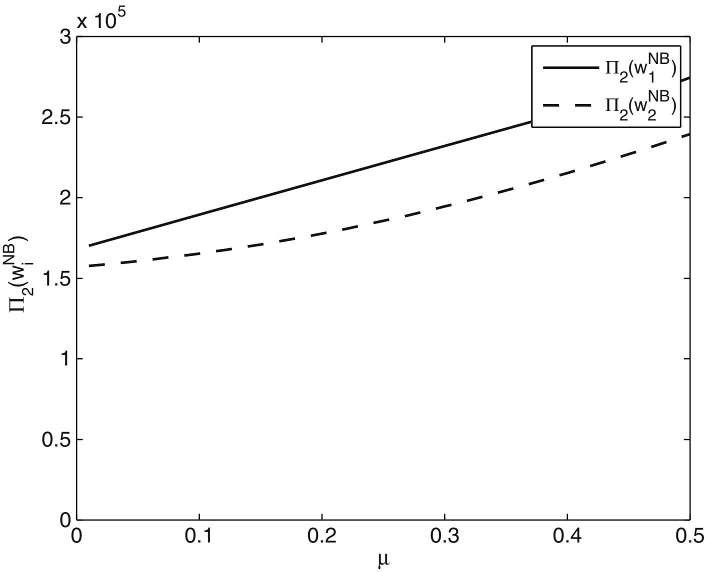

Fig. 7.6 The optimal profit of manufacturer 2 with different negotiated iron ore price

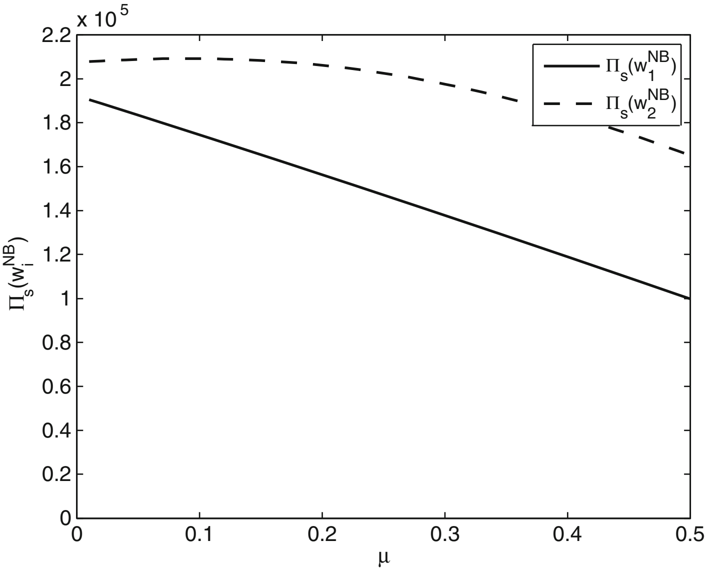

Fig. 7.7 The optimal profit of the supplier with different negotiated iron ore price

7.6 Concluding Remarks

In this chapter we seek to provide some insights for the annual international iron ore price negotiation by studying a one-supplier two-manufacturers supply chain, in which manufacturer 2 has investment on the supplier while manufacturer 1 does not. Firstly we establish a quantity competition model to derive the optimal production quantity of each manufacturer, and further the preferred iron ore price for each player in the supply chain. Next we extend the results of Nash bargaining to two sequences of Nash bargaining. The results show that the investment on iron ore brings more advantages to manufacturer 2 in the quantity competition as well as the iron ore price negotiation. More specifically, relative to manufacturer 1, manufacturer 2 can afford higher iron ore price, and obtain higher profit. Furthermore, when the investment is high enough, the final profit of manufacturer 2 exceeds that of the supplier.

The future research is mainly related to the practice of iron ore price negotiation.

Firstly, in the real steel market, Chinese manufacturers provide steel of low quality while Japanese manufacturers provide high-quality steel. In other words, the two manufacturers compete in different levels of market, and hence obtain different profit margins.

In addition, in the real iron ore price negotiation, there are more than one supplier. A more reasonable model is to consider the two-suppliers and two-manufacturers case. The two suppliers have different production cost and provide different supply in the market. Therefore, new framework should be established for the new relationships among the manufacturers and the suppliers.

Appendix

Proof of Theorem 7.3.5. For manufacturer 1, taking derivative of $\Pi_1(w)$ yields

$$\frac{\partial \Pi_1(w)}{\partial w} = \frac{(4\mu^2 + 8\mu + 13)\alpha^2\beta}{18} w - \frac{\alpha[9M_1 + 4(1+\mu)M_0 + 4\mu(1+\mu)\alpha\beta c_0]}{18}.$$

Clearly the revenue function is convex in w, and the minimizer of the revenue function is $\frac{9M_1+4(1+\mu)(M_0+\mu\alpha\beta c_0)}{(9+4(1+\mu)^2)\alpha\beta}$ where $\frac{9M_1+4(1+\mu)(M_0+\mu\alpha\beta c_0)}{(9+4(1+\mu)^2)\alpha\beta} > \min\{\hat{w}_1, \hat{w}_{10}\}$. Thus $\Pi_1(w)$ is decreasing in w when $c_0 \le w \le \min\{\hat{w}_1, \hat{w}_{10}\}$. Then the optimal iron ore price for manufacturer 1 is $w_1^* = c_0$ and the optimal revenue of manufacture 1 is $\Pi_1(w_1^*) = \frac{(M_1-\alpha\beta c_0)^2}{4\beta} + \frac{(M_0-\alpha\beta c_0)^2}{9\beta}$.

For manufacturer 2, first taking derivatives of $\Pi_2(w)$ yields

$$\begin{aligned}\frac{\partial \Pi_2(w)}{\partial w} &= \frac{(13\mu^2 - 64\mu + 13)\alpha^2\beta}{18} w \\ &\quad + \frac{\alpha}{18}[9\mu M_1 - 9(1-\mu)M_2 + 2(7\mu - 2)M_0 + \mu(32 - 13\mu)\alpha\beta c_0]\end{aligned}$$

and

$$\frac{\partial \Pi_2^2(w)}{\partial w^2} = \frac{(13\mu^2 - 64\mu + 13)\alpha^2\beta}{18}.$$

Thus the revenue function is a quadratic function, or linear function of w when $\mu = \frac{32-3\sqrt{95}}{13}$. The optimal price, w_2^*, depends on the coefficient of the function as follows.

If $0 \leq \mu < \frac{32-3\sqrt{95}}{13}$, then $13\mu^2 - 64\mu + 13 > 0$ and hence $\Pi_2(w)$ is convex. Furthermore, the w that minimizes the revenue function is

$$-\frac{9\mu M_1 - 9(1-\mu)M_2 + 2(7\mu - 2)M_0 + \mu(32 - 13\mu)\alpha\beta c_0}{(13\mu^2 - 64\mu + 13)\alpha\beta}.$$

We seek to prove

$$-\frac{9\mu M_1 - 9(1-\mu)M_2 + 2(7\mu-2)M_0 + \mu(32-13\mu)\alpha\beta c_0}{(13\mu^2-64\mu+13)\alpha\beta} \geq \frac{M_0 + \mu\alpha\beta c_0}{(1+\mu)\alpha\beta} = \hat{w}_{10}. \tag{7.9}$$

It follows from

$$\begin{aligned}
&-[9\mu M_1 - 9(1-\mu)M_2 + 2(7\mu-2)M_0 + \mu(32-13\mu)\alpha\beta c_0](1+\mu) \\
&-(13\mu^2 - 64\mu + 13)(M_0 + \mu\alpha\beta c_0) \\
&= -9\mu(1+\mu)M_1 + 9(1-\mu)(1+\mu)M_2 + (-27\mu^2 + 54\mu - 9)M_0 - 45\mu(1-\mu)\alpha\beta c_0 \\
&\geq 45\mu(1-\mu)M_0 - 45\mu(1-\mu)\alpha\beta c_0 \\
&\geq 0.
\end{aligned}$$

Where the first inequality follows from Lemma 7.3.4. Thus $\Pi_2(w)$ is decreasing when $c_0 \leq w \leq \hat{w}_{10}$, and the maximizer of $\Pi_2(w)$ is $w_2^* = c_0$. Furthermore, the optimal revenue of manufacturer 2 is $\Pi_2(w_2^*) = \frac{(M_2 - \alpha\beta c_0)^2}{4\beta} + \frac{(M_0 - \alpha\beta c_0)^2}{9\beta}$.

If $\mu = \frac{32-3\sqrt{95}}{13}$, since

$$\begin{aligned}
\frac{\partial \Pi_2(w)}{\partial w} &= \frac{\alpha}{18}[9\mu M_1 - 9(1-\mu)M_2 + 2(7\mu-2)M_0 + \mu(32-13\mu)\alpha\beta c_0] \\
&\leq \frac{-(13\mu^2 - 64\mu + 13)(M_0 + \mu\alpha\beta c_0)}{1+\mu} \\
&= 0.
\end{aligned}$$

Where the first inequality follows from (7.9). We find that $\Pi_2(w)$ is decreasing in w and hence the optimal price is $w_2^* = c_0$.

If $\frac{32-3\sqrt{95}}{13} < \mu < \frac{1}{2}$, then $13\mu^2 - 64\mu + 13 < 0$ and hence $\Pi_2(w)$ is concave in w. The maximizer of $\Pi_2(w)$ is $\frac{9\mu M_1 - 9(1-\mu)M_2 + 2(7\mu-2)M_0 + \mu(32-13\mu)\alpha\beta c_0}{(-13\mu^2+64\mu-13)\alpha\beta}$.

If $3M_1 - 3M_2 + 2M_0 - 2\alpha\beta c_0 \leq 0$, then $\frac{9M_2+4M_0-13\alpha\beta c_0}{9M_1+9M_2+14M_0-32\alpha\beta c_0} \geq \frac{1}{2}$ and we find that $\mu < \frac{1}{2} \leq \frac{9M_2+4M_0-13\alpha\beta c_0}{9M_1+9M_2+14M_0-32\alpha\beta c_0}$. Thus it is straightforward to verify that $\frac{9\mu M_1 - 9(1-\mu)M_2 + 2(7\mu-2)M_0 + \mu(32-13\mu)\alpha\beta c_0}{(-13\mu^2+64\mu-13)\alpha\beta} < c_0$. Finally we find that the revenue function is decreasing in w when $w \geq c_0$ and consequently the optimal price is $w_2^* = c_0$.

If $3M_1 - 3M_2 + 2M_0 - 2\alpha\beta c_0 > 0$, then $\frac{9M_2+4M_0-13\alpha\beta c_0}{9M_1+9M_2+14M_0-32\alpha\beta c_0} < \frac{1}{2}$. Further we find that $\frac{9M_2+4M_0-13\alpha\beta c_0}{9M_1+9M_2+14M_0-32\alpha\beta c_0} \geq \frac{13M_0-13\alpha\beta c_0}{32M_0-32\alpha\beta c_0} = \frac{13}{32} > \frac{32-3\sqrt{95}}{13}$ from Lemma 7.3.4. Next when $\frac{32-3\sqrt{95}}{13} < \mu \leq \frac{9M_2+4M_0-13\alpha\beta c_0}{9M_1+9M_2+14M_0-32\alpha\beta c_0}$, we find that

$$\frac{9\mu M_1 - 9(1-\mu)M_2 + 2(7\mu-2)M_0 + \mu(32-13\mu)\alpha\beta c_0}{(-13\mu^2+64\mu-13)\alpha\beta} \leq c_0.$$

Thus the revenue function is decreasing in w when $w \geq c_0$ and consequently the optimal price is $w_2^* = c_0$. When $\frac{9M_2+4M_0-13\alpha\beta c_0}{9M_1+9M_2+14M_0-32\alpha\beta c_0} < \mu < \frac{1}{2}$, in following ways we verify

$$c_0 < \frac{9\mu M_1 - 9(1-\mu)M_2 + 2(7\mu-2)M_0 + \mu(32-13\mu)\alpha\beta c_0}{(-13\mu^2+64\mu-13)\alpha\beta} \leq \min\{w_1^0, w_{10}^0\}. \tag{7.10}$$

The first inequality is straightforward. The proof for the maximizer less than or equal to $\hat{w}_{10}$ follows from inequality (7.9). The rest proof is as follows

$$\begin{aligned}
&9\mu M_1 - 9(1-\mu)M_2 + 2(7\mu-2)M_0 + \mu(32-13\mu)\alpha\beta c_0 - (-13\mu^2+64\mu-13)M_1 \\
&= (13\mu^2 - 55\mu + 13)M_1 - 9(1-\mu)M_2 + 2(7\mu-2)M_0 + \mu(32-13\mu)\alpha\beta c_0 \\
&\leq (13\mu^2 - 55\mu + 13)\frac{2M_0 + \alpha\beta c_0}{3} + (23\mu - 13)M_0 + \mu(32-13\mu)\alpha\beta c_0 \\
&= \frac{26\mu^2 - 41\mu - 13}{3}(M_0 - \alpha\beta c_0) \\
&\leq 0.
\end{aligned}$$

Where the first inequality follows from Lemma 7.3.4 and noting that $13\mu^2 - 55\mu + 13 < 0$ when $\mu > \frac{9M_2+4M_0-13\alpha\beta c_0}{9M_1+9M_2+14M_0-32\alpha\beta c_0} \geq \frac{13}{32}$. Therefore, inequality (7.10) is true when $\frac{9M_2+4M_0-13\alpha\beta c_0}{9M_1+9M_2+14M_0-32\alpha\beta c_0} < \mu < \frac{1}{2}$, and the optimal price for manufacturer 2 is $w_2^* = \frac{9\mu M_1 - 9(1-\mu)M_2 + 2(7\mu-2)M_0 + \mu(32-13\mu)\alpha\beta c_0}{(-13\mu^2+64\mu-13)\alpha\beta}$. □

Chapter 8
Conclusions and Further Research Topics

Today's operating environment calls for a supply chain design that is both secure and resilient. The field of risk management in supply chain is young, growing, and promising.

As we have discussed in previous chapters, a large amount of the literature have been devoted to the study on risk analysis of supply chain models. Hence, we first review the existing literature which is categorized into three kinds of perspectives: modeling of decision maker's risk preference, supply disruption management, and financial risk measurement in supply chains. We further point out that within the three perspectives of risk management in supply chains, the modeling of decision maker's risk preference has been studied a lot and many results have been obtained to guide the practice. However, the analysis on the other two kinds of topics is still in its infancy, and more efforts are needed from academia.

Under this background, this book mainly discusses the problems of risk analysis of supply chain uncertainty and financial risk measurement. For each problem, we provide feasible solutions and insights for managing risk in supply chains. However, the new problem springsup on how such solutions can be evaluated. Works that simply help managers to find a proper trade-off between the cost of solution and the revenue are not enough. Therefore, it will become a hot topic to quantify the benefits of these various solutions.

In this book we address a two-periods inventory control problems faced by a retailer who is served by two unreliable suppliers. The retailer facing stochastic demand needs to determine the sourcing strategy, i.e., which supplier to select and further how much to order. For each period, we identify the conditions under which the retailer will choose single sourcing or dual sourcing, and find that the supplier selection process is the trade-off between the ordering cost and the randomicity of the yield rate. Along this direction, we could further consider the Stackelberg game in which the suppliers determine the wholesale price and then the retailer can choose the sourcing strategy as well as the ordering quantity. In addition, issue of yield information update could be incorporated into the supplier selection model. Specifically, the retailer can collect the yield information after the first period,

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based on how many units the retailer ordered and actually delivered by the supplier. The more the ordered units, then the more information about the yield rate of the supplier is acquired. The optimal sourcing strategy and the ordering quantity need to be computed in the new setting. We investigate not only the sourcing strategy of a retailer but also the pricing strategies of the two suppliers under an environment of supply disruption. We characterize the sourcing strategies of the retailer in a centralized and a decentralized system. Based on the assumption of a uniform demand distribution, we obtain an explicit form of solutions when the suppliers are competitive in nature. Finally we devise a coordination mechanism to maximize the profits of both the suppliers.

The book also studies the dynamic inventory control problem with cash flow constraints and short-term financing. We derive the optimal inventory policy for each period, and characterize the dependence of the firm's optimal operational policy on its financial status. As we have shown, many interesting issues remain to be investigated. To make the model suit practice better, a holding cost rate h and shortage cost rate b could be incorporated, and demands over periods could be assumed not to be identically distributed. Furthermore, the modeling of decision maker's risk preference could be applied to study the risk-averse retailer here. We will find that the optimal inventory control policy is still capital-dependent policy, but more structural results are hard to obtain. We considered the model of inventory financing. In reality, there is always a borrowing limit such that the lender could avoid the risk of borrower bankruptcy.

By studying a one-supplier and two-manufacturers supply chain, the book further dealt with the annual international iron ore price negotiation. The negotiation process is modeled as two sequences of Nash bargaining. The results show that the investment on iron ore brings more advantages to manufacturer 2 in the quantity competition as well as the iron ore price negotiation. It also demonstrates the importance of steel manufacturer considering more investment on iron ore. The future research could be related to the practice of iron ore price negotiation. For example, Chinese and Japanese steel manufacturers provide steel of different quality. Hence, they compete in different market. In addition, there are more than one supplier in the market as well as the negotiation. Therefore, more general model should be established for the new relationships among the manufacturers and the suppliers.

In recent years, the development of B2B online exchanges has brought high liquidity and hence more risks. Since it provides an alternative channel for the traditional supply chain participants, it also could be applied to handle the supply chain risks. Firms have began doing business through these markets. Also in academia, we have seen some research works along this direction. Please see Seifert et al. (2004); Wu et al. (2002) and Peleg and Lee (2002) for reference.

Finally, it can be found that implementing a supply chain–wide risk assessment is a complex and difficult task. It is important for managers to understand risk assessment along the supply chain and developing more practicable approaches to guide the process. Therefore, it will be the future direction to investigate risk implications of different network structures and to develop effective tools for identifying and mitigating network-related risks, to quantify the benefits of the tools and to find a proper trade-off between the revenue and the cost of tools.

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